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A new method for evaluating rock joint roughness based on power spectral density

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Abstract: To further improve accuracy and reliability of describing the roughness feature of the structural plane, and more comprehensively reflect influences of the morphological differences, the fluctuation of the profile was decomposed into three principal characteristics: amplitude, inclination, and curvature, which represent the change characteristics of height, angle, and bending of the profile. The data of the three characteristics were obtained by 10 standard profiles established by Barton. And the distribution curves of the three characteristics were analyzed. Then, the power spectral density (PSD), which can well describe the microscopic morphology, was used to analyze the spectral characteristics of amplitude, inclination and curvature. The fractal parameters D_j (corresponding to the slope of the PSD curve) and roughness parameters A_j (corresponding to the intercept of the PSD curve) were got from the logarithmic PSD curves. The relationship among D_j, A_j and the joint roughness coefficient (JRC) was studied, and the expressive forms of JRC with the new index were established. Meanwhile, the new index was extended to reveal the characterization of three-dimensional morphology, which was also used to calculate surface roughness of a marble surface obtained by the shear test. The results show that the new index has an extremely slight evaluation error, and can reflect the anisotropic property and effectively character morphology features of the structural plane, which provides a reference for characterizing the roughness of the structural plane and it is of great significance to study shear mechanical properties of the structural plane.

Keywords: structural plane; roughness characterization; power spectral density; curvature

1 Introduction

The stability of rock mass cut by discontinuities is always an important problem in the field of rock mass engineering. Since 1973, Barton [1] has obtained the empirical formula of peak shear strength of rock mass discontinuity through Barton [1] obtained an empirical formula for the peak shear strength of the structural face of a rock mass through extensive experimental studies and proposed an important parameter to quantify the roughness of joint surfaces—joint roughness coefficient (JRC). Many scholars have carried out extensive research on the influence of roughness on the shear mechanical behavior and characteristics of structural planes[2–4], but how to accurately and reasonably characterize the roughness of structural planes is the basis for carrying out correlational research.

Chen et al.[5–6] believed that the quantitative parameters of the profile roughness mainly include statistical parameters, fractal dimensions, comprehensive parameters, and straight edge diagrams. Magsipoc et al.[7] also summarized similar results and discussed the characteristics of various roughness quantification methods from statistical characteristics, fractal characteristics, anisotropy, and other aspects, where there are many research achievements in roughness characterization methods dominated by the amplitude characteristics of the profile or structural planes. For example, Tse and Cruden[8] analyzed the functional relationship between parameters of the profile and JRC, including the mean square (MS), root mean square (RMS), and center line average (CLA); Whitehouse[9] utilized skewness and kurtosis to describe the relationship between the amplitude characteristics of the profile and roughness, where the above parameters are all developed from statistics, and the calculation method is simple. However, many scholars have also noticed that the inclination of asperity has an important influence on shear strength, which should be considered. Therefore, more studies have used the spatial length characteristic or the inclination (angle) characteristic including length and amplitude to characterize the roughness of the profile. For example, Tang et al.[10] proposed the profile length ratio (L/L_0), that is, the ratio of the profile curve length to the baseline length; Du et al.[11] improved the mathematical expression of the modified straight edge method, which characterized the roughness of the structural plane by using the inverse trigonometric function of the surface relative amplitude R_A and the strength coefficient K. Yu and Vayssade [12] studied the

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relationship between standard deviation (SD) of fluctuation under the different sampling intervals and JRC. Belem et al.[13] studied the roughness characteristics of the profile by using the combined parameters of average slope (θ_{p+} and θ_{p-}) and average inclination (θ_s). Grasselli et al.[14] also used the ratio of the maximum inclination on the shear direction to the roughness parameter of the joint surface ($\theta^*_{\max} / C$) to directly characterize the roughness of the profile, whose research ideas and research results on the inclination characteristics are widely cited and used for reference. The above parameters can well describe the roughness characteristics, and facilitate the research progress of shear strength of rough discontinuities, and also prove the importance of inclination characteristics of surface profile. Myers[15] believed that the angle and curvature of the profile are important factors affecting the characterization of roughness. In 1962, he proposed the first derivative of the root mean square Z_2 and the second derivative of the root mean square Z_3 , and calculated the root mean square values of the average slope and average curvature of the profile. The $\mathrm{d}^2 y / \mathrm{d}x^2$ in the Z_3 expression formula is the approximation of curvature under the $\mathrm{d}y / \mathrm{d}x$ approaching 0 [9], which can characterize the curve bending characteristics to a certain extent. Whitehouse[9] also put out the roughness indicators to directly calculate the mean of curvature κ of the profile.

