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Stratum identification based on multiple drilling parameters and probability classification

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Abstract: The conventional geological prediction method of advanced drilling usually takes the change rate of one specific drilling parameter as the main basis for stratum identification. The rock breaking of drill bit is a complicated mechanical process. Stratum identification with single drilling parameter results in great uncertainty. Thus the combined effect of multiple parameters in drilling process should be considered. Firstly, the advanced drilling data were preprocessed by SPSS, including standardization, frequency distribution analysis and sensitivity analysis, to select the key drilling parameters that are sensitive to stratum changes. Secondly, based on the principles of energy conservation, binary disordered logistic regression analysis and multi-parameter variability analysis, three comprehensive identification indices including rock breaking energy, logistic regression probability and stratum hardness were established respectively. Finally, the stratum identification model was established by probability classification method based on Bayesian principle, the model parameters were determined by ROC analysis method, and the stratum identification based on multiple drilling parameters and probability classification method was realized. Taking the tunnel project with complex geological conditions as an example, the application of the proposed stratum identification method is introduced. The results show that three comprehensive indices perform well in cross-hole stratum identification, and the identification accuracy exceeds 80%. The rock breaking energy and the logistic regression probability are suitable for the cross-hole stratum identification with short distance, and the average identification accuracies are 86.3% and 84.1%, respectively. The logistic regression probability index has strong identification capability for the weak interlayer, and the identification accuracy reaches 94.2%. The stratum hardness index is suitable for the cross-hole stratum identification with long distance, and the maximum identification accuracy of limestone is 93.2%.

Keywords: advanced drilling; rock breaking energy; logistic regression probability; stratum hardness index; probability classification; stratum identification

1 Introduction

The advanced geological prediction during tunnel construction can identify the basic geological structures and the properties of adverse geological bodies in front of the tunnel face, and provide guidance for the prevention of potential geological disasters in the tunnel. According to the working principle, the advanced geological prediction can be classified into two categories, i.e. geological analysis method and geophysical method. Geophysical method infers the geological condition by detecting the change in the physical properties of surrounding rocks, but the conclusion is usually not quantitative and unique. However, the advanced drilling which belongs to the geological analysis method obtains the surrounding rock condition by direct exposure, and the result is reliable.

Since the 1980s, with the development of science and technology, multi-functional drilling rigs capable of advanced geological prediction have been developed. This type of drilling rig possesses a variety of sensors, which can record and analyze the working data of the drilling rig in real-time to determine the hardness of rock, the integrity of the rock mass, and the distribution positions of faults and holes in the survey area. The advanced drilling and prospecting technology based on multi-functional drilling rigs has the features of quick disassembly and assembly, short working hours and reliable data sources, and it has gradually gained considerable attention in the industry. In 2018, the notice of the General Office of the Ministry of Transport of China on carrying out special rectification actions for the quality and safety of highway tunnel construction projects clearly stipulated that advanced geological

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prediction should be carried out during the tunnel construction in adverse geological conditions.

