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Bin LI
School of Environment and Resource, Southwest University of Science and Technology, Mianyang, Sichuan 621010, China

Da-guo WANG
School of Civil Engineering and Geomatics, Southwest Petroleum University, Chengdu, Sichuan 610500, China, dan_wangguo@163.com

Zhi-liang HE
School of Environment and Resource, Southwest University of Science and Technology, Mianyang, Sichuan 621010, China

Peng WANG
School of Environment and Resource, Southwest University of Science and Technology, Mianyang, Sichuan 621010, China

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Modification of linear regression method for rock shear strength parameters under triaxial condition

LI Bin[1], WANG Da-guo[2], HE Zhi-liang[1], WANG Peng[1]

1. School of Environment and Resource, Southwest University of Science and Technology, Mianyang, Sichuan 621010, China
2. School of Civil Engineering and Geomatics, Southwest Petroleum University, Chengdu, Sichuan 610500, China

Abstract: The triaxial strength envelope of rocks is usually nonlinear, and the shear strength parameters obtained by the linear regression method (LRM) are highly sensitive to confining pressure. In order to enable LRM to consider the influence of confining pressure on the estimation of shear strength parameters, the confining pressure effect coefficient of triaxial strength of rocks is defined. An exponential function is constructed to express the relationship between the coefficient and confining pressure, which is also introduced into the correction of LRM. A linear regression method considering confining pressure effects (CCPE-LRM) is proposed. At the same time, a rationality test method is proposed, and a distance coefficient is defined as an index to characterize the difference between the estimated and actual values of shear strength parameters. Through the verification and analysis of the triaxial strength data of various types of rocks in the published literature, the results show that the distance coefficients of various rocks are small, and the shear strength envelopes obtained by CCPE-LRM are all close to the Mohr circles in an approximately tangent state. It indicates that the shear strength envelope obtained by CCPE-LRM can replace the ideal shear strength envelope to a certain extent, and the shear strength parameters estimated by this method are in good agreement with the theoretical shear strength parameters. These prove that CCPE-LRM LRM has a good applicability.

Keywords: rock shear strength parameters; linear regression method; confining pressure effect; triaxial strength of rocks; shear strength envelope

1 Introduction

The shear strength parameters of rocks, cohesion c and internal friction angle φ (or internal friction coefficient $f=\tan\varphi$) are important parameters for rock engineering[1–4] and the input information for design and stability analysis of almost all rock engineering projects. These parameters can usually be determined using the conventional triaxial compression strength test on rocks, which is based on the Mohr-Coulomb strength criterion. On the basis of obtaining a sequence of triaxial strength data for rocks, the moment estimation method and the linear regression method (LRM) are the most basic methods for deriving shear strength parameters. Since the moment estimation method ignores the variability of the tests in a given group, it makes LRM a commonly used method to acquire shear strength parameters of rocks[5–6].

The application of LRM in determining the shear strength parameters of rocks have been extended by different scholars. Chen et al.[7–8] improved LRM to estimate the mean and variance of parameters. Liu et al.[9] and Cao and Zhang[10] introduced symmetric and asymmetric triangular fuzzy number into the LRM, respectively, so that it can determine the interval range of parameters. Li et al.[11] and Gong et al.[12] selected triaxial test data of different groups for linear regression analysis based on the permutation and combination theory, and they established a small sample information base for the shear strength parameter of rocks.

Since the triaxial strength envelope of rocks is usually nonlinear, the shear strength parameters are highly sensitive to the confining pressure σ_3 in linear regression analysis. The aforementioned achievements extend the application of LRM to the uncertainty of shear strength parameters of rocks, but the effect of confining pressure σ_3 on parameter estimation is ignored, leading to the weakening in the prediction of rock strength. At present, the studies considering the effect of σ_3 on parameter estimation are limited. Among these limited published papers, Shen et al.[13] used linear regression analysis by increasing triaxial strength data of coal rocks at higher confining pressures to find that c and φ gradually increases and decreases with increasing σ_3, respectively. An empirical formula for the parameters regarding uniaxial compressive strength

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First author: LI Bin, male, born in 1988, PhD, Reader, research interests: reliability of geotechnical engineering and mining rock mechanics and engineering. E-mail: binli@swust.edu.cn
Corresponding author: WANG Da-guo, male, born in 1975, PhD, Professor, PhD supervisor, research interests:computational mechanics. E-mail: dan_wangguo@163.com

σ_c and maximum confining pressure $\sigma_{3\max}$ at each regression analysis was also presented, but this formula cannot accurately derive the parameter values at a given confining pressure.

Based on the triaxial strength envelope characteristics of rocks, the variation trend of shear strength parameters with confining pressure was theoretically analyzed. The concept of confining pressure effect coefficient of triaxial strength of rocks was defined and then applied to the correction of LRM, and the estimation model of shear strength parameters considering the confining pressure effect was thus deduced. A method to test the rationality of the model was also proposed. In addition, the applicability of the model is verified by the triaxial strength data of various types of rocks in the published literature.

2 Linear regression method of shear strength parameters of rocks in triaxial tests

2.1 Methods and principles

Triaxial compression test is an important way to determine the shear strength parameters of rocks. Its theoretical basis is Mohr-Coulomb strength criterion, which can be expressed in the form of principal stress as

$$\sigma_1 = \frac{1+\sin\varphi}{1-\sin\varphi}\sigma_3 + \frac{2c\cos\varphi}{1-\sin\varphi} \tag{1}$$

where c is the cohesion; φ is the internal friction angle; and σ_3 and σ_1 are the minimum principal stress and the maximum principal stress corresponding to the failure. In the test, σ_3 and σ_1 are the applied confining pressure and the triaxial strength of rocks at failure under the given confining pressure.