To fully depict the roughness characteristics of the structural plane from different aspects, Ban et al. [16] calculated the fractal dimension parameters and roughness parameters of the profile based on the amplitude characteristic data, and proposed a comprehensive index to characterize the roughness of the profile. Brown and Scholz[17] studied the relationship between spectral parameters and roughness based on the power spectral density of amplitude data. Sun et al.[18] comprehensively analyzed the research results of Belem[13] and Grasselli[14] and proposed a combined index form consisting of fluctuation distribution parameters, inclination fractal parameters, and inclination roughness parameters based on the inclination characteristic data of the profile. Chen and Zeng[19] also summarized similar roughness indicators. Previous studies have shown that amplitude, inclination, and curvature are the most important three elements characterizing the roughness of the structural plane, and existing studies lack indicators to characterize these three elements simultaneously.

To this end, based on the power spectral density method, this paper analyzes the spectral characteristics of the amplitude, inclination, and curvature of the typical profile, obtains the evolution law of the power spectral density curve, proposes a new indicator characterizing the roughness of structural planes, where the new indicator incorporates the characteristics of amplitude, inclination, and curvature, and finally verifies the reliability of the new indicator to describe the roughness of structural planes.

2 A new index for describing the roughness of structural planes

2.1 Deficiency of existing comprehensive evaluation indicators

In view of the problem that the evaluation indicators of a single characteristics (such as MS, Z_2 , Z_3) cannot accurately reflect the complex morphology of the actual profile, some scholars try to obtain multifactor comprehensive indicators to describe the roughness of the structural plane after comprehensive analysis of the parameters representing different characteristics.

Table 1 lists some typical existing comprehensive characterization indicators of roughness. In general, the combination methods of comprehensive characterization indicators can be divided into two categories: one is to obtain new comprehensive indicators after combining with different indicators acquired based on a certain characteristic data (amplitude, inclination or curvature) according to a certain rule; the other is to obtain new comprehensive indicators after combining with different indicators acquired based on different characteristic data. Some of these comprehensive indicators attach importance to the characteristics of amplitude or inclination, while others attach importance to the characteristics of curvature, most of which are the former. The amplitude, inclination and curvature are important characteristics of structural plane morphology, which have an important impact on its shear mechanical properties. The absence of any of them will affect the accuracy of the roughness evaluation.

2.2 Amplitude, inclination, and curvature

To intuitively understand the characteristics of amplitude, inclination, and curvature, the standard profile proposed by Barton et al.[2] was resampled with a sampling interval of 0.5 mm. The data curves of the amplitude, inclination, and curvature of the profile were obtained from the 10 profiles calculated according to the formula in Table 2.

In this subsection, the case of the standard profile at JRC = 18.7 is analyzed, and the variation curves of the three characteristic factors are plotted in Fig. 1. The amplitude Z_i is the height distribution of the profile, and a large Z_i value represents a higher the peak value at the x_i . The inclination θ_i is the distribution of the inclination angle of the profile, implying the larger the value is, the higher the degree of inclination (slope) at the x_i . The curvature κ_i is the curvature distribution of the profile, representing the deviation degree from the linearity of the curve, and is numerically equal to the reciprocal of the arc radius of the profile at the x_i . It can be seen from Fig. 1 that the maximum (minimum) of the inclination mainly occurs at the stages when the profile increases (decreases) significantly, and its zero value corresponds to the turning point of the profile amplitude. The maximum value of curvature, that is, the minimum value of curvature radius, mainly appears at the turning point of the profile, while the minimum value of curvature basically corresponds to the extremum of the inclination.

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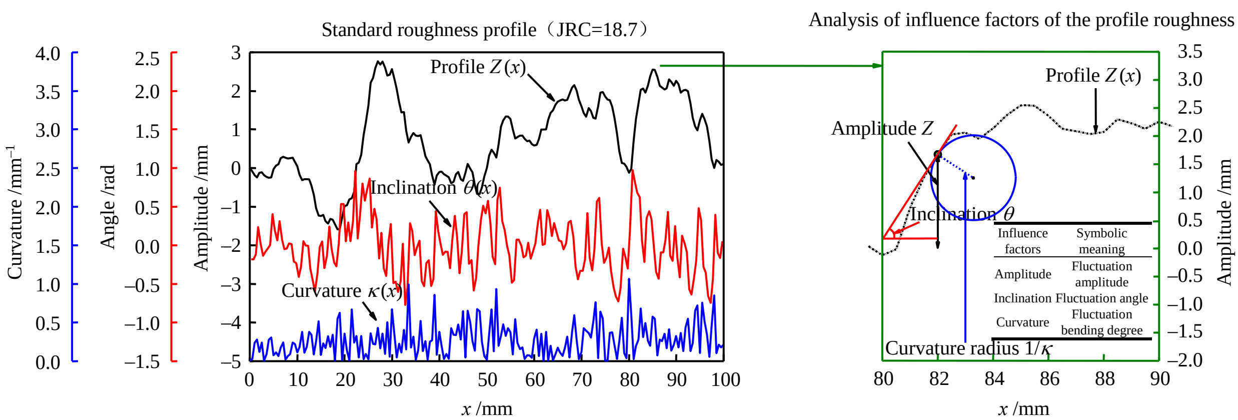
Table 1 Roughness characterization index