The predicting procedure, predicting principle, and data interpretation standard of the geological prediction based on advanced drilling have always been the focus of scholars and engineers[1]. In terms of predicting methods, Li et al.[2], Shu et al.[3] and Zhou et al.[4] provided hole arrangement suggestions based on engineering experience and geophysical approaches. In the analysis of the rock breaking mechanism based on the energy method and mechanical balance, He et al.[5] realized the quantitative prediction of the surrounding rock grade in front of the tunnel face based on the concept of drilling energy. Li et al.[6] proposed the concept of power-to-speed ratio, considering the synergistic effect of four drilling parameters including drilling speed, rotation speed, thrust and torque on rock breaking. Nong[7] employed the wavelet analysis method to transform and analyze the power-to-speed ratio. Compared with core data and TSP prediction, it presents higher accuracy in stratum identification. Li et al.[8] and Wang et al.[9−11] developed a large-scale indoor drilling test platform. Based on the rock breaking mechanism of the PDC bit, the quantitative analysis model between the drilling parameters and the uniaxial compressive strength and shear strength of rock was established with the energy analysis method. Tian et al.[12] focused on the surface properties of surrounding rocks and established a quantitative relationship between the parameters and the property changes of the surrounding rocks. The mechanism analysis requires a large number of assumptions, however, the in-situ working conditions of advanced drilling are complex and changeable, and too many assumptions make the result deviate from the actual situation of the project. Numerous scholars tried to use probability and mathematical statistics to compromise the mechanism analysis. For example, Dong et al.[13] established the equation of DPM drilling rate and rock core RBI value in sandstone formation using linear regression. Xu[14] classified the lithology and structure of surrounding rocks based on the wavelet theory. Yue et al.[15−16] established a time-series analysis method for spatiotemporal data during the drilling process and concluded that the drilling speed is the same in the same stratum. Zhao[17] analyzed the correlation between the drilling parameters and the condition of the tunnel face, and pointed out that the drilling speed can be used to identify the rock hardness and the distribution of joints and fractures. Qin et al.[18] identified the rock mass by analyzing the vibration spectrum and acoustic spectrum during the drilling process. Kahraman et al.[19] established the regression relationship of the drilling speed with rock mass density, uniaxial compressive strength and rock mass quality index through statistical analysis. Qi[20] investigated the distribution features and fractal dimension of the test data during drilling and found that the local prediction result of the lithologic change of surrounding rocks by advanced drilling is more accurate than conventional TSP and GPR techniques. Fang et al.[21] established a neural network-based stratum classification method and pointed out that considering the standard deviation of drilling parameters can improve the accuracy of stratum classification. Currently, the research of probability and mathematical statistics on stratum identification based on drilling parameters has following deficiencies: (i) usually, only the effect of a single drilling parameter is considered[22]; (ii) geological predictions obtain mostly qualitative conclusions and lack quantitative analysis; (iii) the peak value is usually considered as a noise, and the denoising process loses the stratum information at the corresponding depth.

Therefore, the stratum identification based on advanced drilling should comprehensively consider multiple drilling parameters, and can predict the stratum conditions of other drill holes in the site. Meanwhile, on the basis of ensuring the integrity of the data, multiple drilling parameters should be considered for stratum identification. Due to the spatial variability and the complexity of the rock breaking process of the drill bit, the theoretical analysis combined with the mathematical statistics is used to perform the stratum identification.

In this paper, the advanced drilling data are standardized, and the key drilling parameters sensitive to stratum changes are filtered out with the frequency distribution analysis and correlation analysis mode in the statistical product and service solutions (SPSS) software. Based on the principles of energy conservation, binary disordered logistic regression analysis and multi-parameter variability analysis, three comprehensive stratum identification indices, including rock breaking energy, logistic regression probability and stratum hardness index, are established. The probability classification method based on the Bayesian principle is used to establish the stratum identification model, and the receiver operating characteristic (ROC) curve analysis method is used to determine the model parameters. Taking the advanced drilling of Jiudingshan Tunnel of Chu-Da Expressway as an example, the stratum identification method based on multiple drilling parameters and probability classification method is introduced. The prediction results show that the three comprehensive stratum identification indices perform well for cross-hole stratum identification, and have certain advantages compared with the conventional geological prediction method of advanced drilling during tunnel construction, which provide a basis for

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the digital, efficient, and accurate geological survey of future tunnel engineering.

2 Multiple drilling parameters based stratum identification indices

2.1 Sensitivity analysis of drilling parameters

The geological prediction of advanced drilling usually takes the changing rate of drilling speed as the basis for stratum classification. A series of engineering examples has proved that the drilling speed is sensitive to the stratum change, which reflects the difficulty of the drill bit cutting into the rock mass, and the drilling speed is fast in the rock or soil layer with poor integrity or low strength. The breaking of the weak rock layer mainly depends on the bit pressure, and the drilling in the soil layer mainly depends on the shearing of the drill teeth. Consequently, the rock and soil layers with the same drilling speed have different drill bit torques. Therefore, the geological prediction of advanced drilling based on a single drilling parameter for stratum classification has limitations, and the stratum identification method considering the combined effect of multiple drilling parameters should be established.