Since the Eq.(1) is a linear equation, all triaxial test data (σ_1, σ_3) are fitted by linear regression, as follows:

$$\sigma_1 = \alpha\sigma_3 + \beta + \varepsilon \tag{2}$$

where α and β are the fitting parameters, determined by the least squares regression fitting method; ε is the deviation between the test triaxial strength σ_1 and the predicted triaxial strength $\hat{\sigma}_1$, and it is assumed to obey a normal distribution with a mean of 0 and a variance of a constant. Predicted triaxial strength is

$$\hat{\sigma}_1 = \alpha\sigma_3 + \beta \tag{3}$$

Ignore the error term ε and let $\hat{\sigma}_1 = \sigma_1$. Since 0< φ<π/2, according to Eqs. (1) and (3), the cohesion c and the internal friction coefficient $f = \tan\varphi$ can be calculated as [14]

$$\left.\begin{aligned} c &= \frac{\beta}{2\sqrt{\alpha}} \\ f &= \frac{\alpha-1}{2\sqrt{\alpha}} \end{aligned}\right\} \tag{4}$$

Equation (4) is the estimation formula of shear strength parameters of rocks.

This regression analysis method for obtaining shear strength parameters of rocks under triaxial compression test conditions is LRM.

2.2 Prerequisites for the application of LRM

The LRM must meet a basic premise in practical application, i.e., the relationship between triaxial strength of rocks and the confining pressure is approximately linear. That is to say, Eq.（2） should satisfy the correlation test of the univariate linear regression equation, as shown in Fig. 1 (taking three sets of triaxial test data as an example, the numbers outside the brackets are the test serial numbers, and the data pairs inside the brackets are the corresponding confining pressure and triaxial strength, respectively). All triaxial test data (σ_1, σ_3) were subjected to regression analysis according to Eq.(2), and the parameters α and β were obtained by the least squares method.

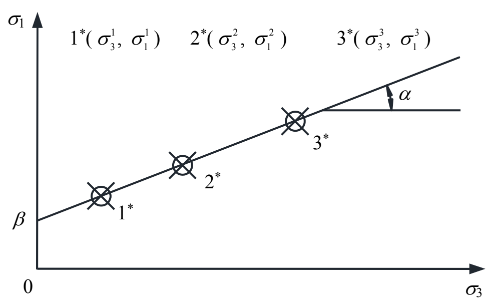

Fig. 1 LRM under ideal condition

During the test, the triaxial strength of rocks was obviously confining pressure dependent. A large number of test results [15–25] show that the relationship between the triaxial strength of rocks and confining pressure exhibits an upward convex nonlinear trend, which is weak at low confining pressure and can be approximated as linear, and this nonlinear relationship is significantly enhanced with increasing confining pressure[26], as shown in Fig. 2. Due to the confining pressure effect, the parameters α and β calculated from the test data of two groups under a similar confining pressure according to Eq. (2) are no longer unique values, as shown in Fig. 3 (with three groups of triaxial test data as an example). In Fig. 3, the dashed line is the linear regression equation of triaxial tests 1^* and 2^* with accordance to Eq.(2), and the regression parameters are α^{12} and β^{12}. The solid line is the regression equation of triaxial tests 2^* and 3^*, and the regression parameters are α^{23} and β^{23}. From Fig. 3, it can be seen that $\alpha^{23}<\alpha^{12}$ and $\beta^{23}>\beta^{12}$. Therefore, the confining effect of the triaxial strength of rocks makes the parameters α and β show a gradual decreasing and increasing tendency with the increasing σ_3, respectively.

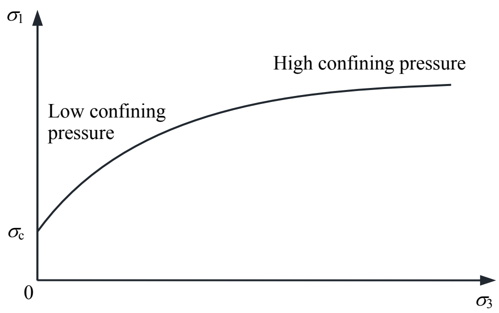

Fig. 2 Triaxial strength envelope of rocks

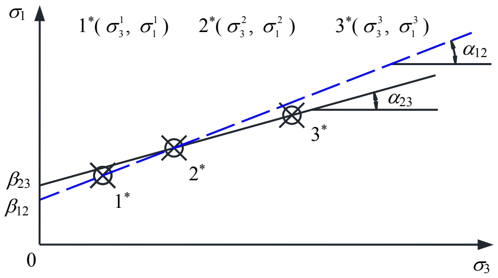

Fig. 3 LRM under the effect of confining pressure on the triaxial strength of rocks

According to the abovementioned trend of the parameters α and β, through Eq.(4), the cohesive *c* and the internal friction coefficient *f* perform a gradual increasing and decreasing trend with increasing σ_3, respectively, indicating that the shear strength parameters of rocks have confining pressure effect.

The LRM can only obtain a unique value of the shear strength parameter of rocks, which should be modified to extend its applicability to characterize the confining effect on the shear strength parameter.