Scholars and sources	Evaluation indicators	Profile characteristic included in analysis		
		Parameters calculated with amplitude data	Parameters calculated with inclination data	Parameters calculated with curvature data
Myers[15]	$Z_2=\sqrt{\frac{1}{L}\int_L\left(\frac{dz}{dx}\right)^2 dx}$	—	the first derivative (slope) $\tan(\theta)=dz/dx$	—
Myers[15]	$Z_3=\sqrt{\frac{1}{L}\int_L\left(\frac{d^2z}{dx^2}\right)^2 dx}$	—	—	the second derivative d^2z/dx^2 , indicating angle change rate and curve asperity
Spragg et al.[20]	$R_{\lambda q}=2\pi RMS/Z_2$	RMS of amplitude RMS	root mean square of slope $\tan(\theta)$ Z_2	—
Brown et al.[17]	$S(f)=\alpha f^{\beta}$	amplitude roughness α; spectral index associated with fractal characteristics and Hurst index β; frequency f	—	—
Whitehouse[9]	$R'_{\Delta a}=\frac{1}{L}\int_L\kappa(x)dx\approx\frac{1}{L}\int_L\left\lvert\frac{d^2z}{dx^2}\right\rvert dx$	—	—	curvature κ, implying the bending degree of curve
Chen et al.[6]	$W_d^{0.97}(D-1)^{1.46}$	profile fluctuation W_d ; fractal dimension D	—	—
Sun et al.[18]	$SRI=D_{\theta^+}^{-0.47}(\alpha A_{\theta^+})$	—	fractal dimension $D_{\theta+}$; fractal roughness parameter $A_{\theta+}$; distribution parameter of fluctuation angle α	—
Zhang et al.[21]	$\lambda=(h/L)^{\alpha}(Z_2')^{1-\alpha}$	height (amplitude) parameter h; profile length L	root mean square value of the slope corrected z_2'	—
Ban et al.[16]	$c_n=c_0D$	fractal dimension D; fractal roughness parameter c_0	—	—
Yang et al.[22]	$M_{hl}=a(\bar{h}/L)+bR_{vh}$	average fluctuation $\bar{h}$	anisotropy parameters $R_{vh}\propto(dz/dx)$	—

Table 2 Calculation formulae of amplitude, inclination, and curvature

Type	Calculation formula
Amplitude	$Z_i=Z(x_i)$
Inclination	$\theta_i=\arctan(dy_i/dx_i)$
Curvature	$\kappa_i=\left\lvert d^2y_i/dx_i^2\right\rvert\Big/\left[1+(dy_i/dx_i)^2\right]^{3/2}$

The independence of curve amplitude, inclination, and curvature is the basis for further calculation and analysis, and a good non-correlation can ensure the complementary ability of the three factors to characterize roughness. Therefore, the statistical chi-square independence test was used to evaluate the correlation of the three factors [23], and the calculation results are shown in Table 3. It can be seen from Table 3 that the chi-square distribution probability of the three factors obviously exceeds the confidence level (0.05), indicating that there is no significant correlation among the three factors and the independence is good. The roughness can be analyzed based on the curve amplitude, inclination, and curvature.



(a) Variation characteristics of amplitude, inclination, and curvature of the profile

(b) Relationship among the amplitude, inclination, and curvature and morphology

Fig. 1 Characteristics curves of amplitude, inclination, and curvature of the profile

Table 3 Independence test results

Factors	Chi square distribution probability	Confidence level	Conclusion
Curve and inclination	0.243 3	0.05	No significant correlation
Curve and Curvature	0.240 5	0.05	No significant correlation
Inclination and curvature	0.239 9	0.05	No significant correlation

2.3 New index based on the power spectral density

The power spectral density (PSD) method is widely used in the evaluation and analysis of micro and meso-surface, such as ultra-precision device processing[24–25], optical elements[26], laser milling[27], and ultrasonic vibration grinding[28]. This method can comprehensively analyze the structural and spectral characteristics of surface fluctuation, accurately reflect the roughness, and is not affected by the selected specific scanning size and pixel resolution. Some scholars[29–31] have used this method to evaluate the rough characteristics of the profile and obtained good research results.