The multi-functional drilling rig can record the data such as drilling speed, torque, number of revolutions, thrust, striking energy, striking number, inflow rate, inflow pressure, outflow rate, outflow pressure, and rock breaking energy. Under normal circumstances, the absolute values of different drilling parameters are quite different, and the sensitivity of drilling parameters to stratum change should be determined by the changing rate of drilling parameters. Therefore, the Min-Max standardization is used to preprocess the data set, so that the result is mapped to the range of 0 to 1, and the value of the same order of magnitude is obtained. The formula is written as follows:

$$x_{\min-\max}=\frac{x-x_{\min}}{x_{\max}-x_{\max}} \tag{1}$$

where $x_{\max}$ and $x_{\min}$ are the maximum and minimum of the data set, respectively.

For the standardized data, the influence of peak value on the data is reduced, and the parameters are comparable.

The normal distribution is of great significance in probability and mathematical statistics. Many of the commonly used statistical methods are based on the assumption that “the quantity under study follows or approximately follows a normal distribution”. Both experience and theory (the central limit theorem) verify the feasibility of this assumption. Therefore, analyzing whether the data set approximately follows or can be transformed into a normal distribution through the frequency distribution histogram is an important part of drilling data filtering. If the variable satisfies the log-normal distribution, exponential distribution, Cauchy distribution, Laplace distribution, and other non-normal distributions, it can be transformed by equivalent normalization. For example, when the variable satisfies the log-normal distribution, it can be converted to a normal distribution by taking the logarithm.

In the drilling process, each drilling parameter contains stratum information. When considering these parameters comprehensively, the sensitivity of different parameters to stratum change should also be considered. Several drilling parameters that are most sensitive to the change in stratum properties are filtered out through correlation analysis, which improves stratum identification efficiency. The stratum is a categorical variable, and the drilling parameters are generally discrete numerical variables, thus the Kendall method can be used to calculate the correlation coefficient and concomitant probability. The positive value of the correlation coefficient represents the positive correlation between the two variables, and the negative value represents the negative correlation. The concomitant probability represents the significant level of the correlation, and the concomitant probability less than 0.05 indicates the correlation between the two variables is significant. According to the magnitude and the positiveness of the correlation coefficient between two variables, the drilling parameters that are sensitive to stratum change are filtered out.

2.2 Stratum identification indices

2.2.1 Rock breaking energy index

For the down-the-hole (DTH) drill bit, the rock is broken by the impact of the drill bit, and the energy required for rock breaking is the work done by the drill bit impacting the rock, which has a clear physical meaning. Based on the principle of energy conservation, the energy of breaking per unit volume of rock is calculated according to the recorded drilling parameters including striking energy, striking number, and drilling speed, which is used as a comprehensive index for stratum identification. The rock breaking energy formula is written as follows[23]:

$$E=\frac{Wn}{Sv} \tag{2}$$

where E is the rock breaking energy; W is the striking energy; n is the striking number; v is the drilling speed; and S is the sectional area of the drill hole.

2.2.2 Logistic regression probability index

Logistic regression, known as multivariate disordered logistic regression analysis, is a generalized linear regression analysis method, which is suitable for the case where the dependent variable is categorical, and the independent variable is composed of categorical or numerical variable. The drilling parameters sensitive to stratum change are taken as the independent variables, and the stratum type is taken as the dependent variable.

The influence of each drilling parameter on stratum identification is evaluated by the logistic regression model. The logistic regression model containing n independent variables is written as follows[24]:

$$P = \frac{1}{1+\mathrm{e}^{-z}} \tag{3}$$

$$z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_n x_n \tag{4}$$

where P is the probability of a certain stratum expressed by the sigmoid function; z is the logistic regression characteristic equation; x_i is the drilling parameter; β_i is the regression coefficient of the drilling parameter and represents the degree of influence of each parameter on the stratum, which is determined by the maximum likelihood estimation. The resulting logistic regression value represents the probability that the group of samples belongs to a certain stratum.

2.2.3 Stratum hardness index

Due to the complexity of the rock breaking mechanism of the drill bit, there is usually a nonlinear relationship between the drilling parameters. According to the multivariable instability index analysis method[25], and considering the significance of each drilling parameter, a stratum hardness index in the form of an exponential multiplication function is proposed. According to the variability analysis, the weights of different drilling parameters are determined and used as the powers of the drilling parameters, and the stratum hardness index is obtained by multiplying the powers. When the drilling parameters increase, the input power of the drilling rig to the surrounding rocks increases, and the hardness index increases. The stratum hardness index formula is written as follows:

$$D = x_1^{W_1} x_2^{W_2} x_3^{W_3}, \ldots, x_n^{W_n} \tag{5}$$

where D is the stratum hardness index; x_i is the drilling parameter value; and W_i is the weight of the drilling parameter.

