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Inversion iterative correction method for estimating shear strength of rock and soil mass in slope engineering

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Abstract: For the slopes that have failed or deformed significantly, the shear strength of rock and soil mass is frequently inversely estimated based on a factor of safety assumed. For the slope with a sliding surface passing through multi-layer rock and soil mass, it is unreasonable to achieve this goal by blind trial. To solve this issue, back propagation (BP) neural network is constructed using shear strength of multi-layer rock and soil mass as the input, and the factor of safety of slope, the entrance and exit positions of the sliding surface obtained by Geoslope as the outputs. Then, based on the assumed factor of safety and the entrance and exit positions measured in site, the shear strength is acquired by carrying out the "reverse back analysis–error check–sample correction" procedure repeatedly. The result of a case study verifies that the shear strength obtained by this method is reasonable and can be used as a reference when designing reinforcement measures for small-scale slopes. BP neural network usually considers the known information as the input, and the information to be determined as the output, which will induce a mathematical underdetermined problem when solving this issue. The proposed method avoids this demerit successfully, and has a lower requirement on the number of samples in the library and a higher precision compared to the classical BP neural network.

Keywords: slope reinforcement; neural network; parameter inversion; reverse iteration; underdetermined problems

1 Introduction

Estimating the shear strengths of rocks and soils is a fundamental task in slope engineering, and it has an important significance for the stability assessment and reinforcement design of slopes. In engineering practice, three methods, i.e. laboratory test, in situ test and inverse analysis, are commonly utilized to estimate the shear strengths of rocks and soils. Usually, the results of laboratory tests are not reliable due to inevitable disturbance in the sampling process of rocks and soils[1–3], limited number of samples[4], and discrepancies between small-size samples and real site conditions[4–5]. Furthermore, it is impossible to collect undisturbed samples for some special rock or soil. Meanwhile, in situ test has not been widely employed in small-scale slopes as it is time-consuming, expensive and limited in measurement range[1].

For the slopes that have failed or deformed obviously, the inverse analysis with an assumed factor of safety is an important method to estimate the shear strengths of rocks and soils. According to *Code for investigation of geotechnical engineering* (GB50021－2001)[6], the factor of safety can be assumed as 0.95–1.00 for a sliding slope, and 1.00–1.05 for a slope that is temporarily stable. To inversely estimate the shear strengths of rocks and soils for small-scale slopes, the technicians often assume a series of possible values of the shear strengths and obtain the factors of safety using slope stability analysis commercial software, such as Geoslope and Lizheng Geotechnical Software, and then they compare the results with the assumed values to determine a rough range of shear strength. Finally, the recommended values of shear strength for the reinforcement design are determined based on a comprehensive consideration of the inverse analysis results, the test results and the engineering experience. For a homogeneous slope containing a single soil layer or a distinct sliding zone, the inverse analysis is easy to be accomplished. However, if the composition of the slope is complicated, the sliding surface is possible to pass through multi-layer rocks and soils when the slope fails. In this case, blind trial is inadvisable in inverse analysis performance, and it is necessary to introduce progressive mathematical tools into inverse analysis.

Back propagation (BP) neural network is one of the hot topics in machine learning recently. The universal approximation theorem[7] points out that the neural networks have the ability to approximate arbitrarily complex nonlinear functions. BP neural network and its improvements had achieved more abundant research results in terms of the inverse analysis of the physico-mechanical parameters for rocks and soils. Li et al.[8] used the monitoring data obtained from Baishan hydropower station to evaluate the rock permeability coefficient. Zhou et al.[9] used BP neural network to acquire the mechanical parameters of surrounding rocks based on the data of surrounding rock deforma-

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tion at Pingshi tunnel of Beijing–Zhuhai Expressway. Hu et al.[10] proposed an intelligent method for the inverse analysis of geotechnical creep parameters, which is a combination of genetic algorithm, neural network and finite difference method. Using radial basis function (RBF) neural network, Jin et al.[11] acquired the rock mechanical parameters and the initial stress field based on the measured stress data. Li et al.[12] designed parameter samples using uniform design theory, gathered training samples through finite element method (FEM) analysis, optimized neural network parameters based on the genetic algorithm, and successfully obtained parameters such as the elastic modulus of the representative rocks at Geheyan hydropower station. Replacing FEM analysis with a neural network and optimizing the neural network parameters based on the particle swarm optimization algorithm, Li et al.[13] developed an improved particle swarm optimization algorithm, and performed the parameter inverse analysis for the concrete face rockfill dam at Shuibuya hydropower station. Wang et al.[14] introduced the genetic algorithm and the simulated annealing algorithm into BP neural network, and proposed a new neural network model GSA-BP. This model has solved the issues that BP neural network is easy to fall into the local optimal solutions and the convergence rate of BP neural network is relatively slow, and it has been successfully applied to inverse analysis of mechanical parameters of the surrounding rocks in the underground plant at Wudongde hydropower station. Combined the neural network inverse analysis with the interval analysis theory, Wei et al.[15] acquired the intervals of deformation moduli of the dam and foundation, determined the monitoring indices of dam deformation according to the most adverse effect of the parameter interval combinations, and achieved better application effects.