Based on the research results of Brown et al. [17], the power spectral density S is a power function of frequency f (or wavelength λ):

$$S(f)=\alpha f^{-\beta} \tag{1}$$

Taking the logarithm of Eq.(1), we get

$$\lg(S)=\lg(\alpha)-\beta\lg(f) \tag{2}$$

where there is a certain conversion relationship between the parameter β and Hurst exponent H , fractal dimension D [30, 32], as shown below:

$$\beta=1+2H \tag{3}$$

$$\beta=2D-5 \tag{4}$$

Xie [33], Brown and Scholz[17], Kulatilake et al.[34] and others believed that the rock joint morphology presents the better self-affine fractal characteristics, and its fractal dimension D has a good correlation with JRC, which can characterize the roughness of joint profiles. According to Eqs. (3) and (4), the linear fitting parameters β of the logarithmic curve of the power spectral density also have obvious fractal characteristics.

The n^{th} derivative of the spectral density function S_n of the profile $z(x)$ can be expressed as[35]

$$S_n(f)=f^{2n}S_{z(x)}(f) \tag{5}$$

where $S_{z(x)}$ is the power spectral densities of the profile $z(x)$.

The power spectral density of the profile $z(x)$, its first derivative $\mathrm{d}z/\mathrm{d}x$ and second derivative $\mathrm{d}^2z/\mathrm{d}x^2$ can be obtained from Eq. (1):

$$\left.\begin{array}{l} S_0(f)=S(f)=\alpha f^{-\beta}, \\ \text{The spectral density of the profile} \\ S_1(f)=f^2S_1(f)=\alpha f^{2-\beta}, \\ \text{The spectral density of the first derivative} \\ S_2(f)=f^4S_1(f)=\alpha f^{4-\beta}, \\ \text{The spectral density of the second derivative} \end{array}\right\} \tag{6}$$

Taking the logarithm of Eq.(6), we get

$$\left.\begin{array}{l} \lg(S_0)=\lg(\alpha)-\beta\lg(f) \\ \lg(S_1)=\lg(\alpha)+(2-\beta)\lg(f) \\ \lg(S_2)=\lg(\alpha)+(4-\beta)\lg(f) \end{array}\right\} \tag{7}$$

One can see from Eq. (6) that the spectral density of the profile and its first derivative and second derivative present similar characteristics in the form of power exponential function, and the logarithm of the profile also has an obvious linear trend, which indicates that they have fractal characteristics. At the same time, the first derivative $\mathrm{d}z/\mathrm{d}x$ is the tangent value of the inclination, and the curvature is obtained through the second derivative and the first derivative (as seen in Table 2). There is an approximate power exponential function relationship between the spectral density of the profile amplitude Z, inclination θ, curvature κ and the frequency f:

$$S_j(f)=\alpha_j\times f^{-\beta_j} \tag{8}$$

Taking the logarithm of Eq.(8) yields:

$$\lg(S_j)=\lg(\alpha_j)-\beta_j\lg(f) \tag{9}$$

Referring to the expression forms of roughness indicators from Ban et al. [16] and Sun et al. [18], a new roughness evaluation indicator HAC (height, angle and curvature) was proposed as follows:

$$\mathrm{HAC}=\mathrm{HAC}\left(A_j,D_j\right) \tag{10}$$

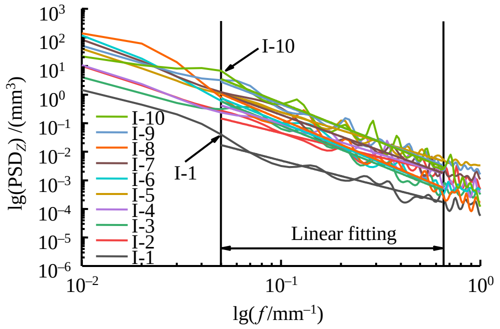
$$\left.\begin{array}{l} A_j=\left|\lg(\alpha_j)\right| \\ D_j=(5+\beta_j)/2 \end{array}\right\} \text{ where, } j=(Z,\theta,\kappa) \tag{11}$$

where A_j and D_j are fractal parameter.

