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## Calculation method and analysis of horizontal frost heave effect of L-shaped retaining wall in permafrost regions

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**Abstract:** In view of the imperfection of the calculation method of horizontal frost heaving force in the design of L-shaped retaining wall in permafrost area, based on the Lifkin foundation model and the cooperative deformation principle between retaining wall and soil, a simplified calculation model of horizontal frost heave effect of L-shaped retaining wall with or without soil replacement behind the wall is established, and the proposed calculation model is solved by superposition principle and finite difference method. Furthermore, the water-thermal-mechanical coupling analysis software is developed by MATLAB. Combined with an engineering example, the horizontal frost heaving force obtained by using the proposed calculation method is compared with the field measured value, the corrected earth pressure value, the code empirical value and the simulation value of the coupling software. The results show that: (1) the horizontal frost heaving force obtained by the proposed calculation method is in good agreement with the field measured value and numerical simulation value, while the corrected earth pressure value and the code empirical value underestimate the effect of horizontal frost heaving force on the retaining wall; (2) compared with the code empirical value and field measured value, the horizontal frost heaving force obtained by the proposed calculation method presents two distribution modes of parabola and trapezoid along the wall height, which is more general; (3) the multifield coupling analysis shows that the frost heaving force trend obtained by the proposed calculation method is similar to the trend obtained by the coupling method, which indicates the feasibility of the calculation method and can provide some theoretical support and guidance for the design of L-shaped retaining wall in permafrost area.

**Keywords:** L-type retaining wall; horizontal frost-heaving forces; frost heave; deformation compatibility; finite difference method

### 1 Introduction

With the rapid development of economic construction in cold regions, more and more railways and highways are being built in permafrost regions, exposing frozen slope works to natural environment<sup>[1]</sup>. These frozen slopes are prone to collapse and damage due to freezing and thawing. To make them safe and stable, the L-shaped retaining wall has become an indispensable support structure in the roadbed and slope along the Qinghai–Tibet Railway due to its advantages of light weight, flexibility and prefabrication<sup>[2]</sup>. However, under the background of global warming and the continuous degradation of permafrost table, the L-shaped retaining wall is seriously damaged in practical application, which causes not only the loss of national economy, but also the waste of design and materials. Fundamentally, the damage of L-shaped retaining wall is mainly caused by frost heaving force—it is also the most prominent external load on other buildings in permafrost area<sup>[3]</sup>.

Existing studies show that the horizontal frost heaving force generated by the frozen fill behind the L-shaped retaining wall is generally several times or even dozens of times of its active earth pressure, and the frost heaving force is related to constraints. The greater the external constraints, the greater the frost heaving force<sup>[4]</sup>. Therefore, the key to design and calculation of L-shaped retaining wall in permafrost region is to determine the horizontal frost heaving force of retaining wall.

To fully understand the frost heaving characteristics of L-shaped retaining wall and its stability during freezing–thawing in permafrost area, substantial laboratory tests and field measurements have been carried out on L-shaped retaining wall. Sheng<sup>[5]</sup> took the retaining wall as a cantilever beam and assumed that the horizontal frost heaving force was distributed in an inverted triangle to check the stability of the retaining wall, indicating that the horizontal frost heaving force

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played a controlling role in the stability calculation of the retaining wall. Liang et al.<sup>[6–7]</sup> conducted field tests on the L-shaped retaining wall and found that the coarse filling behind the wall hardly produced frost heaving force, and proposed earth pressure modification models with and without the influence of permafrost frost heaving considered. Zhang<sup>[8]</sup> simulated the horizontal frost heaving force of shallow-buried buildings based on the existing temperature field, and concluded that the horizontal frost heaving force presented an approximate parabolic distribution along the height of buildings. Hu<sup>[9]</sup> used finite element analysis software to simulate the frost heaving force of L-shaped retaining wall under different stiffness and constraints, and pointed out that the greater the constraints on the retaining wall, the greater the frost heaving force. Guan et al.<sup>[10]</sup> analyzed the influencing factors of horizontal frost heaving force, summarized the distributions of horizontal frost heaving force under different factors, and proposed methods to eliminate or weaken horizontal frost heaving force in different ways. Sui et al.<sup>[11]</sup> provided the design strength diagram of horizontal frost heaving force of the cantilever retaining wall, which provided the basis for the design load of retaining wall in seasonal frost area. Zhang<sup>[12]</sup> verified the rationality of the distribution of horizontal frost heaving force by carrying out laboratory tests, established a numerical model of laboratory tests, analyzed the temperature field and horizontal frost heaving force of retaining wall, and summarized the development law of horizontal frost heaving force at different heights.

In the above studies, although scholars have made detailed analyses of the influence factors, distribution rules and weakening methods of horizontal frost heaving force of L-shaped retaining wall, and combined with numerical simulation, laboratory tests and field observations, there are still two key issues: (1) From the perspective of supporting effect, L-shaped retaining wall will still be damaged in large numbers under freeze-thaw action, resulting in huge economic losses. The main reason is the limitation of quantitative evaluation of the frost heaving force of L-shaped retaining wall. (2) There are many distributions of horizontal frost heaving force given in the code and field measurement, each of which has its limitations. The success or failure of design and calculation depends on the rationality of the distribution selected in the specific engineering application. And a unified calculation method of frost heaving force has not yet been found. Therefore, it is of great guiding significance to theoretically determine the value of horizontal frost heaving force of retaining wall for slope support

engineering in frozen area.

In this paper, the calculation model of L-shaped retaining wall in permafrost area is established based on Lifkin model and solved by finite difference method with the combination of the cooperative deformation principle between retaining wall and soil. And the calculation software is developed by MATLAB. By comparing with field measured values, the rationality of the proposed calculation method of horizontal frost heaving force of L-shaped retaining wall is analyzed. It is expected to provide a useful reference for the design and calculation of L-shaped retaining wall and similar support structures in permafrost regions.

## 2 Calculation model

### 2.1 Assumptions

In slope support engineering, the longitudinal dimension of L-shaped retaining wall is much larger than the transverse dimension, which accords with the assumption of the plane strain problem. In this study, the L-shaped retaining wall is simplified as a vertically placed elastic foundation beam, the continuity of foundation soil and the interaction between retaining wall and soil are considered, and the Lifkin model is introduced. To simplify the calculation, the following assumptions are made:

- (1) The backfill and retaining wall are isotropic homogeneous elastic.
- (2) The bottom of the wall is regarded as the fixed end, and the horizontal frost heaving forces on the toe and heel of the wall cancel each other out.
- (3) In calculation, the retaining wall is always in close contact with the soil, and there is no separation between the wall and soil, that is, the compatibility of deformation is satisfied.
- (4) The influence of the self-weight of retaining wall on interaction is ignored.