In the previous studies, the input information of BP neural network is mostly a large amount of field monitoring data, e.g. displacement and stress, and the output information generally has a dimensionality significantly less than the input information. In this case, it forms an overdetermined problem in BP neural network, whose solution is ensured to be reliable and accurate by mathematical theory[16]. When the sliding surface passes through multi-layer rocks and soils, the available information includes factor of safety assumed based on the slope deformation feature, and the entrance and exit positions of the sliding surface observed in the field investigation. In this case, the mechanical parameters to be estimated often have a larger dimensionality than the available information. If the BP neural network selects the factor of safety, the entrance and exit positions as the inputs, and takes the shear strength of the rock and soil mass as the output, an underdetermined problem is formed in the BP neural network. Though it can be solved by some classical methods, the accuracy of the results is generally low. For instance, Zhou et al.[17] utilized BP neural network in the calibration of meso-mechanical parameters of particles in Particle Flow Code (PFC). In the neural network, the inputs contain three macro-mechanical parameters, i.e. elastic modulus, uniaxial compressive strength and Poisson's ratio of rocks and soils, and the outputs include four meso-mechanical parameters of particles. Ten extra samples are used to test the validity of the neural network, and a few samples have results with a low numerical accuracy, although the training library consists of 400 samples. A reliable strategy is to transform the underdetermined problem into a suitable problem or an overdetermined problem by introducing some extra constraints. However, more investigation work should be carried out to ensure the reasonability of these additional constraints to be involved into inverse analysis of the shear strength of rocks and soils.

A novel inversion method to estimate the shear strengths of rocks and soils is proposed in this study for the case that the sliding surface passes through multi-layer rocks and soils during the design of small-scale slope reinforcement. This method is capable of solving the limits of neural network in addressing the underdetermined problem. First, a BP neural network is established by taking the shear strengths of rocks and soils as the input and the known factor of safety of the slope, the entrance and exit positions of the sliding surface as the outputs. Then, through setting the given output information as the target, the shear strength is obtained by the reverse analysis, and the precision of the shear strength obtained is verified through error check. If the result fails to pass the error check, sample correction is executed, the neural network is retrained, and then the reverse back analysis is performed once again. The above process is repeated to obtain a shear strength satisfying the specific requirements. Compared with the neural network that takes known information as the input and information to be deduced as the output, the new method requires a library of less samples and has a higher accuracy.

2 Establishing BP neural network for inversion of mechanical parameters

2.1 Basic principle of BP neural network

BP neural network, put forward by Werbos[18] and improved by Rumelhart et al.[19] and other researchers, is a multi-layer feed forward neural network. Through simulating the information processing mechanism of the brain, it ascertains nonlinear relation between the input parameters and the output results. Nowadays, it is the most commonly used neural network.

As shown in Fig. 1, a typical BP neural network is composed of three parts, which are the input layer, the hidden layers, and the output layer. Once the structure of a BP neural network is determined, the output value of each neuron in forward propagation is

$$u_i^l = \sigma_i^l \left(\sum_{j=1}^{k_{l-1}} w_{ji}^l u_j^{l-1} + b_i^l \right) \tag{1}$$

where u_i^l is the output value of the i-th neuron in the l-th layer; σ_i^l is the activation function for the i-th neuron in the l-th layer; k_{l-1} is the number of neurons in the $(l-1)$-th layer; w_{ji}^l is the weight of the j-th neuron in the $(l-1)$-th layer regarding to the i-th neuron in the l-th layer; and b_i^l is the bias term for the i-th neuron in the l-th layer.

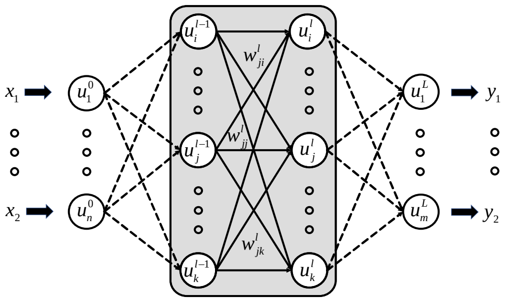

Fig. 1 Structure of BP neural network

The forward propagation of the neural network in fact defines a complicated nonlinear function $y(x;\theta)$, where x denotes the input variable and the parameter $\theta=\{w,b\}$. Through updating the parameter, the BP of the neural network minimizes the error between the output value and the true value, i.e.

$$\min E=\frac{1}{2}[y(x;\theta)-\bar{y}]^2 \tag{2}$$

where $\bar{y}$ is the true value of the output information; and E is the loss scalar, which can be defined in various ways such as mean absolute error, mean square error, and cross entropy loss. In this study, the mean square error is used.