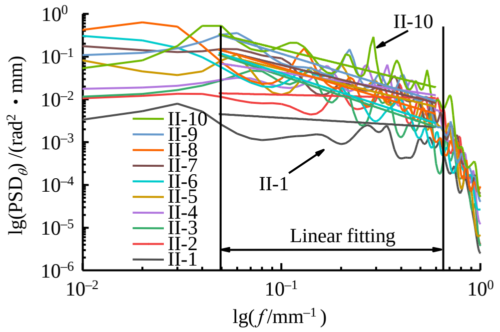
The multi window method proposed by Thomson[36] in 1982 was adopted as the power spectral density calculation method, where the fractal parameter data of amplitude Z_i, inclination θ_i and curvature κ_i were calculated by the integration function。

3 Calculation results of standard curve parameters

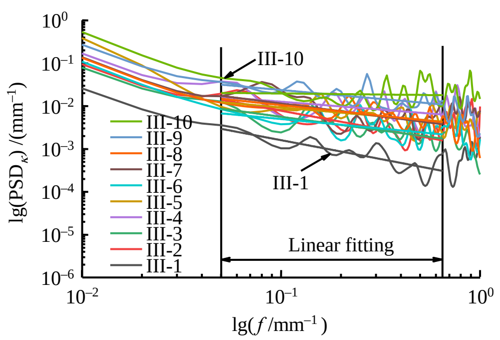
The power spectral density distributions of amplitude (I-1 to 10), inclination (II-1 to 10), and curvature (III-1 to 10) data of 10 standard profiles are shown in Fig. 2. The power spectral density of the profile amplitude in Fig. 2 (a) is a common analysis method for analyzing surface roughness[24–31]. The middle frequency data after taking the logarithm has obvious linear characteristics, and is the main area for linear fitting in Eq.(2), while the high- frequency nonlinear section at the right is unreliable noisy data[31] and the regularity of the amplitude, inclination and curvature power spectral density curves of low-frequency section at the left is also contrary to the medium frequency linearity. Therefore, to ensure the validity and consistency of the fitting data, the linear fitting area of the inclination and curvature power spectral density curves is consistent with the amplitude. In Fig. 2 (b), the power spectral density of the inclination at the high-frequency section change dramatically, and there is a rapid decline phase, indicating that the inclination has a great change at the high frequency (short wavelength), which also conforms to the feature that the inclination changes greatly in a short distance at the turning point of the profile asperity. In Fig. 2 (c), the power spectral density curves of curvature are relatively stable at the low-frequency section, and the difference between the high and medium-frequency sections is not remarkable, and its fluctuation amplitude gradually increases with the increase of frequency. The PSD curves of the amplitude, inclination, and curvature of the profile at JCR=18.7 in Fig. 1 are shown in Fig. 3, and it can be clearly observed that the slopes of the three curves gradually tend to be gentle (Z→θ→κ).



(a) Amplitude characteristics (Class I)



(b) Inclination characteristics (Class II)



(c) Curvature characteristics (Class III)

Fig. 2 Logarithmic curves of PSD of three characteristic curves

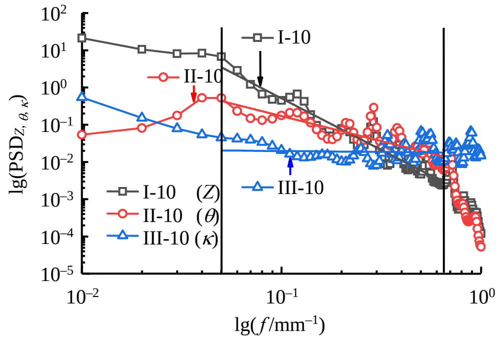


Fig. 3 Logarithmic curves of three PSD of the profile (JRC=18.7) of the 10th standard curve

By the statistics of the power spectral density data of amplitude, inclination, and curvature when the inclination of the positive and opposite profiles is more than 0, the slope and intercept obtained by the statistics were converted into the fractal parameter A_j and D_j according to Eq. (11), and the calculation results are shown in Table 4 and Table 5 (blue is the maximum, brown is the minimum). The law of the opposite fractal parameter A_j is the most obvious: the maximum is the profile at the JRC=0.4, and the minimum is the profile at the JRC=18.7; The mean value of minimum of the opposite fractal parameter D_j is the profile at the JRC=14.8.

Table 4 Calculation results of positive (from left to right) parameters about the standard JRC curve

No.	JRC	Amplitude $Z(x)$		Inclination $\theta(x)$		Curvature $\kappa(x)$	
		D_Z	A_Z	D_θ	A_θ	D_κ	A_κ
1	0.4	**1.660**	**3.983**	**2.644**	2.863	2.216	**3.486**
2	2.8	1.474	3.802	2.263	2.735	2.138	2.739
3	5.8	1.440	3.138	1.868	**3.079**	2.361	2.320
4	6.7	1.590	2.843	2.221	2.462	2.324	2.056
5	9.5	1.487	2.672	1.979	2.666	2.402	2.152
6	10.8	**1.108**	3.301	1.841	3.067	**2.484**	2.344
7	12.8	1.286	2.365	1.864	2.652	**2.127**	2.509
8	14.5	1.433	**2.161**	**1.697**	2.928	2.465	1.929
9	16.7	1.376	2.459	2.338	**1.843**	2.279	2.007
10	18.7	1.391	2.478	1.917	2.331	2.457	**1.486**