3 Correction and reasonableness test of LRM

3.1 Correction of LRM

In the triaxial compression test, the failure strength at the confining pressure σ_3=0 is the uniaxial compressive strength of rocks, i.e. $\sigma_1=\sigma_c$. Substituting it into Eq.(2), $\beta=\sigma_c$ can be obtained. Considering that the uniaxial compressive strength of rocks is a key parameter in rock mechanics, let $\beta=\sigma_c$ in the correction and ignore the error term ε, and then Eq. (2) can be rewritten as

$$\sigma_1=\alpha\sigma_3+\sigma_c \tag{5}$$

where α can be written as

$$\alpha=\frac{\sigma_1-\sigma_c}{\sigma_3} \tag{6}$$

Then the physical meaning of the parameter α is clear, representing the increasing rate of the triaxial strength of rocks compared to the uniaxial compressive strength of rocks under confining pressure conditions, which is defined as the coefficient of the confining pressure effect of the triaxial strength of rocks.

According to the characteristics of the confining pressure effect of triaxial strength of rocks, the angle of depression of triaxial strength relative to uniaxial compressive strength can be plotted in the σ_1-σ_3 coordinate system, as shown in Fig. 4. We take three sets of triaxial test data as an example, θ_1, θ_2 and θ_3 are the angle of depression of triaxial strength σ_1^1, σ_1^2 and σ_1^3 against uniaxial compressive strength, respectively.

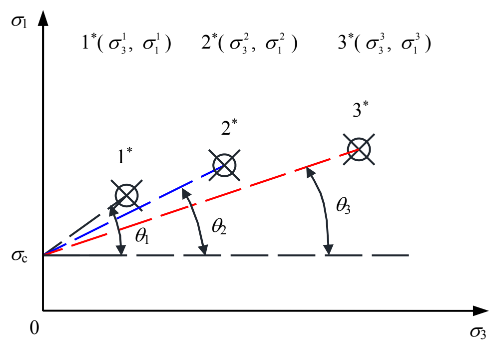

Fig. 4 Confining pressure effect of triaxial strength of rocks

From Eq. (6), it can be seen that the coefficients α of confining pressure effect for triaxial tests 1^*, 2^* and 3* are $\tan\theta_1$, $\tan\theta_2$ and $\tan\theta_3$, respectively. It is clear that $\tan\theta_1>\tan\theta_2>\tan\theta_3$ since $\theta_1>\theta_2>\theta_3$. Therefore, the coefficient of confining pressure effect of triaxial strength of rocks α gradually decreases with increasing confining pressure σ_3. Considering that the exponential function is a commonly used nonlinear function, an exponential function is constructed to characterize the relationship between α and σ_3, as follows:

$$\alpha=\alpha_1+\alpha_2 e^{\alpha_3\sigma_3} \tag{7}$$

where e is the natural exponent; and α_1, α_2 and α_3

are the fitting parameters. When fitting the parameters of Eq. (7), the α values are calculated according to Eq. (6).

Through the correction analysis of LRM, substituting Eq. (7) and $\beta=\sigma_c$ into Eq. (4) yields

$$\begin{aligned} c &= \frac{\sigma_c}{2\sqrt{\alpha_1+\alpha_2 e^{\alpha_3\sigma_3}}} \\ f &= \frac{\alpha_1+\alpha_2 e^{\alpha_3\sigma_3}-1}{2\sqrt{\alpha_1+\alpha_2 e^{\alpha_3\sigma_3}}} \end{aligned} \quad (8)$$

Equation (8) is the estimation equation of shear strength parameters of rocks considering the confining pressure effect.

The modified LRM takes the effect of confining pressure of triaxial strength on the estimation of shear strength parameters into account. Therefore, it is named the linear regression analysis method considering the confining pressure effect (CCPE-LRM).

3.2 Reasonableness test method

In shear stress−normal stress coordinate system, the intercept and slope of the tangent line at a certain point on the shear strength envelope of rocks is on the τ axis are the cohesion c and the internal friction coefficient f under the corresponding stress state (σ_1, σ_3), as shown in Fig. 5. The thick solid line is the ideal shear strength envelope of rocks, and it is tangent to all Mohr circles. The thick dashed line is the non-ideal shear strength envelope, non-intersecting (e.g., Mohr circle 1) or intersecting (e.g., Mohr circle 2) with Mohr circles is possible.

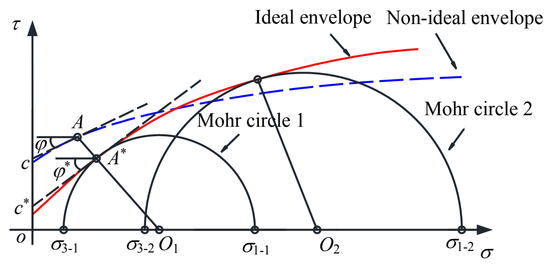

Fig. 5 Diagram of shear strength envelope

We take the Mohr circle 1 as an example (the analysis method of the Mohr circle 2 is the same as the Mohr circle 1), in this case, the tangent point of the ideal envelope and the Mohr circle 1 is point A^* (τ, σ), and the intercept c^* of the tangent line at point A^* on the τ axis and the slope $f^*=\tan\varphi^*$ are the theoretical shear strength parameters in the stress state (σ_1, σ_3). The corresponding point of the non-ideal envelope in the stress state (σ_1, σ_3) is point $A(\hat{\tau}, \hat{\sigma})$, and the intercept c of the tangent line at point A on the τ axis and the slope $f=\tan\varphi$ are the shear strength parameters using CCPE-LRM.