To avoid the effect of dimension inconsistency of the data on the result, the following equation is used to normalize the data before training BP neural network:

$$\hat{x}_i=\frac{x_i-x_{\min}}{x_{\max}-x_{\min}} \tag{3}$$

where x_i and $\hat{x}_i$ are the values of data before and after normalization, respectively; and $x_{\min}$ and $x_{\max}$ are the minimum and maximum values of the data, respectively.

2.2 BP neural network for inversion of mechanical parameters

In the available applications of BP neural network to inverse analysis of mechanical parameters for rocks and soils, the field monitoring data are used as the inputs, and a few critical parameters such as elastic modulus and permeability coefficient are used as the outputs, which is effective because a great deal of monitoring data of various types could be measured. Whereas, as mentioned in Section 1, for inverse analysis of the shear strength in the case of the sliding surface passing through multi-layer rocks and soils, the classical way to establish BP neural network is to take the assumed factor of safety of the slope, the entrance and exit positions of the sliding surface as the inputs, and take the shear strength of each layer of rocks and soils as the output, which in fact forms an underdetermined problem that the dimensionality of the output exceeds that of the input.

Here, the slope model with a height of 15 m and a slope angle of 60° in Fig. 2 is taken as an example for analysis. In addition to the bed rock, the slope is composed of three soil layers. Each soil layer has a dip angle of 15° and a thickness of 3.6 m. The unit weight of all the soils is 19 kN/m^3. The slope's factor of safety, the entrance and exit positions of the sliding surface are given, and the shear strengths of three soil layers are to be estimated. The classical way to establish BP neural network will result in the input layer of three neurons and the output layer of six neurons, which are the cohesions and the friction angles of soil layers #1, #2 and #3. In this case, an underdetermined problem is formed, and the result obtained by BP neural network may have a low accuracy.

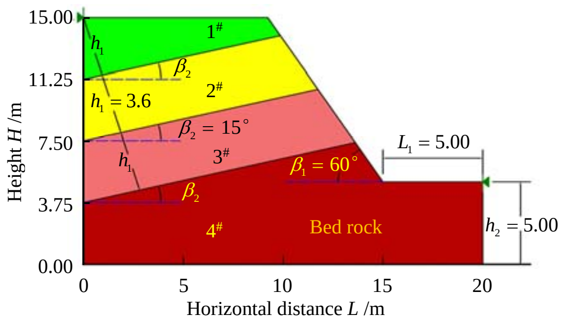

Fig. 2 Schematic diagram of a slope example

To avoid resolving the underdetermined problem, a BP neural network to perform inverse analysis of the shear strength of soil is constructed by taking the shear strengths of three soil layers as the input information. The output information is the factor of safety K of the slope, the entrance position S^{in} and the exit position S^{out} of the sliding surface. As a consequence, for the slope model shown in Fig. 2, the input layer has six neurons and the output layer has three neurons. The dimensionalities of the input and output information induce a mathematical overdetermined problem. It is noticed that the entrance and exit positions of the sliding surface is marked with their coordinates (x, y), which have two data but can only be considered as one neuron. Actually, both the entrance and exit positions of the sliding surface must outcrop at the ground surface, and they certainly can be marked with a relative scale of the slope surface line, indicating that they can only be considered as one data. In views of that in this study they are marked with their coordinates, the errors of two corresponding neurons in Eq. (2) are computed based on Euclidean distance.

The shear strengths of the three soil layers in the slope model as Fig. 2 shows are assumed to vary in the following ranges: $c_1,c_3\in[20,25]$ kPa; $c_2\in[10,15]$ kPa; $\varphi_1,\varphi_3\in[20^\circ,30^\circ]$; $\varphi_2\in[5^\circ,15^\circ]$. Here, c and φ represent the cohesion and internal friction angle of soil, respectively; and the subscript denotes the

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number of the soil layer. Neural network generally ranks variables in their value range, and then generates the input samples through the complete orthogonal combination, which will result in a great number of the input samples in this study. Even though each variable is roughly divided into two levels, 64 input samples will be generated. To reduce the workload, 50 combinations of the shear strengths are generated as the input samples based on their value range by the uniform random method, which is realized through using the uniform function in TensorFlow software. It has been mathematically proved that the uniform random array is capable of covering the entire value range uniformly. A series of methods, such as the linear congruence method[20] and feedback shift register method[21], can be employed in the generation of the uniform random array at present. After generation of the input samples, the Bishop method in Geoslope software is adopted to calculate the factor of safety of the slope, the entrance and exit positions of the sliding surface for each combination of the shear strengths, which form the output samples. At the moment, both the input and output samples are available, which means that the sample library of the BP neural network is ready. In order to verify whether the input samples generated by the uniform random method affect the results of the BP neural network or not, two different sample libraries are prepared in this study for the subsequent comparison on the inverse analysis results, as listed in Table 1.