Table 5 Calculation results of opposite (from right to left) parameters about the standard JRC curve

No.	JRC	Amplitude $Z(x)$		Inclination $\theta(x)$		Curvature $\kappa(x)$	
		D_Z	A_Z	D_θ	A_θ	D_κ	A_κ
1	0.4	1.179	**4.658**	**2.456**	**3.217**	2.456	**3.143**
2	2.8	1.293	4.046	2.166	2.991	2.126	2.738
3	5.8	1.409	3.198	1.675	3.757	2.285	2.682
4	6.7	1.471	3.153	2.072	2.453	**2.603**	1.531
5	9.5	1.442	2.788	2.044	2.694	2.579	1.791
6	10.8	**1.478**	2.094	1.759	3.180	2.231	2.651
7	12.8	1.245	2.968	2.062	2.515	2.232	2.448
8	14.5	**1.011**	3.086	**1.629**	3.129	**2.231**	2.406
9	16.7	1.374	2.346	1.924	2.421	2.278	1.826
10	18.7	1.542	**1.868**	2.199	**1.848**	2.559	**1.338**

4 Determination of roughness evaluation system

4.1 New indicator expression form

The characteristics of the power spectral density parameters (fractal parameters D_j and roughness parameters A_j) of the three characteristic curves were discussed above, and the roughness characteristics of the structural plane can be described comprehensively using the above parameters. To verify the rationality of the roughness evaluation index proposed, a comprehensive analysis of the relationship between fractal parameters D_j, roughness parameters A_j and roughness is needed to determine its specific expression. In this subsection, positive parameter data is analyzed as an example. JRC and HAC of 10 standard roughness curves were fitted with power exponential function by using the Levenberg-Marquardt method. The forms of HAC mainly include: ① $D_Z D_\theta D_\kappa$ and $A_Z A_\theta A_\kappa$, ② $D_Z D_\theta$ and $A_Z A_\theta$, and ③ D_Z and A_Z, which represent gradual superposition of three characteristic factors, i.e., amplitude Z, inclination θ, and curvature κ.

The data fitting results are shown in Fig. 4. With the increase of characteristic factors superimposed, the fitting coefficients of HAC and JRC increase gradually: 0.965→0.968→0.980. The main reason is that the inclination and curvature reflect the differential relationship between the profile and space, which represents the angular and bending characteristics of the profile; amplitude reflects the distribution relationship between the profile and space, representing the height characteristics of the profile. The above HAC form containing three characteristic factors (blue data points in Figure 4) has the highest fitting correlation with JRC, indicating that the roughness of the structural plane can be effectively characterized by the comprehensive index HAC considering the amplitude, inclination, and curvature. Obviously, it is not comprehensive to use a single feature of the profile to describe the roughness, and a combination of multiple factors can reflect the profile roughness more accurately.

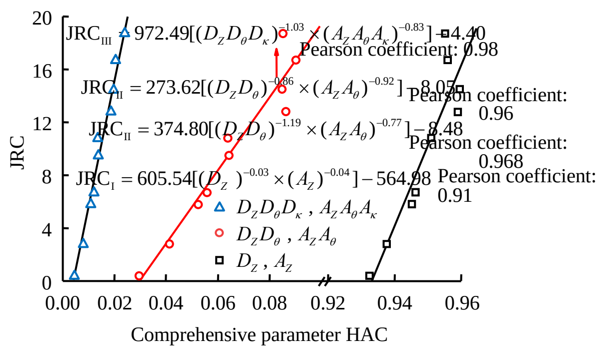


Fig. 4 Relationship between comprehensive parameters (positive) HAC and JRC

Therefore, the effective form of HAC can be determined as

$$\mathrm{HAC}=(D_Z D_\theta D_\kappa)^{-1.03}\times(A_Z A_\theta A_\kappa)^{-0.83} \tag{12}$$

The fitting result between the new HAC and JRC is shown in Fig. 5, and its relationship can be expressed as

$$\left.\begin{aligned}\mathrm{JRC}&=972.41\times\mathrm{HAC}-4.40, (\text{Positive})\\ \mathrm{JRC}&=784.01\times\mathrm{HAC}-2.27, (\text{Opposite})\end{aligned}\right\} \tag{13}$$

Further, the form of the HAC can be simplified as

$$\mathrm{HAC}=(D_Z D_\theta D_\kappa)^{-1}\times(A_Z A_\theta A_\kappa)^{-0.8} \tag{14}$$

As shown in Fig. 5, the Pearson coefficient of positive linear fitting is 0.98, while that of the opposite linear fitting is 0.94, which indicates that the new index can better reflect the directivity of roughness.