The smaller the distance between the point A and the Mohr circle, and the closer the non-ideal envelope is to the ideal envelope, the closer the shear strength parameters using CCPE-LRM are to the theoretical values. Therefore, the distance between the non-ideal envelope and the ideal envelope can indirectly characterize the difference between the shear strength parameters using CCPE-LRM and the theoretical values. For this purpose, the distance coefficient η between the non-ideal envelope and the ideal envelope is defined as

$$\eta=\frac{|d-r|}{r}\times 100\% \quad (9)$$

where r is the distance from point A^* to the Mohr circle center O_1, i.e., the radius of the Mohr circle; and d is the distance from point A to the Mohr circle center O_1. r and d can be expressed as follows, respectively:

$$r=\frac{\sigma_1-\sigma_3}{2} \quad (10)$$

$$d=\sqrt{\left(\frac{\sigma_1+\sigma_3}{2}-\hat{\sigma}\right)^2+\hat{\tau}^2} \quad (11)$$

where $\hat{\tau}$ and $\hat{\sigma}$ are the estimated values of τ and σ respectively. According to the formulas of τ and σ in Ref. [27−28], $\hat{\tau}$ and $\hat{\sigma}$ can be obtained respectively:

$$\hat{\sigma}=\sigma_3+\frac{\hat{\sigma}_1-\sigma_3}{\dfrac{\partial\hat{\sigma}_1}{\partial\sigma_3}+1} \quad (12)$$

$$\hat{\tau}=\frac{\hat{\sigma}_1-\sigma_3}{\dfrac{\partial\hat{\sigma}_1}{\partial\sigma_3}+1}\sqrt{\frac{\partial\hat{\sigma}_1}{\partial\sigma_3}} \quad (13)$$

Substituting Eq. (7) and $\beta=\sigma_c$ into Eq.(3) results in $\hat{\sigma}_1$ and $\partial\hat{\sigma}_1/\partial\sigma_3$, respectively:

$$\left.\begin{aligned} \hat{\sigma}_1 &= \left(\alpha_1+\alpha_2 e^{\alpha_3\sigma_3}\right)\sigma_3+\sigma_c \\ \frac{\partial\hat{\sigma}_1}{\partial\sigma_3} &= \left(\alpha_3\sigma_3+1\right)\alpha_2 e^{\alpha_3\sigma_3}+\alpha_1 \end{aligned}\right\} \quad (14)$$

Hence, the distance coefficient η can be used as an indicator to test the reasonableness of CCPE-LRM. The smaller the distance coefficient η is, the closer the non-ideal envelope is to the ideal envelope, and the more consistent the shear strength parameters obtained by CCPE-LRM are with the theoretical values.

4 Calculation process

The CCPE-LRM calculation process mainly consists of the following 7 steps:

(1) Triaxial strength data of rocks (σ_1, σ_3) are obtained by triaxial compression tests, including data

(σ_c, 0).

(2) Calculate the coefficient of confining pressure effect of triaxial strength of rocks α at different confining pressures σ_3 according to Eq.(6) to obtain the sequence data (α, σ_3).

(3) The sequence data (α, σ_3) are nonlinearly fitted according to Eq. (7) to determine the parameters α_1, α_2 and α_3, and to obtain the coefficient of determination R^2.

(4) Substitute the parameters determined in Step (3) into Eq. (8) to obtain the cohesion c and the internal friction coefficient f at different confining pressures.

(5) Substitute the parameters determined in Step (3) into Eq. (14) to obtain ($\hat{\tau}$, $\hat{\sigma}$) at different values of σ_3 according to Eqs. (12) and (13).

(6) Substitute ($\hat{\tau}$, $\hat{\sigma}$) into Eq. (11) to calculate the distance d at different values of σ_3, and to calculate the distance coefficient η at different values of σ_3 according to Eqs. (9) and (10).

(7) Calculate the average value $\overline{\eta}$ of the distance coefficients η at different values of σ_3 according to Step (6).

The parameters in Step (3) can be determined by nonlinear fitting with commonly used commercial software (e.g., MATLAB, 1stOpt, SPSS, etc.).

5 Applicability verification

The applicability of CCPE-LRM is mainly verified from two aspects: methodological soundness; and universality of application.

5.1 Reasonableness verification

Based on the objective principle, the triaxial strength data of Indiana limestone in Ref. [15] were used as an example to verify the rationality of CCPE-LRM, as shown in Table 1.

A linear fit of the data in Table 1 to Eq. (2) yields α =1.52 and β =59.67 MPa. Substituting α and β into Eq.(4) yields the shear strength parameters of Indiana limestone by LRM as c= 24.20 MPa and f = 0.21.

Using CCPE-LRM, the data in Table 1 were fitted to calculate the parameters according to the method in Section 4, and the values of the fitted parameters, the determination coefficient R^2 and the related calculation results were listed in Table 2.