### 2.2 Coordination of frost heave deformation

Normally, free frost heave occurs in natural soil due to freezing, and constrained frost heave occurs in soil behind the wall due to the restriction of retaining wall under the support of L-shaped retaining wall. In this case, the actual frost-heave capacity of soil behind the wall is the free frost-heave capacity minus the constrained frost-heave capacity. The frost heave deformation of the retaining wall and soil is shown in Fig. 1.

In Fig. 1, I is the ground before freezing, II is the ground after free frost heave of soil without support structure, III is the ground after freezing with support structure,  $u$  is the deformation amount of retaining wall, and  $u_s$ ,  $u'$ ,  $\Delta u$  are respectively the free frost-heave capacity, the actual frost-heave capacity and the

constrained frost-heave capacity of soil behind wall. The deformation coordination relationship between retaining wall and soil behind wall is as follows [13]:

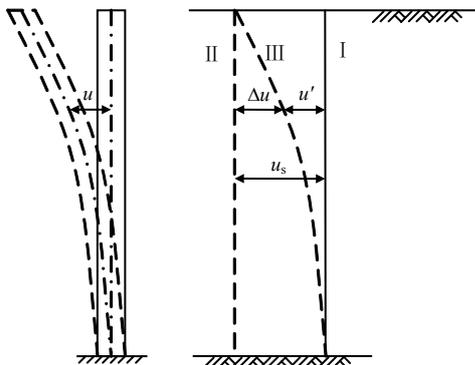


Fig. 1 Frost heave deformation diagram

$$u = u' = u_s - \Delta u \quad (1)$$

Usually, coarse soil without frost heave or with weak frost heave will be replaced behind the L-shaped retaining wall during construction. Compared with the L-shaped retaining wall without soil replacement, the frost heaving force on L-shaped retaining wall after soil replacement is obviously different. Therefore, the calculation model of horizontal frost heaving force is established according to the two working conditions of whether there is coarse fill behind the wall.

### 2.3 Calculation model without soil replacement

According to the above assumptions and analysis, when the soil behind the wall is subjected to frost heave deformation, both the retaining wall and the soil are deformed, and the deformation between them satisfies the deformation compatibility. The L-shaped retaining wall is simplified as an Euler-Bernoulli beam with one end fixed and the other free. The Lifkin model is used to consider the continuity of soil behind the wall. The simplified model of L-shaped retaining wall is shown in Fig. 2.

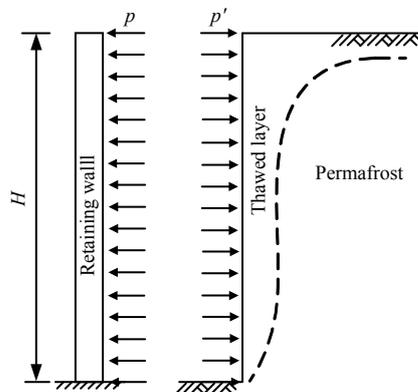


Fig. 2 Diagram of retaining wall support system

As shown in Fig.2, the frost heaving force  $p$  acting on the simplified model of L-shaped retaining wall

and the frost heaving force  $p'$  generated by the fill behind the wall are a pair of acting force–reactive force.

$$p = -p' \quad (2)$$

For the Euler-Bernoulli beam shown in Fig. 2, the basic differential equation is as follows:

$$E_w I_w \frac{d^4 u}{dy^4} = -p \quad (3)$$

where  $p$  is the frost heaving force acting on the simplified model of retaining wall;  $E_w$  is the elastic modulus of retaining wall; and  $I_w$  is the moment of inertia of retaining wall.

When the frost-heaving soil with behind the wall is not replaced by coarse-grained soil, the back of the retaining wall directly contacts the frost-heaving soil. To fully consider the interaction between soil and the simplified model of L-shaped retaining wall, the Lifkin model is introduced to analyze the fill behind the wall. The model calculation diagram is shown in Fig.3.

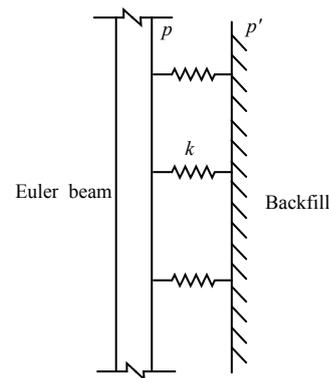


Fig. 3 Calculation diagram of micro element section of retaining wall

The Lifkin equation of soil behind the wall is as follows:

$$p' = k' [1 + \beta e^{-\alpha(1-\xi)}] b \Delta u \quad (4)$$

$$k' = k \frac{\zeta + 0.5}{1.5\zeta} \quad (5)$$

where  $k$  is the coefficient of subgrade reaction;  $k'$  is the modified coefficient of subgrade reaction;  $\zeta$  is the influence coefficient of the shape of retaining wall;  $\alpha$  and  $\beta$  are dimensionless parameters of foundation, which are associated to soil properties; and  $\xi$  is the relative coordinates of the points considered on the interface,  $\xi = y/b$ ,  $b$  is the width of the retaining wall.

By combining Eqs. (1)–(4), the simplified model of L-shaped retaining wall without coarse fill behind the wall can be obtained:

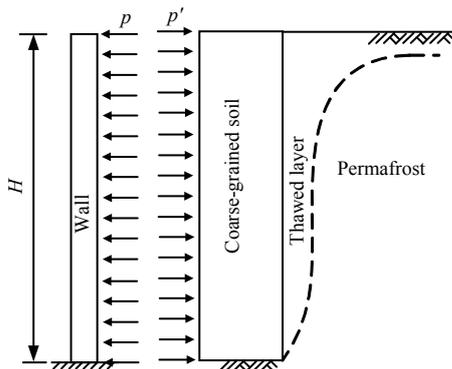
$$E_w I_w \frac{d^4 u}{dy^4} + k' [1 + \beta e^{-\alpha(1-\xi)}] b(u_s - u) = 0 \quad (6)$$

$\psi$  is defined as the Lifkin coefficient, and let

$$\psi = k' [1 + \beta e^{-\alpha(1-\xi)}] \quad (7)$$

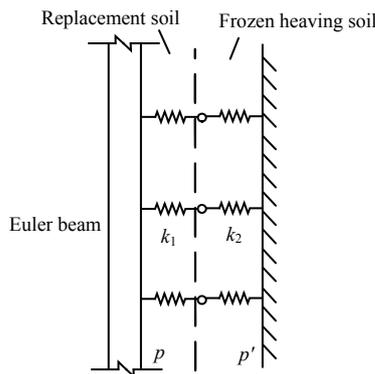
**2.4 Calculation model with soil replacement**

When part of the soil behind the wall is replaced with coarse-grained soil, it is equivalent to adding a heave-reducing layer between the retaining wall and the frost-heaving soils. The schematic diagram of retaining wall support is shown in Fig. 4.