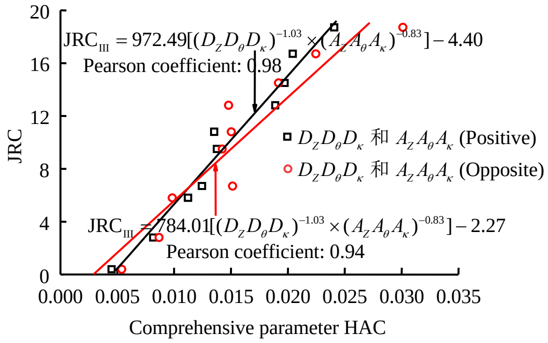


Fig. 5 Fitting results of positive and opposite HAC with JRC

4.2 JRC evaluation error and comparison

To evaluate the practicability and accuracy of this method, the calculation results were compared with the previous research results for error analysis. The probability distribution of JRC estimation error is shown in Fig. 6. It can be seen that the probability density curves of calculation results derived from the three expressions of HAC are more concentrated, and the mean and standard deviation are far less than those of Ünlüsoy and Süzen [29], Tse and Cruden [8], Tatone and Grasselli [37] and Jang et al. [38]. The specific data is listed in Table 6 (upper right). The calculation results of the mean and standard deviation obtained from the first form of HAC, namely Eqs. (12) and (14), are also the least, which implies the error of estimation results of the indicator form is the least and has better precision.

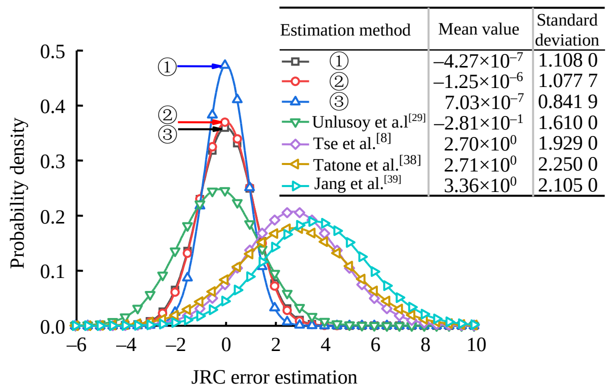


Fig. 6 Error analysis of JRC

5 Anisotropy characterization of structural plane roughness

Two methods for describing 3D surface PSD are proposed by Jacobs et al.[31] for studying the quantitative characterization of surface structure characteristics by using the power spectral density, where the first is the average method based on rectangular coordinates: the PSDs of multiple profiles are calculated at first, and then the PSD of the whole surface is obtained by using the average method; the second is the pseudo 2D PSD method based on the polar coordinate system. Considering that the proposed HAC is based on the two-dimensional profile, the average method is used as the calculation method for the new HAC of the real structural plane roughness.

As shown in Fig. 7, the marble shear plane is examined as an example. The plane is obtained by using 3D laser scanning after the shear failure of the complete marble, whose size is 150 mm×150 mm. The profiles were extracted at an interval of 0.1 mm along the *Y* direction of the surface, and the fractal parameters D_j and A_j of each profile was calculated by using Eqs. (10) and (11), the roughness indexes of the structural plane in this direction were calculated after averaging, and then the structural plane was rotated counterclockwise to obtain the roughness indexes in different directions, as shown in Fig. 8.

In Fig. 8, the positive shear direction represents the direction in which the upper and lower walls are extruded and staggered each other after shear fracture (Fig. 7 is rotated about 90° counterclockwise), and the negative shear direction indicates the opposite direction in which the upper and lower walls are extruded and staggered each other after shear fracture (Fig. 7 is rotated about 270° counterclockwise). It can be seen from Figs. 8 (a) and 8 (b) that the fractal parameter *D* in all directions of the shear plane are relatively uniform, fluctuating mainly in the range of 6 to 10, and the minimum value appears in the shear direction. The polarization of roughness parameter is more prominent. It can be observed that the shear direction (positive and negative) parameter values are significantly greater than the other directions, indicating that the anisotropy of parameter *A* is more prominent; the extremum distribution results of *A* and *D* are opposite, and the maximum of parameter *D* often corresponds to the minimum of parameter *A*. Fig. 8 (c) illustrates the JRC value estimated according to the HAC，and it shows the direction regions with the lower roughness are located in the ranges of 60° to 90° and 240° to 270°; at angles close to 90° and 270°, i.e., in the shear direction domain, JRC minima of 2.02 and 4.00 occur in both positive and negative shear directions, respectively. It is worth noting that the JRC value in the positive shear direction is lower than that in the negative shear direction, namely the roughness in the positive (staggered and extruded direction) direction is lower, which is consistent with the actual situation of the high grinding degree in the positive shear direction after the sample was extruded and rubbed. It is shown from the above analysis that this method can more effectively distinguish anisotropy and characterize the roughness characteristics of 3D structural planes.