Table 1 Triaxial test data for Indiana limestone[15]

Group	σ_3 / MPa	σ_1 / MPa	Group	σ_3 / MPa	σ_1 / MPa
1	0.0	44.0	7	41.2	128.2
2	6.5	66.0	8	48.4	135.1
3	13.7	85.0	9	55.4	141.9
4	20.3	99.0	10	62.3	149.1
5	27.9	109.0	11	68.4	156.5
6	34.4	119.0			

Table 2 Parameters and results of Indiana limestone

σ_3 / MPa	α	Parameters			R^2	Calculated results							
		α_1	α_2	α_3		c /MPa	f	$\hat{\tau}$ /MPa	$\hat{\sigma}$ /MPa	r /MPa	d /MPa	η /%	$\overline{\eta}$ /%
0.0	–					11.17	0.73	17.77	9.02	22.00	22.00	0.00	
6.5	3.38					11.93	0.65	25.87	21.50	29.75	29.78	0.09	
13.7	2.99					12.75	0.57	32.78	35.49	36.65	35.59	0.18	
20.3	2.71					13.47	0.51	37.27	48.01	39.35	39.04	0.78	
27.9	2.33					14.26	0.45	40.55	61.48	40.55	41.15	1.47	
34.4	2.18	1.35	2.53	-3.21×10^{-2}	0.998	14.87	0.40	42.14	71.89	42.30	42.41	0.26	0.53
41.2	2.04					15.47	0.36	43.00	81.59	43.50	43.11	0.90	
48.4	1.88					16.03	0.32	43.39	90.65	43.35	43.40	0.12	
55.4	1.77					16.50	0.29	43.52	98.53	43.25	43.52	0.62	
62.3	1.69					16.91	0.27	43.56	105.68	43.40	43.56	0.37	
68.4	1.64					17.22	0.25	43.59	111.69	44.05	43.60	1.03	

As shown in Table 2, the determination coefficient R^2 is 0.998, indicating that Eq.(7) has a high goodness of fit for the Indiana limestone. The coefficient of confining pressure effect of triaxial strength α gradually decreases with increasing confining pressure σ_3, which is consistent with the analysis in Section 3.1. The cohesion c and internal friction coefficient f gradually increase and decrease with increasing confining pressure, which are consistent with the confining pressure effect characteristics of shear strength parameters analyzed in Section 2.2. The maximum value of the distance coefficient η under various confining pressures is 1.47%, and the average value is only 0.53%, indicating that the non-ideal envelope obtained by CCPE-LRM is very close to the ideal envelope. This also indirectly indicates that the shear strength parameters of Indiana limestone obtained by this method are in good agreement with the theoretical counterparts.

To observe the position relationship between the non-ideal envelope obtained by CCPE-LRM and Mohr circles, the envelope is plotted in the $\tau-\sigma$ coordinate system according to the stress combination points ($\hat{\tau}$, $\hat{\sigma}$) in Table 2, as shown in Fig. 6, where the dot marks the stress combination points ($\hat{\tau}$, $\hat{\sigma}$), the thick solid line is the shear strength envelope determined by the CCPE-LRM through the 11 stress combination points ($\hat{\tau}$, $\hat{\sigma}$) in Table 2, and the dashed line is the envelope plotted with accordance to the shear strength parameters c = 24.20 MPa and f = 0.21 determined by LRM.

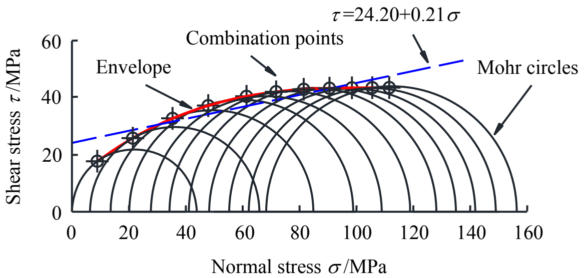

Fig. 6 Shear strength envelope and Mohr's circles of Indiana limestone

It can be observed from Fig.6 that 11 stress combination points ($\hat{\tau}$, $\hat{\sigma}$) are very close to corresponding Mohr circles, indicating that the shear strength of the rock can be calculated more accurately based on the triaxial strength by CCPE-LRM. The envelope determined by CCPE-LRM is nearly tangent to all Mohr circles, indicating that this envelope can replace the ideal envelope to a large extent. The envelope determined by LRM runs through the Mohr circles and overestimates the shear strength on both sides, while the envelope underestimates the shear strength in the middle.

The results of the above analysis show that the CCPE-LRM can determine the shear strength envelope of rocks more accurately, which proves that the method is reasonable for determining the shear strength parameters.

5.2 Universality verification

To verify the universality of CCPE-LRM in applications, triaxial strength data of eight types of rocks from the Refs. [16−18, 20−24] were analyzed as an example. The triaxial strength data of eight types of rocks are listed in Table 3.