**Fig. 4 Schematic diagram when replaced soil is behind the wall**

The replacement soil behind the wall and the seasonal thawing layer are simplified as a double-layer medium, and the double-layer medium is abstracted into two series springs. The calculation model is shown in Fig. 5.



**Fig. 5 Calculation model diagram when the replaced soil is behind the wall**

To consider the interaction between the two layers of different soils behind the wall, the coefficients of subgrade reaction  $k_1$  and  $k_2$  of the two layers of soil are firstly modified according to the shape of the retaining wall:

$$k'_1 = k_1 \frac{\zeta + 0.5}{1.5\zeta} \quad (8)$$

$$k'_2 = k_2 \frac{\zeta + 0.5}{1.5\zeta} \quad (9)$$

where  $k_1$  is the coefficient of subgrade reaction of the coarse-grained soil;  $k_2$  is the coefficient of subgrade reaction of soil of the active layer.

Then, according to the calculation method of the stiffness coefficient of series spring,  $k^*$ —the equivalent coefficient of subgrade reaction of the soils behind the wall—can be obtained as follows:

$$k^* = \frac{k'_1 k'_2}{k'_1 + k'_2} \quad (10)$$

Equation (10) describing the equivalent coefficient of subgrade reaction in the case of soil replacement behind the wall is substituted into Eq. (4) to obtain frost heaving force  $p'$ , and then combined with Eq. (3), the basic equation with soil replacement can be obtained as follows:

$$E_w I_w \frac{d^4 u}{dy^4} + k^* [1 + \beta e^{-\alpha(1-\xi)}] b(u_s - u) = 0 \quad (11)$$

**3 Solution of model**

**3.1 Determination of free frost-heave capacity**

It can be seen from Eqs. (6) and (11) that the horizontal free frost heave deformation  $u_s$  should be determined first, and then be substituted into Eqs. (6) and (11) to obtain the horizontal displacement of L-shaped retaining wall caused by frost heave. Therefore, how to determine the free frost heave deformation is in priority to be solved.

**3.1.1 Without soil replacement**

When there is no soil replacement behind the wall, the thickness of the active layer can be calculated as below according to the thermal characteristics of the soil freezing<sup>[4]</sup>:

$$Z_d = 0.95 (\sum T_t)^{1/2} + 0.882 \quad (12)$$

where  $\sum T_t$  is the standard value of the thawing index, which is generally set according to the code.

For the permafrost slope, the water distribution in the soil is uneven due to gravity. Without considering water migration, The free frost heave of the soil is the in situ frost heave, which can be obtained according to the unfrozen water content of soil:

$$u_s = 0.09 (\omega_0 - \omega_u) Z_d \quad (13)$$

where  $\omega_0$  is the initial water content of soil;  $\omega_u$  is unfrozen water content; and  $Z_d$  is the depth of seasonal thawing layer.

The unfrozen water content of soil is mainly affected by soil property, temperature and external conditions, among which the influence of temperature is the most basic. And the basic equilibrium equation is as follows:

$$\omega_u = \omega_0 (T_f / T_s)^B \tag{14}$$

where  $T_f$  is the absolute value of the freezing temperature of soil;  $T_s$  is the absolute value of the frozen soil temperature; and  $B$  is the parameter related to the soil property.

3.1.2 With soil replacement

When there is soil replacement behind the wall, the coarse-grained soil is regarded as an insulation layer, and Eq. (12) is no longer available. And the thickness of the active layer considering thermal resistance of the insulation layer is calculated as below:

$$Z_d^* = \left[ \frac{2\kappa_t \sum T_t}{L_0} + S^2 \right]^{1/2} - S \tag{15}$$

where  $\kappa_t$  is the thermal conductivity of soil;  $L_0$  is the phase transition heat of water; and  $S$  is the equivalent thickness of the thermal resistance of soil surface:

$$S = \kappa_t \left( \frac{1}{\chi} + \frac{Z_s}{\kappa_s} \right) \tag{16}$$

where  $\kappa_s$  is the thermal conductivity of the coarse-grained soil;  $\chi$  is the exothermic coefficient of soil; and  $Z_s$  is the thickness of coarse-grained soil.

That is, the free frost-heave capacity in the case of replacement soil is

$$u_s^* = 0.09(\omega_0 - \omega_u)Z_d^* \tag{17}$$

3.2 Solution of frost heaving force and frost-heave capacity without soil replacement

For Eq. (6), due to the existence of relative coordinates  $\xi = y / b$ , Lifkin coefficient changes with the height of retaining wall. This equation is a fourth-order variable-coefficient differential equation, which is difficult to obtain completely through analytical methods. Therefore, the free frost-heave capacity is calculated by Eq. (13), and the finite difference method is used to solve the deformation of the retaining wall, and then the internal force induced by frost heave is obtained. As shown in Fig. 6, the height of the beam on the foundation is set as  $H$ , and it is divided into  $n$  sections according to equal length  $\lambda$  from the bottom to the top of the beam. For the convenience of calculation, two virtual nodes are added at each end of the beam, and the length between the virtual nodes is  $\lambda$ . At point  $i$ , the horizontal displacement of the beam is  $u_i$ .

Taylor expansion is carried out for the five interpolation points before and after the deflection of the beam, and one-dimensional center interpolation method is adopted to obtain the difference format:

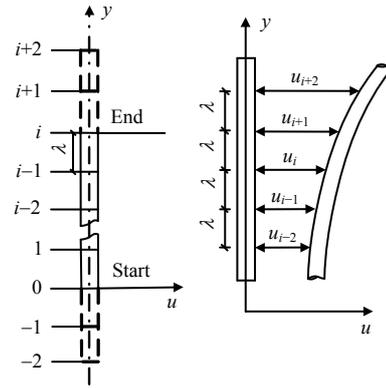


Fig. 6 Beam deflection and differential node diagram

$$\left. \begin{aligned} \left( \frac{du}{dy} \right)_i &= \frac{u_{i+1} - u_i}{\lambda} \\ \left( \frac{d^2u}{dy^2} \right)_i &= \frac{u_{i-1} - 2u_i + u_{i+1}}{\lambda^2} \\ \left( \frac{d^3u}{dy^3} \right)_i &= \frac{-u_{i-2} + 2u_{i-1} - 2u_{i+1} + u_{i+2}}{2\lambda^3} \\ \left( \frac{d^4u}{dy^4} \right)_i &= \frac{u_{i-2} - 4u_{i-1} + 6u_i - 4u_{i+1} + u_{i+2}}{\lambda^4} \end{aligned} \right\} \tag{18}$$