The detailed procedure is described as follows:

Step 1: Variability analysis

The coefficient of variation (CV) obtained from the normalized drilling parameters reflects the sensitivity of each parameter to the variation of stratum hardness. The larger the CV is, the more sensitive the parameter is to stratum changes. The CV is calculated by[26]:

$$\mathrm{CV} = \frac{\sigma}{\mu} \tag{6}$$

where σ is the standard deviation and μ is the mean value.

Step 2: Weight calculation

The weight of each factor is obtained by dividing the corresponding CV by the sum of all coefficients of variation, as expressed in Eq.(7)[27]. The weight indicates how much the parameter contributes to stratum identification.

$$W_i = \frac{\mathrm{CV}_i}{\mathrm{CV}_1 + \mathrm{CV}_2 + \mathrm{CV}_3 + \ldots + \mathrm{CV}_n} \tag{7}$$

3 Probabilistic classification method for stratum identification

3.1 Bayesian theory

The Bayesian theory[28−29] applies the observed phenomena to modify the subjective judgments (i.e. marginal probability) following probability distribution. Assuming that there are random events A and B, the Bayesian theory can explain the relationship between the AB marginal probability and the conditional probability. The formula is written as follows:

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)} = \frac{P(B|A_i)P(A_i)}{P(B)} \tag{8}$$

$A_1 - A_n$ is a complete set of events, representing all situations of A, i.e. $\bigcup_{i=1}^{n} A_i = \Omega$, $A_i A_j = \phi$, $P(A_i) > 0$.

In Bayesian theory, each expression has a certain meaning:

$P(A)$ indicates the marginal probability of A;

$P(B)$ indicates the marginal probability of B;

$P(A|B)$ represents the conditional probability of A when the event B occurred;

$P(B|A)$ represents the conditional probability of B when the event A occurred;

$P(A, B)$ represents the joint probability of the co-occurrence of AB.

The symmetric form of the Bayesian theory can be obtained from the concept of conditional probability:

$$P(A,B) = P(B|A)P(A) = P(A|B)P(B) \tag{9}$$

In the stratum identification problem, i represents the stratum category, and $\boldsymbol{x}$ is the attribute vector (drilling parameters, rock breaking energy, logistic regression probability, and stratum hardness index), then the Bayesian theory can be written as follows:

$$P(\boldsymbol{x}, i) = P(i|\boldsymbol{x})P(\boldsymbol{x}) = P(\boldsymbol{x}|i)P(\boldsymbol{x}) \tag{10}$$

The Bayesian theory based probability classification method can estimate the joint probability $P(\boldsymbol{x}, i)$ or conditional probability $P(i|\boldsymbol{x})$, and the category can be calculated by maximum likelihood:

$$c(\boldsymbol{x}) = \arg\max_i P(i|\boldsymbol{x}) = \arg\max_i P(\boldsymbol{x}, i) \tag{11}$$

where c is the maximum likelihood estimate of the category, i.e. the maximum value of the conditional probability. argmax is the maximum likelihood estimation function. For a given measurement point, when $P(x)$ is the same, and the calculation result of joint probability or conditional probability is the same.

3.2 Stratum identification model

The stratum identification is a process of first establishing a classification model and then clarifying a decision function. Using numbers to represent different strata (e.g. +1 for limestone, −1 for weak interlayer), the decision function can be written as follows[30]:

$$f(\boldsymbol{x})=\frac{P(+1|\boldsymbol{x})-P(-1|\boldsymbol{x})}{P(+1|\boldsymbol{x})+P(-1|\boldsymbol{x})} \tag{12}$$

The typical stratum identification model is written as follows:

$$c(\boldsymbol{x})=\begin{cases}-1 & f(x)<\theta \\ +1 & f(x)>\theta\end{cases} \tag{13}$$

$c(\boldsymbol{x})$ represents the predicted stratum category, and the sensitivity of the model changes accordingly as the threshold θ changes. In the case of $f(\boldsymbol{x})=\theta$, the function is meaningless and returns a random value.