After the establishment of the sample library, the BP neural network is trained. In this study, the number of hidden layers of the neural network is set to be one. The determination of the number of neurons in hidden layer is still an unsettled question[22]. Kolmogorov's theorem suggests that the number of neurons in the hidden layer could be preliminarily determined according to the "$2N+1$" method, in which N is the number of neurons in the input layer. On the basis of the result obtained by the "$2N+1$" method, five schemes for the number of neurons in the hidden layer are attempted, which are 9, 11, 13, 15 and 17, respectively. Through the error comparison, the number of neurons in the hidden layer is taken as 15 eventually. The program is coded in TensorFlow software, in which Sigmoid function is used as the training activation function, the Adam optimizer is adopted, and the learning rate is set to be 0.001.

Table 1 Library of neural network training samples

Sample library	No.	c_1 /kPa	φ_1 /(°)	c_2 /kPa	φ_2 /(°)	c_3 /kPa	φ_3 /(°)	Factor of safety K	Entrance position S^{in} (x, y) /m	Exit position S^{out} (x, y) /m
	1	22	26	13	7	23	29	0.810	(5.97, 15.0)	(12.43, 9.44)
	2	23	29	10	12	22	26	0.850	(7.60, 15.0)	(12.72, 8.94)
	3	23	22	12	10	22	28	0.910	(5.97, 15.0)	(12.35, 9.58)
	4	23	24	11	13	20	20	0.960	(6.48, 15.0)	(12.40, 9.48)
	5	23	24	10	12	21	22	0.990	(7.02, 15.0)	(12.97, 8.51)
1#						...	...	...		
	46	23	29	13	13	22	20	1.000	(6.04, 15.0)	(12.61, 9.13)
	47	23	26	12	12	24	27	1.030	(5.53, 15.0)	(11.97, 10.23)
	48	21	29	13	14	23	22	1.090	(6.08, 15.0)	(13.60, 7.41)
	49	24	23	13	14	21	25	1.100	(6.71, 15.0)	(13.61, 7.41)
	50	22	22	12	14	24	22	1.110	(6.08, 15.0)	(13.60, 7.41)
	1	23	22	10	10	24	21	0.880	(6.01, 15.00)	(12.17, 9.89)
	2	22	22	14	8	23	22	0.902	(5.22, 15.00)	(12.52, 9.28)
	3	22	25	10	10	21	25	0.960	(6.51, 15.00)	(12.78, 8.83)
	4	23	23	10	13	23	21	0.986	(6.07, 15.00)	(13.50, 7.72)
	5	22	27	10	12	24	22	1.007	(6.87, 15.00)	(12.59, 9.25)
2#						...	...	...		
	46	24	29	12	11	23	25	1.010	(7.45, 15.00)	(12.98, 8.50)
	47	23	24	13	12	24	27	1.041	(5.64, 15.00)	(12.26, 9.66)
	48	24	22	13	13	22	23	1.071	(6.66, 15.00)	(13.44, 7.70)
	49	22	28	12	14	24	27	1.106	(6.05, 15.00)	(11.88, 10.51)
	50	24	20	14	14	23	29	1.219	(6.17, 15.00)	(13.19, 8.20)

3 Inversion of mechanical parameters with reverse iteration and sample correction

3.1 Reverse back analysis

The forward propagation process of BP neural network built in Section 2 can be regarded as a substitution of the calculation of the factor of safety and determination of the sliding surface in GeoSlope based on the shear strengths of three soil layers. If the forward propagation rule of BP neural network is denoted by f, the transformation from the input $x = (c_1, \varphi_1, c_2, \varphi_2, c_3, \varphi_3)$ to the output $y = (K, S^{\text{in}}, S^{\text{out}})$ can be expressed as

$$(K, S^{\text{in}}, S^{\text{out}}) = f(x) \tag{4}$$

After the training of BP neural network based on the sample library is accomplished, all the biases and weights in the BP neural network have been determined, which means that the forward propagation rule f is available at present. The rule f is generally a nonlinear mapping. The known factor of safety is denoted by K^*, and the target entrance and exit positions of the sliding surface are denoted by $S^{\text{in}*}$ and $S^{\text{out}*}$, respectively, thus the inverse analysis of the shear strengths for the three soil layers means that the following equation is to be solved.

$$(K^*, S^{\text{in}*}, S^{\text{out}*}) = f(x^*) \quad (5)$$

where x^* is the shear strength of the three soil layers to be back analyzed.

The factor of safety of the slope and the entrance and exit positions of the sliding surface are known in advance, and the shear strengths of rocks and soils are required to be determined in this study. To achieve this goal, the conventional way to establish BP neural network will set $(K, S^{\text{in}}, S^{\text{out}})$ and $(c_1, \varphi_1, c_2, \varphi_2, c_3, \varphi_3)$ in the input layer and the output layer, respectively. Once the training of neural network is accomplished, x^* will be obtained by directly inputting $(K^*, S^{\text{in}*}, S^{\text{out}*})$, which actually is a forward calculation on the basis of the neural network rule. To avoid resolving the underdetermined problem, this study sets $(c_1, \varphi_1, c_2, \varphi_2, c_3, \varphi_3)$ as the inputs and $(K, S^{\text{in}}, S^{\text{out}})$ as the outputs. This will benefit the numerical stability and accuracy of neural network. When the establishment and training of the BP neural network are completed, x^* will be solved for a given $(K^*, S^{\text{in}*}, S^{\text{out}*})$. To distinguish the proposed approach from the conventional one, the new way to obtain x^* is called "reverse back analysis" in this study.