Substituting Eq. (18) into Eq. (6), the controlling finite difference equation at any node can be obtained as follows:

$$E_w I_w \left( \frac{u_{i-2} - 4u_{i-1} + 6u_i - 4u_{i+1} + u_{i+2}}{\lambda^4} \right) + \psi_i (u_s - u_i) b = 0 \tag{19}$$

L-shaped retaining wall is a beam with free top and fixed bottom, so the boundary conditions are as follows:

$$\left. \begin{aligned} \frac{d^2u}{dy^2} \Big|_{y=H} = 0 ; \quad \frac{d^3u}{dy^3} \Big|_{y=H} = 0 \\ u \Big|_{y=0} = 0 ; \quad \frac{du}{dy} \Big|_{y=0} = 0 \end{aligned} \right\} \tag{20}$$

For node  $i = 0$ , considering the endpoint condition, the equations are formulated as follows:

$$\left( \frac{d^4u}{dy^4} \right)_{i=0} = \frac{16u_{i-1} - 9u_i + \frac{8}{3}u_{i+1} - \frac{1}{4}u_{i+2}}{\lambda^4} \tag{21}$$

For nodes  $i = n - 1$  and  $i = n$ , the following equations are formulated as

$$\left( \frac{d^4u}{dy^4} \right)_{i=n-1} = \frac{16u_{i-3} - 60u_{i-2} + 72u_{i-1} - 28u_i}{17\lambda^4} \tag{22}$$

$$\left( \frac{d^4u}{dy^4} \right)_{i=n} = \frac{-12u_{i-3} + 96u_{i-2} - 156u_{i-1} + 72u_i}{17\lambda^4} \tag{23}$$

And  $n + 1$  equations can be derived from the difference governing equations of each node, and the finite difference equations can be expressed as

$$Cu = B \tag{24}$$

with

$$B = -\frac{\lambda^4 b}{E_w I_w} u_s \begin{bmatrix} \psi_0 & & & & & & & & & 0 \\ & \psi_1 & & & & & & & & \\ & & \ddots & & & & & & & \\ & & & \psi_i & & & & & & \\ & & & & \ddots & & & & & \\ & & & & & \psi_{n-1} & & & & \\ 0 & & & & & & & & & \psi_n \end{bmatrix} \tag{25}$$

$$u = \{u_0, u_1, u_2, u_3, \dots, u_{n-1}, u_n\}^T \tag{26}$$

$$C = \begin{pmatrix} 16 & -9 & \frac{8}{3} & -\frac{1}{4} & 0 & 0 & \dots & 0 & 0 \\ -4 & a & -4 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & -4 & a & -4 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -4 & a & -4 & 1 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & 0 & 1 & -4 & a & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & a & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{16}{17} & -\frac{60}{17} & \frac{72}{17} & -\frac{28}{17} \\ 0 & 0 & 0 & 0 & 0 & \frac{12}{17} & \frac{96}{17} & -\frac{156}{17} & \frac{72}{17} \end{pmatrix} \tag{27}$$

$$\text{with } a = 6 - \frac{\lambda^4 b}{E_w I_w} \psi_i.$$

Assuming that the reaction force at the bottom of each beam is applied to node  $i$ , the average frost heave reactive force  $R_i$  at node  $i$  is:

$$R_i = b\lambda p_i = b\lambda \psi_i (u_s - u_i) \tag{28}$$

Based on the Eq. (6), Eq. (27) can be expressed as the following matrix equation:

$$R = K_S u_s - K_S u \tag{29}$$

where  $K_S$  is the stiffness matrix of Lifkin foundation, which can be represented by the following  $n+1$  order diagonal matrix.

$$K_S = b\lambda \begin{bmatrix} \psi_0 & & & & & & & & & 0 \\ & \psi_1 & & & & & & & & \\ & & \ddots & & & & & & & \\ & & & \psi_i & & & & & & \\ & & & & \ddots & & & & & \\ & & & & & \psi_{n-1} & & & & \\ 0 & & & & & & & & & \psi_n \end{bmatrix} \tag{30}$$

### 3.3 Solution of frost heaving force and frost-heave capacity with soil replacement

When there is coarse-grained soil behind the wall, the solution method of Eq. (11) is similar to that in the case of no soil replacement. According to Eq. (17), the free frost heave of soil with soil replacement is calculated, and the simplified finite difference format of beam of retaining wall with soil replacement is consistent with that without soil replacement. According to Eq. (11) and Eq. (18), the controlling finite difference equation at any node in the case of no soil replacement is

$$E_w I_w \left( \frac{u_{i-2} - 4u_{i-1} + 6u_i - 4u_{i+1} + u_{i+2}}{\lambda^4} \right) + \psi_i^* (u_s - u_i) b = 0 \tag{31}$$

The coefficient matrices  $C$  and  $B$  in Eq. (24) are

$$B = -\frac{\lambda^4 b}{E_w I_w} u_s \begin{bmatrix} \psi_0^* & & & & & & & & & 0 \\ & \psi_1^* & & & & & & & & \\ & & \ddots & & & & & & & \\ & & & \psi_i^* & & & & & & \\ & & & & \ddots & & & & & \\ & & & & & \psi_{n-1}^* & & & & \\ 0 & & & & & & & & & \psi_n^* \end{bmatrix} \tag{32}$$

$$C = \begin{pmatrix} 16 & -9 & \frac{8}{3} & -\frac{1}{4} & 0 & 0 & \dots & 0 & 0 \\ -4 & a^* & -4 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & -4 & a^* & -4 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -4 & a^* & -4 & 1 & \dots & 0 & 0 \\ \vdots & \vdots \\ 0 & 0 & 0 & 1 & -4 & a^* & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & a^* & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{16}{17} & -\frac{60}{17} & \frac{72}{17} & -\frac{28}{17} \\ 0 & 0 & 0 & 0 & 0 & \frac{12}{17} & \frac{96}{17} & -\frac{156}{17} & \frac{72}{17} \end{pmatrix} \tag{33}$$

$$\text{with } a^* = 6 - \frac{\lambda^4 b}{E_w I_w} \psi_i^*.$$

Similarly, the average frost heave reactive force  $R_i^*$  at node  $i$  of each section of L-shaped retaining wall with soil replacement can be obtained as follows:



diagram and formula are shown as follows<sup>[6]</sup>:

$$\sigma_{\max} = \xi K_0 \gamma H \quad (39)$$

where  $\xi$  is a coefficient modified with the shape of the replacement surface, generally 1.0–1.5;  $\gamma$  is the unit weight of fill;  $H$  is the height of retaining wall;  $K_0$  is the coefficient of earth pressure at rest,  $K_0 = 1 - \sin \varphi$  or  $K_0 = \mu / (1 - \mu)$ ,  $\mu$  and  $\varphi$  are Poisson's ratio and internal friction angle of replaced soil respectively.