Table 3 Triaxial test data of eight types of rocks

Ezhou granite[16]		Carrara marble[18]		Daye marble[20]		Dunham dolomite [17]		Jinping marble[21]		Nanyang marble [22]		Jinping sandstone [23]		Mizuho trachyte [24]	
σ_3 /MPa	σ_1 /MPa	σ_3 /MPa	σ_1 /MPa	σ_3 /MPa	σ_1 /MPa	σ_3 /MPa	σ_1 /MPa	σ_3 /MPa	σ_1 /MPa	σ_3 /MPa	σ_1 /MPa	σ_3 /MPa	σ_1 /MPa	σ_3 /MPa	σ_1 /MPa
0	83.22	0.0	137	0	96.2	0	262	0	199.20	0	84.1	0	61.6	0	100
10	155.96	25.0	234	10	145.4	25	400	10	268.53	5	131.7	5	109.5	15	193
20	212.41	50.0	314	20	193.4	45	487	20	312.04	10	168.3	10	138.6	30	253
30	250.29	68.4	358	30	232.9	60	540	30	358.11	20	226.8	20	174.6	45	300
40	276.85	85.5	404	40	246.4	65	568	40	398.49	30	266.2	30	209.0	60	339
50	299.43	161.8	558	50	272.4	85	620	50	422.72	40	301.9	40	240.5	75	365
60	328.67			60	308.0	105	682	60	464.96			50	263.0	100	419
70	343.83			70	348.7	125	725	70	499.26			60	288.5		
				80	353.6							70	305.4		
				90	367.6										
				100	402.7										

The triaxial strength data for the eight rocks in Table 3 were fitted to obtain corresponding parameters α_1, α_2 and α_3 and the determination coefficients R^2 according to the method in Section 5.1. The average distance coefficients $\overline{\eta}$ were also calculated, as listed in Table 4.

The fitted determination coefficients R^2 of the eight types of rocks in Table 4 are very close to 1, indicating that Eq. (7) can better describe the variation of coefficient of confining pressure effect of triaxial strength α with the confining pressure σ_3. By substituting the fitting results of parameters α_1, α_2 and α_3 into Eq. (8), the shear strength parameters of rocks under different confining pressures can be obtained.

As can be seen from Table 4, the average distance coefficients $\overline{\eta}$ of the eight types of rocks are small, with the maximum value of 1.42% for Daye marble and the minimum value of 0.10% for Nanyang marble, indicating that the shear strength envelopes of these eight types of rocks determined by CCPE-LRM are very close to their Mohr circles.

Table 4 Fitting parameters and distance coefficients of eight types of rocks

Rock category	Fitting parameters			R^2	$\overline{\eta}$ /%
	α_1	α_2	α_3		
Ezhou granite	2.50	6.10	-2.31×10^{-2}	0.996	0.49
Carrara marble	2.17	2.21	-1.01×10^{-2}	0.996	0.23
Daye marble	2.51	3.08	-1.82×10^{-2}	0.932	1.42
Dunham dolomite	2.42	3.84	-8.67×10^{-3}	0.992	0.25
Jinping marble	4.17	4.35	-4.78×10^{-2}	0.982	0.33
Nanyang marble	4.14	6.53	-4.02×10^{-2}	0.999	0.10
Jinping sandstone	3.64	8.08	-6.56×10^{-2}	0.994	1.12
Mizuho trachyte	2.72	4.92	-2.35×10^{-2}	0.999	0.28

For further verification, the eight envelopes are plotted in the $\tau-\sigma$ coordinate system according to the method in Section 5.1, as shown in Fig. 7. It can be found that for these eight types of rocks, the shear strength envelopes determined by CCPE-LRM are closely tangent to their corresponding Mohr circles, demonstrating that it is feasible to indirectly characterize the difference between the estimated and actual values of the shear strength parameters using the distance coefficient η.

Therefore, it can be concluded from the above analysis that CCPE-LRM has good applicability for different types of rocks as well.