3.3 Determination of model parameters

The stratum identification is divided into positive and negative categories, and all possible results of the stratum identification model are represented by a confusion matrix, as shown in Table 1. The number of samples for each situation can be approximated by a joint probability as follows[31]:

$$N\begin{bmatrix}P(-1,-1) & P(-1,+1)\\ P(+1,-1) & P(+1,+1)\end{bmatrix}\approx\begin{bmatrix}N_{\mathrm{TN}} & N_{\mathrm{FP}}\\ N_{\mathrm{FN}} & N_{\mathrm{TP}}\end{bmatrix} \tag{14}$$

where N is the total number of test samples; $P(-1,-1)$ represents the probability of the event when the actual stratum category is −1, and the predicted stratum category is −1 (true negative); N_{TN} represents the number of true negative events. The other situations are defined similarly. The confusion matrix of the perfect stratum identification model is diagonal.

Table 1 Confusion matrix of stratum identification model

	Positive category	Negative category	Sum
Positive category	True positive category（TP）	False positive category（FP）	Predicted positive category
Negative category	False negative category（FN）	True negative category（TN）	Predicted negative category
Sum	Actual positive category	Actual negative category	TP+FP+FN+TN

ROC is an effect evaluation method for classification models based on the Bayesian theory. According to the key parameters such as the area under the curve (AUC), Youden index, sensitivity, specificity and optimal critical point, the threshold θ for the stratum identification model can be determined. The ROC curve is shown in Fig.1. The main indices of ROC are obtained according to the confusion matrix, and the concepts are shown in Table 2.

AUC, representing the area under the ROC curve, is an index for evaluating the performance of the classification model. The meanings of the AUC values are shown in Table 3. If the AUC value is less than 0.5, the opposite result to the original prediction is feasible.

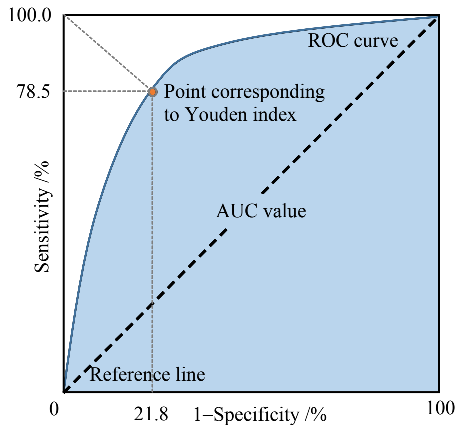


Fig. 1 Diagram of ROC curve

Table 2 ROC analysis indices

Index	Equation	Definition
Sensitivity	$P_{\mathrm{TP}}=N_{\mathrm{TP}}/(N_{\mathrm{TP}}+N_{\mathrm{FN}})$	P_{TP} is the proportion of the samples that both the predicted value and the actual value are positive to all positive category samples
Specificity	$P_{\mathrm{TN}}=N_{\mathrm{TN}}/(N_{\mathrm{FP}}+N_{\mathrm{TN}})$	P_{TN} is the proportion of the samples that both the predicted value and the actual value are negative to all negative category samples
Missed diagnosis rate	$P_{\mathrm{FN}}=N_{\mathrm{FN}}/(N_{\mathrm{TP}}+N_{\mathrm{FN}})$	P_{FN} is the proportion of the samples that the predicted value is negative and the actual value is positive to all positive category samples
Misdiagnosis rate	$P_{\mathrm{FP}}=N_{\mathrm{FP}}/(N_{\mathrm{FP}}+N_{\mathrm{TN}})$	P_{FP} is the proportion of the samples that the predicted value is positive and the actual value is negative to all negative category samples

Table 3 AUC value meaning

AUC value	Meaning
[0.95, 1.00]	Perfect classification model
[0.70, 0.95）	Good
[0.50, 0.70）	Medium
0.50	Random guess
[0.00, 0.50）	Opposite prediction

The effect of stratum identification model ultimately depends on the selection of threshold θ. Fig.2 shows the schematic diagram of the influence of θ on the stratum identification effect. The maximum value of the sum of sensitivity and specificity minus 1 corresponding to the points on the ROC curve is called the Youden index, and the optimal classification effect can be obtained by taking the value corresponding to this point as the threshold θ.