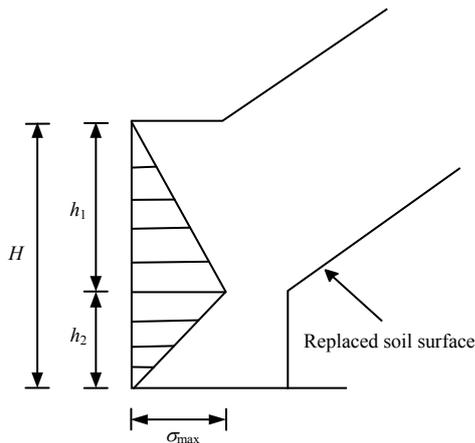


Fig. 7 Revised earth pressure distribution

According to geological conditions, retaining structure and fill type, relevant parameters in the earth pressure modification model are as follows:  $H=5$  m,  $\mu = 0.28$ ,  $\varphi=40^\circ$ ,  $\gamma=19$  kN/m<sup>3</sup>,  $\xi=1.25$ .

#### 4.4 Code empirical method

According to Section 8.2.11 of *Code for design of soil and foundation of building in frozen soil region* (JGJ118–2011)<sup>[17]</sup>, the magnitude and distribution of horizontal frost heaving stress acting on the back of the wall should be determined by field test. When field test cannot be carried out, the distribution and magnitude of frost heaving force along the wall height of L-shaped retaining wall can be taken as specified in the Code<sup>[17]</sup>. Since the fill in this calculation sample is coarse, combined with geological conditions, retaining structure and Literature [7], it can be deemed that the distribution of frost heaving force is a right triangle, and the maximum horizontal frost heaving force is 107 kPa, as shown in Fig. 8.

#### 4.5 Comparative analysis and verification of the results from different methods

Combined with the geological conditions, retaining structure and fill types of engineering examples, the horizontal frost heaving force or earth pressure at the wall back in cold regions is obtained by the calculation method proposed in this paper, earth pressure modification model in Literature [2, 6], method in the Code<sup>[17]</sup> and field measurement, as shown in Fig. 9.

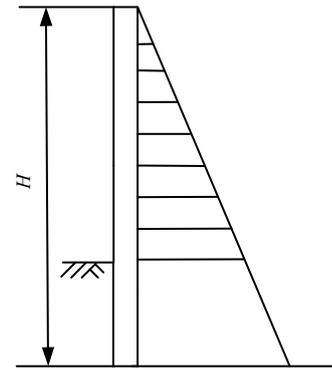


Fig. 8 Distribution of frost heaving force in the Code<sup>[17]</sup>

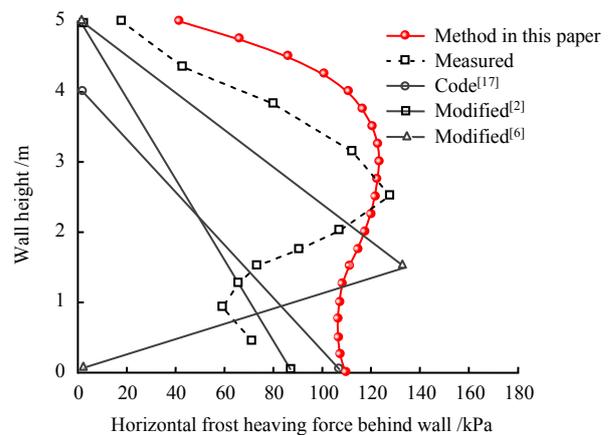


Fig. 9 Comparison of horizontal frost heaving force (earth pressure) behind the wall

It can be seen from Fig. 9 that the horizontal frost heaving force calculated by the method proposed in this paper presents a parabolic distribution along the wall height, and the maximum horizontal frost heaving force 126 kPa appears at 3 m (2/3 height from the bottom of the wall). Meanwhile, the horizontal frost heaving force obtained by this method is consistent with the measured value<sup>[6]</sup>, and the error range between the measured value and the theoretical value of the maximum frost heaving force is no more than 10%, indicating that the horizontal frost heaving force calculated by the proposed method is reasonable for retaining wall design as the design load. However, the distribution of horizontal frost heaving force behind the retaining wall obtained by the proposed method differs from that of the modified earth pressure. The maximum earth pressure obtained by the two modified earth pressure models appears at 1/3 height from the bottom of the retaining wall or near the heel of the wall, which is obviously different from the measured value. The extremum of the horizontal frost heaving force estimated in this paper is larger than the value specified in the Code, and the action points of the maximum horizontal frost heaving forces estimated by these two methods are different. The maximum value

of the horizontal frost heaving force specified in the Code is at the bottom of the wall, while the maximum value of the frost heaving force calculated in this paper is at 2/3 height from the bottom of the wall. Therefore, it can explain the reason why the retaining wall is still damaged when designed in compliance with the Code.

To further verify the rationality of the calculation model of horizontal frost heaving force of retaining wall proposed in this paper, the proposed method is used to calculate the retaining wall measured by Northwest Research Institute Co., Ltd. Of C.R.E.C [4,18]. The comparison of calculation results, Code empirical values and field measured values is shown in Fig. 10. It shows that the distribution of the measured horizontal frost heaving force is roughly triangle, and the maximum value is 253 kPa; the earth pressure distribution estimated by the proposed method is approximately trapezoidal, and the extremum appears at the bottom of the wall, which is 245 kPa. The difference between the measured maximum value of horizontal frost heaving force and the calculated maximum value is 8 kPa, within the allowable error range, indicating that the method proposed is reasonable for the calculation and analysis of horizontal frost heaving force. However, the maximum horizontal frost heaving force given in the Code is 125 kPa, which is significantly 102.4% smaller than the measured maximum value. Therefore, the retaining wall is also very likely to be damaged if designed in compliance with the empirical value given by the Code.