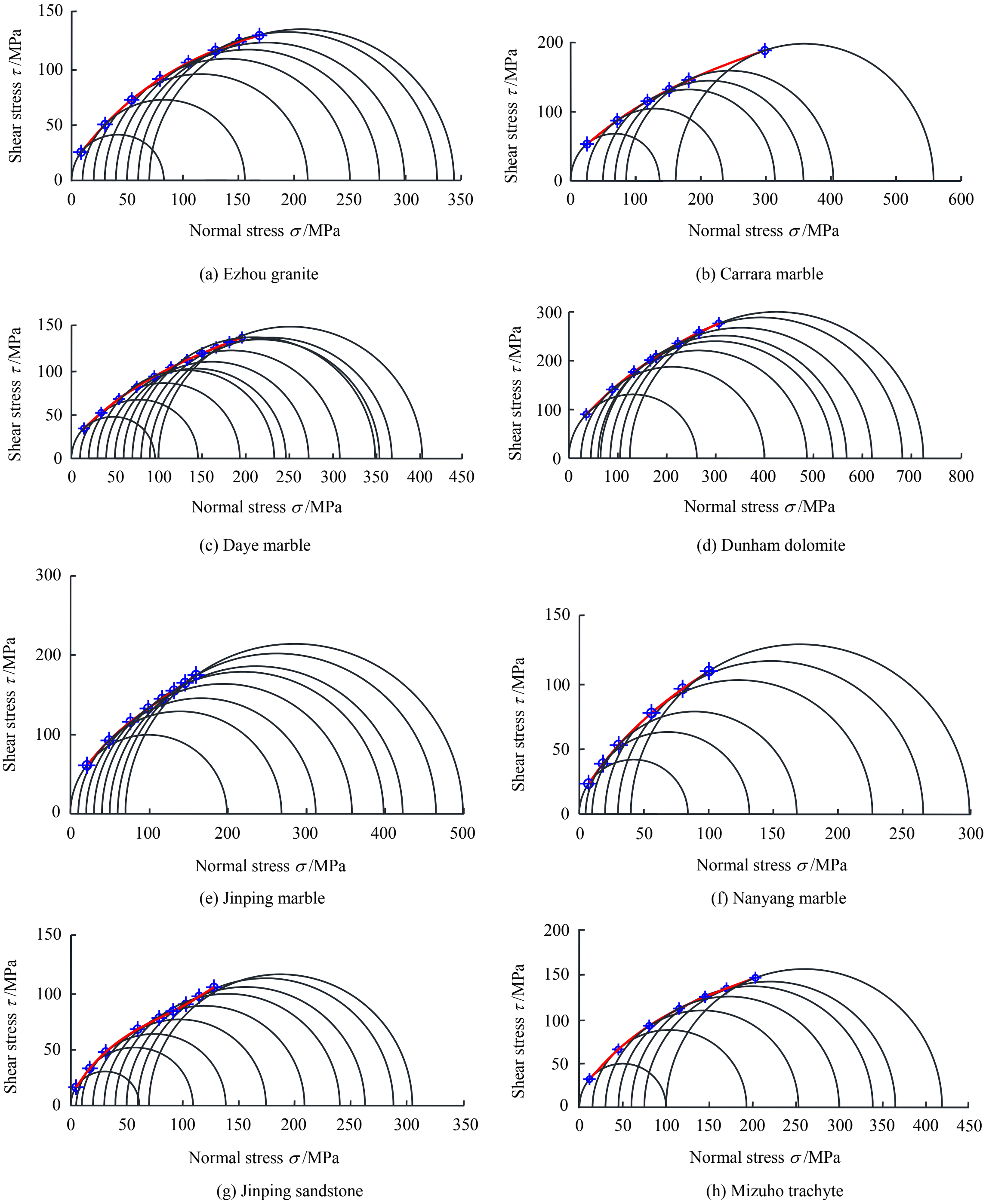

Fig. 7 Shear strength envelopes and Mohr's circles of eight types of rocks

6 Discussion

In Section 2, based on the nonlinear characteristic of the upward convexity of the triaxial strength envelope of the rock, the confining effect characteristics of the shear strength parameters of rocks are theoretically analyzed, i.e., the cohesion c and the internal friction coefficient f gradually increases and decreases with increasing confining pressure, respectively. As the starting point of this paper, it is consistent with the results of Shen et al[13, 29].

The results of the validation analysis in Section 4 show that CCPE-LRM can estimate the shear strength parameters of rocks at different confining pressures more accurately, and the shear strength envelopes determined by CCPE-LRM are very close to Mohr circles. Therefore, it is feasible to use the distance coefficient η between the non-ideal envelope and the ideal envelope as an indicator to characterize the difference between the estimated and actual values of shear strength parameters. In addition, the effect of confining pressure effect can be used as a new research perspective to analyze the uncertainty of shear strength parameters of rocks.

The background condition of CCPE-LRM is that the relation between the coefficient of confining pressure effect of triaxial strength α and the confining pressure σ_3 is negatively correlated, and it is assumed that the negative correlation obeys the exponential function Eq.(7). Therefore, CCPE-LRM has a clear scope of application, i.e., the triaxial strength of the rock must have a significant confining effect. If the triaxial strength is linear or approximately linear with respect to the confining pressure, α can be determined as a constant and CCPE-LRM degenerates to LRM, which is still applicable. The accuracy of CCPE-LRM in estimating the shear strength parameters are directly determined by the relationship function between α and σ_3. Whether the negative correlation is upward or downward convex, and the construction of a better-fitting relationship function to replace Eq.(7) need to be further investigated based on triaxial strength data for numerous types of rocks.

7 Conclusions

(1) Based on the nonlinear characteristic of the upward convexity of the triaxial strength envelope of the rock, the concept of the coefficient of confining pressure effect of triaxial strength is defined. An exponential function is constructed to represent the negative correlation between this coefficient and the confining pressure, and it is introduced into the correction of LRM to propose the CCPE-LRM.

(2) The concept of distance coefficient between the non-ideal shear strength envelope and the ideal shear strength envelope is defined and is used as an index to characterize the difference between the estimated and actual values of shear strength parameters of rocks. By taking triaxial strength data of various types of rocks as examples, it is verified that the exponential function can better describe the relationship between the coefficient of confining pressure effect of triaxial strength and confining pressure. Meanwhile, it is concluded that the shear strength envelope obtained by CCPE-LRM closely follows and is approximately tangent to the Mohr circle, indicating that the shear strength parameters obtained by this method are in good agreement with the theoretical shear strength parameters, which proves that CCPE-LRM has good applicability.

(3) Taking the effect of the confining pressure on the shear strength parameters of rocks into account, the CCPE-LRM can estimate the shear strength parameters of rocks at different confining pressures more accurately. Meanwhile, since the shear strength envelope of the rock obtained by this method closely follows the Mohr circle, the envelope can thus replace the ideal shear strength envelope to a certain extent.

References

[1] GONG Feng-qiang, LUO Song, GE Lin, et al. Evaluation of shear strength parameters of rocks by preset angle shear, direct shear and triaxial compression tests[J]. Rock Mechanics and Rock Engineering, 2020, 53: 2505−2519.

[2] SHEN Jia-yi, SHU Zheng, CAI Ming, et al. A shear strength model for anisotropic blocky rock masses with persistent joints[J]. International Journal of Rock Mechanics and Mining Sciences, 2020, 134: 1−11.

[3] TANG Xiao-song, LI Dian-qing, WANG Xiao-gang, et al. Statistical characterization of shear strength parameters of rock mass for hydropower projects in China [J]. Engineering Geology, 2018, 245: 258−265.