The proposed reverse back analysis uses an expression similar to the loss scalar of neural network. The following objective function is defined in advance:

$$E^* = \frac{1}{2}\left[(\hat{K}^* - \hat{K})^2 + (\hat{S}^{\text{in}*} - \hat{S}^{\text{in}})^2 + (\hat{S}^{\text{out}*} - \hat{S}^{\text{out}})^2\right] \quad (6)$$

where the mark "^" above each variable means that all the quantities are normalized. The normalization of the quantities in Eq. (6) is executed referring to the trained BP neural network. Thus the original problem can be replaced by an alternative problem that an appropriate x^* to minimize the objective function E^* is to be found based on Eq. (5). The resolution of the new problem in fact is a parameter optimization with a given target based on the trained neural network.

The gradient descent method is used to solve this optimization problem[22] due to the nonlinearity of the forward propagation rule f. First, an initial value x^0 is specified, and it can be taken as the mean value of the laboratory test results of rock and soil shear strengths, or be determined referring to the engineering practices. Then the following computational steps are executed:

(1) Using Eq. (4), an output $(K, S^{\text{in}}, S^{\text{out}})$ is obtained by directly inputting x^i into the BP neural network. At the first step, x^i is equal to x^0.

(2) Using Eq. (6), the objective function E^* is calculated and checked. If its value is less than the set convergence threshold of 10^{-5}, the entire computation is terminated and x^* takes the value of x^i. If not, continue to perform step (3).

(3) Calculating the gradient of the objective function E^* at x^i, and updating the input parameter along the descent direction of E^*. The formulation to update the input parameter is written as

$$\Delta x = -\eta_2 \frac{\partial E^*}{\partial x}\bigg|_{x=x^i}, \; x^i + \Delta x \rightarrow x^{i+1} \quad (7)$$

where Δx is the increment used to update x^i and η_2 is the step length. Δx is obtained by multiplying the negative gradient of the objective function by η_2. On account of the nonlinearity of the forward propagation rule f, the gradient descent method should be performed with a step length η_2. Because the value of η_2 directly affects the computational efficiency, the reasonable value of η_2 is still a matter of concern in the algorithm optimization of the gradient descent method[22]. η_2 is taken as 0.001 in this paper.

(4) Let $i+1=i$ and go back to step (1).

3.2 Error check

As mentioned earlier, forward propagation of the BP neural network can be regarded as a substitution of the calculation process of GeoSlope. Considering that the number of the samples in the library is limited, this substitution is certain to be rough, indicating that the shear strengths obtained by the reverse back analysis may be inaccurate. A step called error check is introduced to judge whether the resulted shear strengths are acceptable or not. Through denoting the shear strengths obtained in the last round of reverse back analysis as x_1, the detailed process to execute the error check is given as follows:

(1) Computing the factor of safety of the slope and acquiring the entrance and exit positions of the sliding surface using Geoslope with the obtained shear strengths x_1. This step can be written as

$$(K_1, S_1^{\text{in}}, S_1^{\text{out}}) = \text{Geoslope}(x_1) \quad (8)$$

where K_1 is the factor of safety obtained by Geoslope; and S_1^{in} and S_1^{out} denote the entrance and exit positions of the sliding surface obtained by Geoslope, respectively.

(2) Comparing the results with the target information to judge whether their errors satisfy the requirements or not. The threshold values of their errors are set as follows: The difference between K_1 and K^* is required to be less than 0.01, the difference between S_1^{in} and $S^{\text{in}*}$ is required to be smaller than 0.2 m, and the difference between S_1^{out} and $S^{\text{out}*}$ is required to be smaller than 0.2 m too.

If the error check is satisfied, the shear strengths obtained by the last round of reverse back analysis are considered as solutions satisfying the requirements. While the failure of the error check indicates that the current BP neural network is not accurate enough to be employed in the reverse back analysis. At the moment, the sample library of the BP neural network should be further augmented to improve the accuracy of forward propagation, which is called "sample correction" in this study.

3.3 Sample correction

In view that Eq. (5) is a nonlinear equation, the BP neural network established can provide a rough search direction for the study. Although the search direction is not accurate enough, the shear strengths x_1 obtained by the last round of the reverse back analysis must be closer to the true solution than the specified initial value x^0. Therefore, sample correction is performed as follows in this study. A new sample composed of the shear strengths x_1 and their corresponding K_1, S_1^{in} and S_1^{out} is added to the sample library, and then the neural network is retrained.