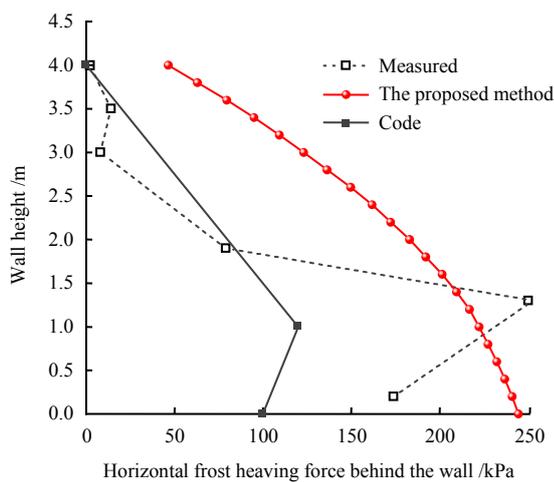


Fig. 10 Comparison of horizontal frost heaving force (earth pressure) behind the wall

### 5 Numerical simulation and verification

To further understand the working characteristics of L-shaped retaining wall under frost heave effect and the rationality of the proposed frost heaving calculation model of retaining wall, it is analyzed by the self-developed hydro-thermal-mechanical coupling software

to further verify the feasibility of the proposed calculation method.

#### 5.1 Governing equation

(1) Temperature field equation

$$C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + L \rho_i \frac{\partial \theta_i}{\partial t} \quad (40)$$

where  $C$  is the volume specific heat capacity of soil;  $T$  is the soil temperature;  $t$  is the time;  $\lambda$  is the thermal conductivity of soil;  $L$  is the latent heat of phase transition;  $\rho_i$  is the density of ice; and  $\theta_i$  is the ice content.

(2) Moisture field equation

$$\frac{\partial \theta_u}{\partial t} = \frac{\partial}{\partial x} \left( D \frac{\partial \theta_u}{\partial x} \right) + \frac{\partial}{\partial y} \left( D \frac{\partial \theta_u}{\partial y} \right) - \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t} \quad (41)$$

where  $\theta_u$  is the volume content of unfrozen moisture;  $D$  is the moisture diffusion coefficient; and  $\rho_w$  is the density of water.

The relation equation between unfrozen moisture content and temperature in frozen soil is

$$\theta_u = \begin{cases} a |T|^{-b} & T < 0 \\ \theta_w & T \geq 0 \end{cases} \quad (42)$$

where  $a$  and  $b$  are parameters related to soil properties;  $\theta_w$  is the total moisture content.

Substituting Eq. (40) into Eq. (42) and combining with hydrothermal Eq. (42) yields the governing equation of hydrothermal coupling:

$$C^* \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda^* \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda^* \frac{\partial T}{\partial y} \right) \quad (43)$$

where  $C^*$  is the specific heat capacity of soil with phase transition considered,  $C^* = C + L \rho_w \frac{\partial \theta_u}{\partial t}$ ;  $\lambda^*$  is the thermal conductivity of soil with phase transition considered,  $\lambda^* = \lambda + L \rho_w D \frac{\partial \theta_u}{\partial t}$ .

(3) Stress field equation

$$\sigma = D (\epsilon - \epsilon^v) \quad (44)$$

where  $\sigma = (\sigma_x, \sigma_y, \tau_{xy})^T$ ,  $\sigma_x, \sigma_y, \tau_{xy}$  are the stress component;  $D$  is the elastic matrix;  $\epsilon = (\epsilon_x, \epsilon_y, \epsilon_z)^T$ ,  $\epsilon_x, \epsilon_y, \epsilon_z$  are the stress component caused by self-weight;  $\epsilon^v = (\epsilon_x^v, \epsilon_y^v, \epsilon_z^v)^T$ ,  $\epsilon_x^v, \epsilon_y^v, \epsilon_z^v$  are the stress component caused by frost heave during freezing.

(4) Coupling of retaining wall structure with soil

The retaining wall is regarded as a beam element, and the combined type is adopted to consider the contribution of the support structure to the soil element [19]. The soil adopts a triangular finite element mesh, so the displacement of any point in the element can be expressed as

$$\begin{bmatrix} u \\ v \end{bmatrix} = \sum_{i=1}^3 \begin{bmatrix} N_i & 0 \\ 0 & N_i \end{bmatrix} \begin{bmatrix} u_i \\ v_i \end{bmatrix} \quad (45)$$

In the case of small rotation, the relationship between the node displacement of triangular element and that of retaining wall beam element is

$$\begin{bmatrix} u \\ v \\ \omega \end{bmatrix} = \begin{bmatrix} N_1 & 0 & \dots & N_n & 0 \\ 0 & N_1 & \dots & 0 & N_n \\ -\frac{\partial N_1}{2\partial y} & \frac{\partial N_1}{\partial x} & \dots & -\frac{\partial N_n}{2\partial y} & \frac{\partial N_n}{\partial x} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ \vdots \\ u_n \\ v_n \end{bmatrix} = \mathbf{S} \boldsymbol{\delta}_p \quad (46)$$

where  $\omega$  is the rotation angle of a certain point in the element;  $\mathbf{S}$  is the relationship between the displacement and rotation angle of a certain point in the retaining wall beam and the displacement of each node of the triangular element; and  $\boldsymbol{\delta}_p$  is the node displacement of triangular element.

Let  $\mathbf{k}_s$  be the stiffness matrix of soil triangular element without retaining wall,  $m$  and  $l$  are the two node labels of beam element;  $\mathbf{k}_w$  is the stiffness matrix of retaining wall beam element, and  $\bar{\mathbf{k}}_w$  is the stiffness contribution of the structural beam element of retaining wall to soil triangular element, which can be obtained by virtual work principle:

$$\bar{\mathbf{k}}_w = \begin{bmatrix} \mathbf{S}_m \\ \mathbf{S}_l \end{bmatrix}^T \mathbf{k}_w \begin{bmatrix} \mathbf{S}_m \\ \mathbf{S}_l \end{bmatrix} \quad (47)$$

Thus, the stiffness matrix of the combined element of retaining wall beam element and soil triangular element can be obtained:

$$\mathbf{k} = \bar{\mathbf{k}}_w + \mathbf{k}_s \quad (48)$$

Combined with Eq. (48), the incremental form of the global equilibrium equation of the cooperation between retaining wall structure and soil can be written:

$$\mathbf{k} \Delta \boldsymbol{\delta} = \Delta \mathbf{F}_V + \Delta \mathbf{F}_T \quad (49)$$

where  $\mathbf{k}$  is the global stiffness matrix after assembly;  $\Delta \boldsymbol{\delta}$  is the displacement increment of the structure in the current time step; and  $\Delta \mathbf{F}_V$ ,  $\Delta \mathbf{F}_T$  are the virtual equivalent nodal load increment generated by the self-weight elastic strain increment  $\Delta \boldsymbol{\varepsilon}_V$  and frost heave strain increment  $\Delta \boldsymbol{\varepsilon}_T$  respectively, with:

$$\left. \begin{aligned} \mathbf{k} &= \sum \int \mathbf{B}^T \mathbf{D} \mathbf{B} dV \\ \Delta \mathbf{F}_V &= \sum \int \mathbf{B}_s^T \mathbf{D}_s \Delta \boldsymbol{\varepsilon}_V dV \\ \Delta \mathbf{F}_T &= \sum \int \mathbf{B}_s^T \mathbf{D}_s \Delta \boldsymbol{\varepsilon}_T dV \end{aligned} \right\} \quad (50)$$

where  $\mathbf{B}$  is the strain matrix after assembly;  $\mathbf{B}_s^T$  is the plane strain matrix of soil;  $\mathbf{D}_s$  is the elastic matrix of soil;  $dV$  is the volume of infinitesimal element.