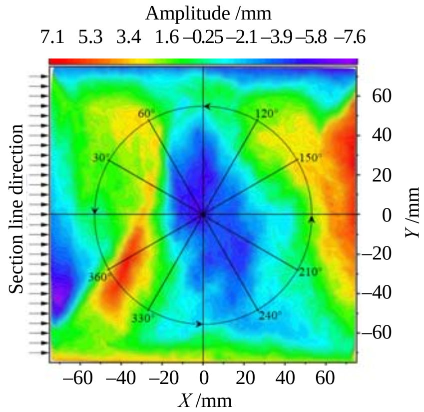
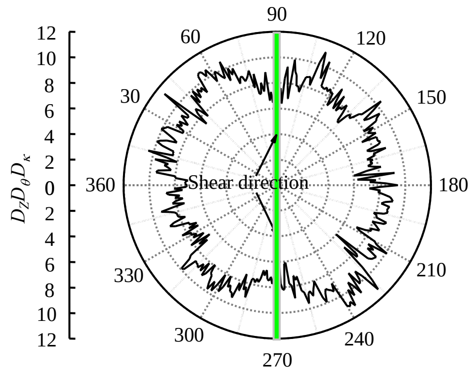
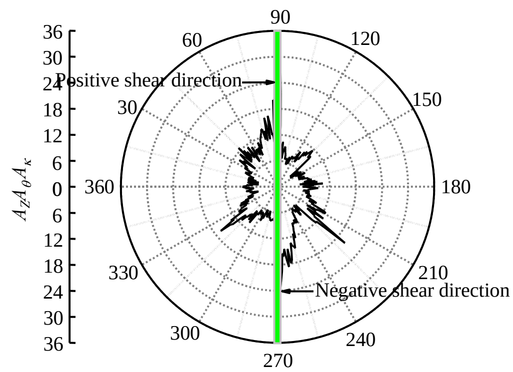


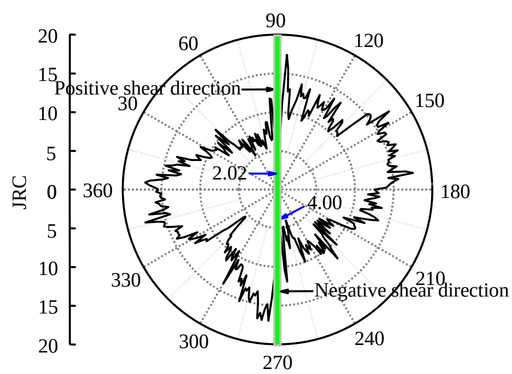
Fig. 7 Three-dimensional scanning diagram of marble shear plane



(a) Calculation results of fractal parameter *D*



(b) Calculation results of fractal parameter *A*



(c) JRC calculation results

Fig. 8 JRC of the marble structure shear plane evaluated by using the HAC index

6 Conclusion

Based on the characteristic curves of the amplitude, inclination, and curvature of the profile, the power spectral density characteristics of the three characteristic curves are described, and the fractal parameters and roughness parameters of each characteristic curve are extracted. A new index characterizing roughness with higher accuracy and clearer theoretical meaning is obtained through the comprehensive analysis.

(1) Curvature characteristic curve is an important parameter that can describe the roughness of the profile, which is different from the amplitude characteristic and the inclination characteristic. It represents the evolution law of the bending degree at the curve asperity. The three parameters are closely related and have their characteristics: the extremum of amplitude corresponds to the extremum of curvature and the zero value of inclination; and the extremum of inclination corresponds to the minimum of curvature.

(2) Based on the power spectral density function, the spectral characteristics of the amplitude, inclination, and curvature characteristic curves of 10 standard profiles are analyzed. The slope degrees of the logarithmic curves of the power spectral densities for the three factors gradually flattens out, and the low-frequency component gradually decreases. The slope has obvious fractal characteristics, and the intercept has more prominent directionality and anisotropy, both of which are important parameters to characterize the roughness of the contour line.

(3) A new roughness evaluation index HAC is proposed, and the relationship between the fractal parameters $D_Z D_\theta D_\kappa$, roughness parameters $A_Z A_\theta A_\kappa$ of HAC and JRC is established. The comparison analysis shows that the evaluation error of the HAC is low, which indicates that the roughness of the profile can be well described by HAC.

(4) The HAC can describe the morphological characteristics of three-dimensional structural surfaces, and also can reflect the differences between the anisotropy and subtle roughness of three-dimensional morphology. It has important practical value.

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