When the retraining of the BP neural network is completed, the forward propagation rule has been altered to f_1. The reverse back analysis in Section 3.1 is used again to solve the following equation:

$$(K^*, S^{\text{in}*}, S^{\text{out}*}) = f_1(x^*) \quad (9)$$

A new combination of shear strengths x_2 will be obtained after solving Eq. (9), and then the error check is performed again. The process of "reverse back analysis–error check–sample correction" is repeated till the error check is satisfied, and then the inverse analysis of mechanical parameters is completed. It is noticed that in each round of the reverse back analysis to solve Eq. (9), the result of the last round is taken as the initial value of the current round. In other words, the initial value x^0 in the reverse back analysis to determine x_i is taken as x_{i-1}. Repeating solution of the shear strengths in multiple rounds of the reverse back analysis is analogous to the iterative solution of the nonlinear equations in mathematics. Considering that, the proposed method is called "an inversion method with reverse iteration and sample correction. The flowchart to perform the proposed inversion method for estimating the shear strengths is plotted in Fig. 3.

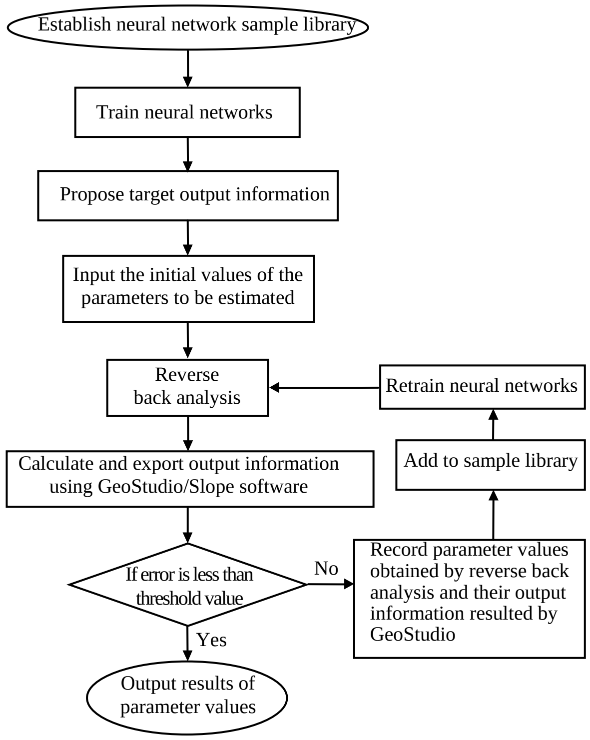

Fig. 3 Flowchart of the proposed method

Each step in the overall process of the proposed inversion method plays different roles. By setting the information to be inversely deduced in the input layer, the BP neural network established in this study avoids the disadvantages of an underdetermined problem. In this case, the true values of the input information are obtained through reverse back analysis according to the known information in the output layer. When the sample number in the library is limited, the forward propagation of the trained BP neural network has an insufficient accuracy for substituting the GeoSlope calculation in the neighborhood of the true solution. Consequently, the accuracy of the reverse back analysis results must be tested using the error check step. When the error check fails, the sample correction will be performed and the BP neural network will be retrained, and then the reverse back analysis will be performed again. The shear strengths obtained in each round of reverse back analysis will be closer to the true solution than the last round. Hence, the repeated sample corrections can be considered as a refinement of the BP neural network in a neighborhood of the true solution. The repeated retraining of the BP neural network actually improves the accuracy of forward propagation as a substitution of the GeoSlope calculation in the neighborhood of the true solution.

3.4 Inversion example

For the model in Fig. 2, if the slope has deformed obviously but is temporarily stable, the factor of safety can be assumed to be 1.02 according to *Code for investigation of geotechnical engineering* (GB50021—2001)[6]. The entrance and exit positions of the sliding surface are given in Table 2.

Table 2 Target factor of safety and sliding surface information

Factor of safety K^*	Entrance position $S^{\text{in}*}$ (x, y) /m	Exit position $S^{\text{out}*}$ (x, y) /m
1.02	(6.19, 15.00)	(13.58, 7.45)

Based on the two sample libraries in Table 1, the shear strengths of the three soil layers in the model are estimated by applying the proposed inversion method. The obtained soil shear strengths of the two libraries both successfully pass the error check after four rounds of reverse back analysis. Table 3 lists the values of the obtained soil shear strengths, the factors of safety of the slope and the entrance and exit positions of the sliding surface obtained by Geoslope for each round. There is little difference in the final results of the soil shear strengths based on sample libraries #1 and #2, indicating that the application of uniform random method to generate the input samples do not affect the ultimate results of the proposed inversion method. During the inversion process, the results in each round of reverse back analysis based on the two sample libraries both gradually approximate to the true solution, but their paths are different due to the difference in their data.