### 5.2 Solution of coupled equation

The hydro-thermal-mechanical coupling model of retaining wall supporting frozen slope is composed of

Eqs. (41), (43) and (49). Since the thermal conductivity of soil is a variable changing with temperature, it is difficult to obtain an analytical solution to the problem. Therefore, the above equations are derived by Galerkin method and virtual work principle, and the formulation of finite element method is [19]

$$\mathbf{M} \mathbf{T} + \mathbf{R} \frac{\partial \mathbf{T}}{\partial t} = \mathbf{P} \quad (51)$$

$$\mathbf{Q} \frac{\partial \boldsymbol{\theta}_i}{\partial t} = \mathbf{G} \boldsymbol{\theta} - \mathbf{H} \frac{\partial \boldsymbol{\theta}_u}{\partial t} \quad (52)$$

$$\mathbf{K} \Delta \boldsymbol{\delta} = \Delta \mathbf{F} \quad (53)$$

where  $\mathbf{M}$  is the stiffness matrix of steady-state temperature field;  $\mathbf{R}$  is the unsteady-state temperature matrix;  $\mathbf{P}$  is the boundary temperature load vector;  $\mathbf{Q}$  is the stiffness matrix of ice content;  $\mathbf{G}$  is the stiffness matrix of total moisture content;  $\boldsymbol{\theta}$  is the vector of total moisture content;  $\mathbf{H}$  is the stiffness matrix of unfrozen moisture content;  $\Delta \boldsymbol{\delta}$  is the displacement increment in the current time step; and  $\Delta \mathbf{F}$  is the equivalent nodal load vector generated by self-weight and frost heave.

Firstly, the corresponding temperature is obtained by solving each period of Eq. (51). And then, combining with Eq. (42), the temperature is substituted into Eq. (52) to obtain the moisture content. Finally, the temperature and moisture content are substituted into Eq. (53) to obtain the stress-strain.

### 5.3 Software development

The traditional finite element software can not directly simulate the frost heave and thaw-subsidence of soil, so it needs to be simplified by equivalence, which brings a lot of inconvenience to engineering. Based on MATLAB, a hydro-thermal-mechanical coupling analysis software of permafrost slope supported by L-shaped retaining wall is developed. The running of the software includes four steps: generating calculation model, setting model parameters, computing analysis and outputting results. The model meshing and results can be displayed in the way of nephogram. The software main interface is shown in Fig. 11.

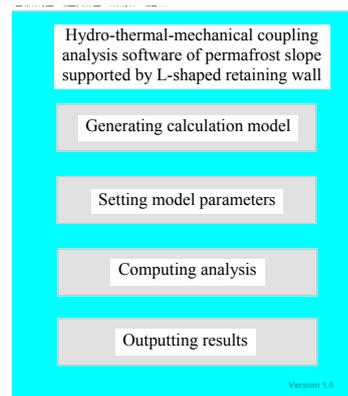


Fig. 11 The main interface of the software

5.4 Example analysis

5.4.1 Model establishment and parameter selection

Taking the L-shaped retaining wall supporting structure in Section 4.1 as an example, the developed finite element software is used to carry out the numerical analysis. For calculation convenience, a two-dimensional model is established, and the soil and retaining wall are discretized and meshed by triangular element and beam element respectively. The calculation time is 3 years. Fig. 12 shows the finite element model of slope supported by L-shaped retaining wall. The material and soil parameters in the simulation process are shown in Table 1. Subscripts f and u represent the freezing and

thawing state of soil respectively.

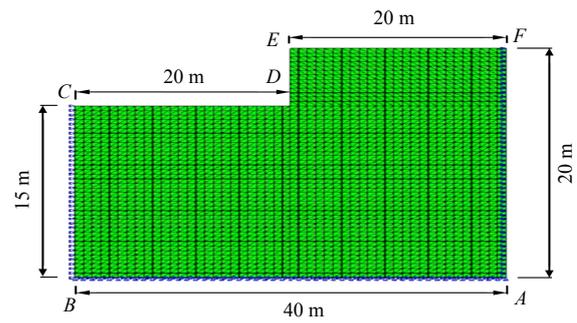


Fig. 12 Finite element model of L-shaped retaining wall

Table 1 Relevant material parameters

Material	Density /(kg · m <sup>-3</sup> )	Thermal conductivity /(J · m <sup>-1</sup> · °C <sup>-1</sup> · d <sup>-1</sup> )		Specific heat /(J · kg <sup>-1</sup> · °C <sup>-1</sup> )		Phase transition latent heat/(J · kg <sup>-1</sup> · °C <sup>-1</sup> )	Moisture diffusion coefficient /(J · kg <sup>-1</sup> · °C <sup>-1</sup> )		Elastic modulus /MPa		Poisson's ratio	
		κ <sub>f</sub>	κ <sub>u</sub>	C <sub>f</sub>	C <sub>u</sub>		D <sub>f</sub>	D <sub>u</sub>	E <sub>f</sub>	E <sub>u</sub>	ν <sub>f</sub>	ν <sub>u</sub>
Concrete	2 500	2×10 <sup>5</sup>	2×10 <sup>5</sup>	960	960	–	–	–	3×10 <sup>4</sup>	3×10 <sup>4</sup>	0.20	0.20
Coarse-grain soil	1 700	2.2×10 <sup>5</sup>	1.68×10 <sup>5</sup>	856	1 102	1.57×10 <sup>4</sup>	–	–	25	25	0.25	0.20
Fine-grained soil	1 600	1.3×10 <sup>5</sup>	8.2×10 <sup>4</sup>	1 560	1 650	1.19×10 <sup>4</sup>	3.1×10 <sup>-11</sup>	1.53×10 <sup>-8</sup>	19	4.5	0.27	0.25

5.4.2 Boundary conditions

In this simulation, soil is regarded as a solid element and the retaining wall as a beam element. The boundary conditions are as follows: the upper boundary of the model is the top of the slope, the lower boundary is the bottom of the model, and the left and right boundaries are the side of the model. Temperature boundary conditions: EF on the upper side of the slope, DE on the left side and CD at the slope foot are loaded according to Eq. (54). The heat flux of AB at the model bottom is 0.04 W/(m · s), and BC and AF are adiabatic boundaries.

$$T_2 = 0.7 + \sin(2\pi t' / 8\ 760 + 1.5\pi) + 2.6t' / (8\ 760 \times 50) \quad (54)$$

The displacement boundary conditions are as follows: the horizontal displacements of BC and AF on the left and right sides of the slope are 0, the horizontal and vertical displacements of AB at the bottom of the slope are 0, and the other boundaries are not limited.