[4] AGHCHAI M H, MAAREFVAND P, RAD H S. Analytically determining bond shear strength of fully grouted rock bolt based on pullout test results[J]. Periodica Polytechnica Civil Engineering, 2020, 64(1): 212−222.

[5] LIN Bin, CHEN Zhi-bo, FANG Qing. Study on determination method for parameters of rock's shear strength through least absolute linear regression based on symmetric triangular fuzzy numbers[J]. Applied Mechanics and Materials, 2012, 170: 804−807.

[6] CHEN Li-hong, CHEN Zu-yu, LI Guang-xin. Discussion of linear regression method to estimate shear strength parameters from results of triaxial tests[J]. Rock and Soil Mechanics, 2005, 26(11): 1785−1789.

[7] CHEN Li-hong, LI Xu, XU Yao, et al. Accurate estimate of soil shear strength parameters[J]. Journal of Central South University, 2019, 26: 1000−1010.

[8] CHEN Li-hong, CHEN Zu-yu, LI Guang-xin. A modified linear regression method to estimate shear strength parameters[J]. Rock and Soil Mechanics, 2007, 28(7): 1421−1426.

[9] LIU Xi-long, CHEN Yi-zeng. A systematic approach to optimizing *h* value for fuzzy linear regression with symmetric triangular fuzzy numbers[J]. Mathematical Problems in Engineering, 2013, 6: 1−9.

[10] CAO Wen-gui, ZHANG Yong-jie. Study on determining method for parameters of rock's shear strength based on asymmetric triangular fuzzy numbers[J]. Chinese Journal of Rock Mechanics and Engineering, 2007, 26(7): 1340−1346.

[11] LI Xi-bing, ZHU Huan-zhen, HUANG Tian-lang. Deduction of probability distribution of M-C strength parameters by Legendre polynomial[J]. Science & Technology Review, 2015, 33(18): 49−55.

[12] GONG Feng-qiang, HOU Shang-qian, YAN Xiao-ming. Probability model deduction method of Mohr-Coulomb criteria parameters based on normal information diffusion principle[J]. Chinese Journal of Rock Mechanics and Engineering, 2013, 32(11): 2225−2234.

[13] SHEN Jia-yi, WAN Ling, ZUO Jian-ping. Non-linear shear strength model for coal rocks[J]. Rock Mechanics and Rock Engineering, 2019, 52: 4123−4132.

[14] BALMER G. A general analytical solution for Mohr's envelope[J]. Proceedings of American Society of Test Materials, 1952, 52: 1260−1271.

[15] HOEK E. Strength of jointed rock masses[J]. Géotechnique, 1983, 23(3): 187−223.

[16] LI Bin. Study on rock failure characteristics and rock strength criteria under high confining pressure[D]. Wuhan: Wuhan University of Science and Technology, 2015.

[17] MOGI K. Experimental rock mechanics[M]. London: Taylor and Francis, 2007.

[18] GOWD T N, RUMMEL F. Effect of confining pressure on the fracture behavior of a porous rock[J]. International Journal of Rock Mechanics and Mining Sciences, 1980, 37(2): 225−229.

[19] VON KARMAN T. Festigkeitsversuche unter allseitigem druck[J]. Zeitschrift Verein Deutscher Ingenieure, 1911, 55: 1749−1757.

[20] OUYANG Z, ELSWORTH D. A phenomenological failure criterion for brittle rock[J]. Rock Mechanics and Rock Engineering, 1991, 24: 133−153.

[21] XIAO Tao-li, HUANG Mei, LI Xin-ping. Research on strength and deformation with marble of deep rock mass considering confining pressure effect[J]. Chinese Journal of Underground Space and Engineering, 2018, 14(2): 362−368.

[22] YOU Ming-qing. Mechanical characteristics of the exponential strength criterion under conventional triaxial stresses[J]. International Journal of Rock Mechanics and Mining Sciences, 2010, 47(2): 195−204.

[23] YOU Ming-qing. Three independent parameters to describe conventional triaxial compressive strength of intact rocks[J]. Journal of Rock Mechanics and Geotechnical Engineering, 2010, 2(4): 350−356.

[24] YOU Ming-qing. Study on mathematical equation and parameter determination of strength criteria for rock [J]. Chinese Journal of Rock Mechanics and Engineering, 2010, 29(11): 2172−2184.

[25] TARASOV B, POTVIN Y. Universal criteria for rock brittleness estimation under triaxial compression[J]. International Journal of Rock Mechanics and Mining Sciences, 2013, 59: 57−69.

[26] LI Bin, WANG Da-guo. Negative power rock strength criterion under conventional triaxial compression[J]. Coal Geology & Exploration, 2020, 48(2): 152−160.

[27] KUMAR P. Shear failure envelope of Hoek-Brown criterion for rock masses[J]. Tunnelling and Underground Space Technology, 1998, 13(4): 453−458.

[28] LI Bin, XU Meng-guo, LIU Yan-zhang. Application of critical state confining pressure to rock strength criteria modification[J]. Rock and Soil Mechanics, 2016, 37(2): 390−398.

[29] SHEN J Y, KARAKUS M. Three-dimensional numerical analysis for rock slope stability using shear strength reduction method[J]. Canadian Geotechnical Journal, 2014, 51(2): 164−172.