Table 3 Results of shear strength and corresponding factor of safety and sliding surface based on Geoslope

Inversion method	Sample library	Round of reverse back analysis	c_1 /kPa	φ_1 /(°)	c_2 /kPa	φ_2 /(°)	c_3 /kPa	φ_3 /(°)	Factor of safety K	Entrance position S^{in} (x, y) /m	Exit position S^{out} (x, y) /m
The proposed method	#1	1	22.0	22.7	13.9	11.5	22.6	21.1	0.99	(6.74, 15.00)	(11.24, 8.03)
		2	22.2	22.2	14.1	11.8	22.5	20.1	1.05	(6.51, 15.00)	(13.25, 8.01)
		3	22.1	21.0	14.2	11.4	22.1	20.0	1.03	(6.29, 15.00)	(13.56, 7.58)
		4	22.5	20.9	14.2	11.2	22.1	20.0	1.02	(6.20, 15.00)	(13.56, 7.58)
	#2	1	23.5	23.0	15.8	10.6	22.6	21.5	1.00	(5.51, 15.00)	(12.86, 8.73)
		2	23.3	22.2	14.7	11.7	22.5	20.8	1.06	(6.20, 15.00)	(13.38, 7.78)
		3	23.0	20.8	13.4	12.5	20.5	21.6	1.03	(6.18, 15.00)	(13.58, 7.45)
		4	22.5	21.1	14.0	11.0	21.8	20.5	1.02	(6.08, 15.00)	(13.60, 7.41)
Underdetermination method	#1		21.9	23.6	12.3	12.7	22.2	23.4	1.04	(7.57, 15.00)	(12.89, 6.83)
	#2		22.4	24.5	12.4	12.2	21.9	23.9	1.06	(7.36, 15.00)	(13.40, 7.77)
	#1 & #2		21.7	23.4	12.2	12.44	22.3	22.7	1.06	(6.06, 15.00)	(13.60, 7.41)

To reveal the demerits of the BP neural networks to solve the underdetermined problem, an additional BP neural network is established by taking the factor of safety of the slope, the entrance and exit positions of the sliding surface as the inputs and the shear strengths of the three soil layers as the output, respectively. The positions of the known and unknown quantities in this neural network conform to the conventional way to build a BP neural network. At this time, the input layer has three neurons and the output layer has six neurons. Using the same layers and neurons in the hidden layers as those in Section 2, the shear strengths of the three soli layers are estimated. Firstly, this BP neural network is trained based on sample libraries #1 and #2 separately, and the shear strengths are obtained by inputting the known information in Table 2 into the BP neural network. The results of soil shear strengths and the information in terms of the corresponding factor of safety and slip surface by Geoslope are provided in the “under-determined method” row in Table 3. The accuracy of the shear strength results is significantly lower than that of the results obtained by the proposed method, which can be attributed to the demerits of the neural network to solve an underdetermined problem. Secondly, a new sample library #1 & #2 containing 96 samples is formed by combining the sample libraries #1 and #2 together and excluding those reduplicated data, and is used to train this BP neural network. In theory, a sample library of a larger size is capable of improving the accuracy of the BP neural network. The obtained shear strengths listed in Table 3 show that increasing the size of the sample library can improve the accuracy of the shear strength obtained by this BP neural network. But, the result still cannot satisfy the specified requirements due to the effect of the underdetermination. Therefore, for the inversely estimation of the soil shear strengths when the sliding surface crossing multi-layer soils, the novel inversion method proposed in this study has a lower requirement on the sample number of library and a higher accuracy than the neural network defined as the classical approach.

The sliding surfaces corresponding to the obtained soil shear strengths by the methods used in this study are drawn in Fig. 4, for a distinct demonstration of the results difference. After four rounds of reverse back analysis, the soil shear strengths obtained by the proposed method in this study induce a sliding surface in close agreement with the target sliding surface, and the factor of safety obtained is equal to the target one. Therefore, the proposed inversion method is effective and accurate in the inverse analysis of the soil shear strengths in case that the sliding surface passes through multi-layer soils.

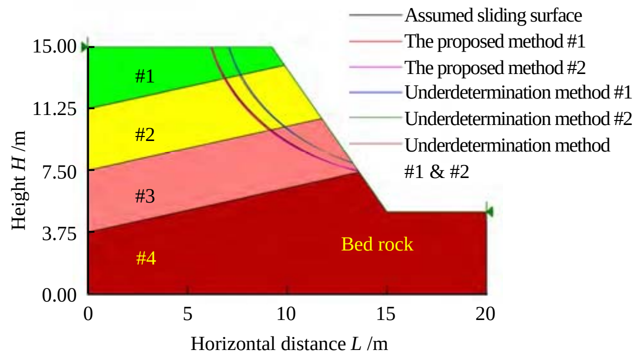

Fig. 4 Sliding surfaces corresponding to mechanical parameters of soil mass estimated by different inversion methods

4 Field application

The slope located on the north side of a planned student apartment in a university has a length of 120 m and a height of 7–10 m. From top to bottom, the slope is composed of miscellaneous fill soil, mucky clay, silty clay, highly weathered sandstone and moderately weathered sandstone. Field investigation found that the slope has deformed significantly before. Figure 5 shows the tensile cracks at the top of the slope and the continuous shear cracks at the toe of the slope. Thus, some engineering measures must be carried out to reinforce the slope.