5.4.3 Results and analysis

Figures 13 and 14 are the temperature and moisture nephograms of the L-shaped retaining wall support structure during the frozen period respectively. It can be seen from the figure that the temperature of the active layer at the surface during the frozen period is about -10 °C, while the internal temperature of the slope is -1.09 °C, which shows the slope is still frozen. The slope temperature contour is an S-shaped parallel line. With the increase of the slope depth, the temperature distribution inside the slope tends to be consistent. This indicates that the interior of the slope is less affected by the external temperature, while the slope surface is more affected by the temperature, and there is a certain thickness of active layer.

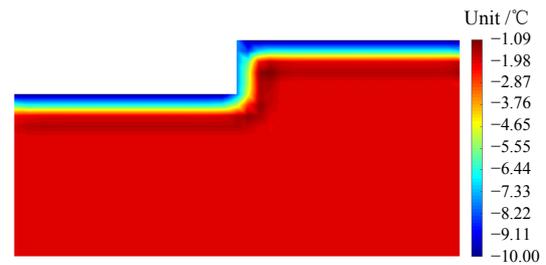


Fig. 13 Temperature nephogram during frozen period

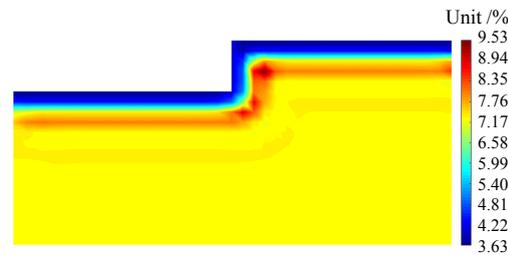


Fig. 14 Moisture nephogram during frozen period

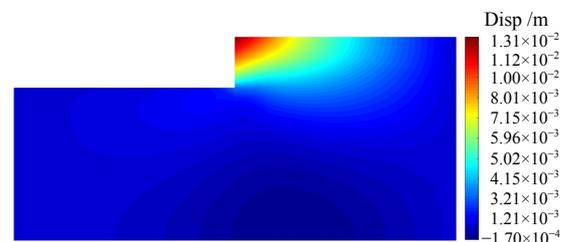


Fig. 15 Horizontal displacement nephogram of the slope supported by L-shaped retaining wall

Figure 16 is a comparison plot of the horizontal frost heaving force behind the retaining wall obtained by the method proposed in this paper and the multi-field coupling numerical simulation. It can be observed from the figure that the horizontal frost heaving force behind the wall obtained by the two methods is

distributed in a parabola along the wall height. The maximum horizontal frost heaving force of the retaining wall estimated by numerical simulation is in the middle and lower part of the retaining wall, while the maximum horizontal frost heaving force behind the wall estimated by the proposed method appears at 2/3 away from the bottom of the wall, and the two positions are relatively close. The maximum theoretical value by the proposed method is less than the numerical simulation value. This is because the calculation method does not consider the hydro-thermal-mechanical coupling effect of soil, but the difference between them is small, which indicates that the proposed method is available.

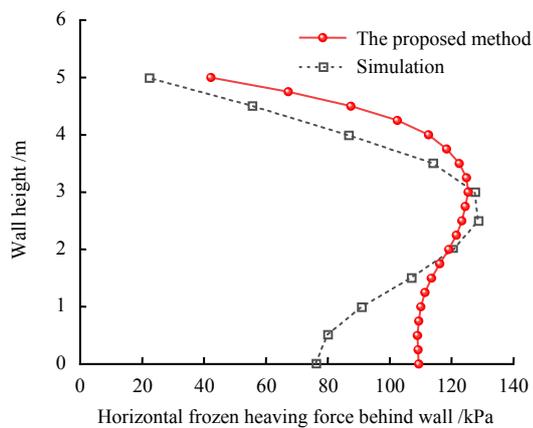


Fig. 16 Frost heaving force calculated by different methods

## 6 Conclusion and suggestion

In view of calculation of horizontal frost heaving force of L-shaped retaining wall in permafrost region, based on the Lifkin model and combined with the deformation coordination between the retaining wall and the fill behind the wall, the calculation model of horizontal frost heaving force of L-shaped retaining wall with or without soil replacement behind the wall is established and solved by the finite difference method. The rationality of the proposed method is verified by field measurement and programming. The main conclusions are as follows:

(1) The frost heaving force estimated by the model in this paper presents parabolic and trapezoidal distribution, which are controlled by retaining wall displacement. Compared with the frost heaving force distribution given by the Code, the method proposed in this paper is more universal. The maximum horizontal frost heaving force of the retaining wall appears at 2/3 away from the bottom of the wall, which can explain the reason why the retaining wall still has a lot of damage when designed according to the provisions of frost heaving force in the code.

(2) The self-developed hydro-thermal-mechanical coupling software is used for multi-field coupling

analysis of slope supported by L-shaped retaining wall. The results show that the horizontal frost heaving force is related to the thickness of the active layer of the slope, the frozen area at the top of the retaining wall is large, and the frost heaving force is controlled by the displacement of the retaining wall. Compared with the calculation method of frost heave effect of retaining wall proposed in this paper, it is found that although the proposed method does not consider hydro-thermal-mechanical coupling, the calculated horizontal frost heaving force is little different from the value obtained by the numerical method with hydro-thermal-mechanical coupling considered. The proposed calculation method can be applied to the design and calculation for horizontal frost heave effect of L-shaped retaining wall.

(3) For better guide on engineering practice, the horizontal frost heaving force should be taken as the main design load in the design of slope support engineering of L-shaped retaining wall in permafrost area, and the thickness of replaced soil behind the wall should be reasonably considered under the dual factors of economy and safety. It is suggested to optimize the height–thickness ratio of retaining wall in design, so that the retaining wall can not only meet the deformation requirements of the flexible shorting, but also bear the frost heaving force caused by soil frost heaving.

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