In the slope investigation, seven sets of soil samples were taken from each representative soil layer in the main deformation range of the slope, which are miscellaneous fill soil, mucky clay and silty clay.

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Their mechanical parameters were measured using laboratory tests. The experimental results of the soil shear strengths had a strong discreteness, and their maximum and minimum values are listed in Table 4. As usually, some unreasonable experimental results were excluded based on the principle of three times mean square deviation when performing the statistic analysis of mechanical parameters for rock and soil. The final statistical results satisfied the requirements that the variation coefficient δ is less than 0.3, and the correction coefficient ψ is larger than 0.75. Ultimately, the standard values of the test results of the shear strengths are also listed in Table 4.

To ensure the reasonability of the values of the shear strength parameters, the mechanical parameters of the three soil layers should be inversely estimated. Though the overall failure of the slope did not occur, the local deformation was apparent, which means that the slope is in the limit equilibrium state currently. By consulting with technicians in the geological exploration unit, the current factor of safety is assumed to be 0.98 for the slope, and the mechanical parameters of soil are inversely estimated. The slope model for the inverse analysis is drawn in Fig. 6.

(a) A photo for the tensile cracks at the slope top

(b) A photo for the continuous shear cracks at the slope toe

Fig. 5 Deformation of the slope studied

Table 4 Test results of shear strength parameters of soils

Soil layer	n	c /kPa					φ /(°)				
		max	min	δ	ψ	Standard value	max	min	δ	ψ	Standard value
Miscellaneous fill soil	7	10	5	0.14	0.92	8	25	18	0.19	0.84	23
Mucky clay	7	28	18	0.15	0.89	22	7	4	0.16	0.88	5
Silty clay	7	52	35	0.13	0.91	39	15	11	0.09	0.94	12

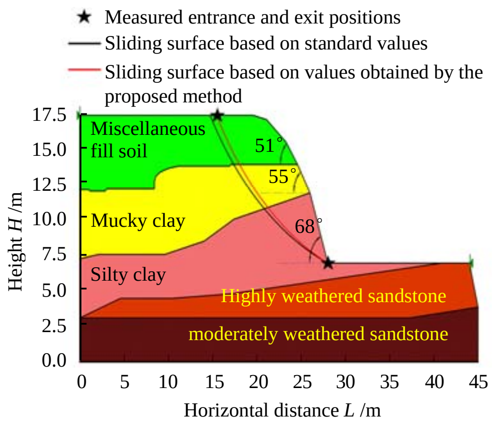

Fig. 6 Calculation model of the slope studied

The locations of the cracks at the top and toe of the slope were measured in the field investigation and marked in the model. The coordinates of the entrance and exit positions of the sliding surface are (15.5 m, 17.5 m) and (28.0 m, 7.0 m), respectively. To establish a BP neural network for inverse analysis of the soil shear strengths, the ranges of the shear strengths are determined by slightly extending the ranges of experimental results listed in Table 4. Taking the standard values of these shear strengths in Table 4 as the initial values for the first round of reverse back analysis, the inversion method proposed in this study is applied to inversely estimating the soil shear strengths, and the results are listed in Table 5 for the three soil layers.

Table 5 Results of shear strength parameters obtained by the proposed inversion method

Soil layer	c /kPa	φ /(°)
Miscellaneous fill soil	7.1	22.3
Mucky clay	21.1	5.3
Silty clay	38.9	12.9

Based on the shear strength parameters listed in Table 5, the slope stability is evaluated using Geoslope. Results show that the factor of safety of the slope is 0.99 and the center of the sliding surface is located in (37.6 m, 31.3 m). The entrance and exit positions are located in (15.44 m, 17.50 m) and (28.02 m, 7.01 m), respectively, which are in close agreement with the locations of those cracks measured in field investigation, as shown in Fig. 6. The slope stability is also assessed using Geoslope with the standard values of the shear strength parameters in Table 4, and the obtained sliding surface is also plotted in Fig. 6. By comparison, the results of the soil shear strength parameters estimated by the proposed method obviously match better with the current stability state and deformation feature of the slope than that of the standard values. Therefore, the shear strength parameters obtained by the proposed inversion method are more reliable for the slope reinforcement design.

5 Conclusions

To solve the difficulty in inverse analysis of the rock and soil shear strength when the sliding surface passes through multi-layer rocks and soils, an inversion method with reverse iteration and sample correction is proposed in this study, and successfully applied in an engineering case. The following conclusions can be drawn:

(1) In the inverse analysis of various parameters using BP neural networks, more attention should be paid to the dimensionalities of the input and output information. An underdetermined problem must be avoided when establishing the BP neural network.

(2) The proposed inversion method based on the BP neural network is effective in inverse analysis of the shear strength parameters of the rocks and soils. The obtained shear strength parameters match well with the stability state and deformation feature of the slope. Thus, the proposed inversion method is capable of providing reasonable shear strength parameters for the rocks and soils, which can benefit the reinforcement design for small-scale slopes.

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