3.4 Specific implementation

The specific implementation of stratum identification based on the probability classification method is shown in Fig.3. Firstly, the data of the calibration hole are preprocessed, including parameter standardization, frequency distribution analysis and sensitivity analysis, to filter out the key parameters sensitive to stratum change. Secondly, based on the principle of energy conservation, logistic regression analysis and variability analysis, three comprehensive indices for stratum identification including rock breaking energy, logistic regression probability and stratum hardness are established. Finally, the Bayesian theory based probability classification method is used to establish the stratum identification model, and the model parameters are determined by ROC analysis. Input the data of the test hole into the model, output the prediction results, and the model effect is tested.

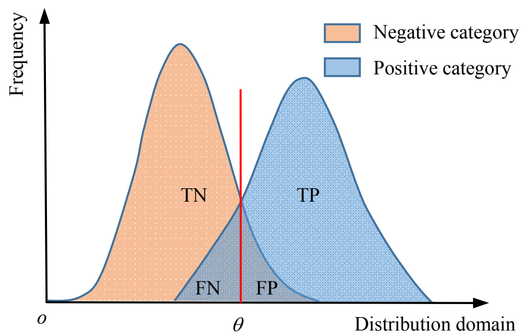


Fig. 2 Schematic diagram of the influence of threshold θ on classification effect

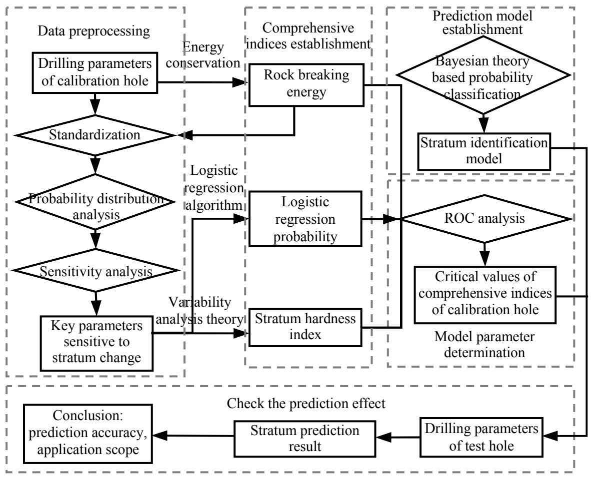


Fig. 3 Stratum identification flow chart based on multiple drilling parameters and probability classification method

4 Field application

4.1 Project description

The stratum identification method based on multiple drilling parameters and probability classification is applied to the advanced drilling geological prediction of the Jiudingshan Tunnel of Chuxiong−Dali Expressway Extension Project.

Jiudingshan tunnel is a control node of the extension project of Chuxiong−Dali Expressway. It is 7 560 m long, of which 1 830 m-long tunnel segment is located at the depth of more than 500 m, with the maximum depth of 731 m. It is a deep and long tunnel. The terrain of the project area is steep, the elevation is between 2 180 m and 3 085 m, and the relative height difference is 905 m. It belongs to the topographic and landform area formed by structural dissolution and structural denudation. The tunnel route map is shown in Fig.4.

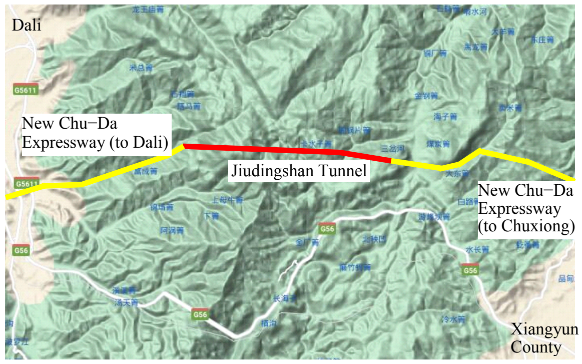


Fig. 4 Route map of Jiudingshan tunnel

As shown in Fig.5, the tunnel is located in the composite part of the north-south structural zone and the Qinghai-Tibet-Yunnan eta-type structural system, and passes through two fault zones. The stability of the surrounding rocks is low. As shown in Fig.6, due to severe tectonic compression and strong metamorphism, the rock mass is fragmented and the lithology is complex and changes frequently.

The design of the deep and long tunnel is limited by objective conditions such as survey approaches, time, and funds. The number and depth of survey holes are limited, and the engineering geology and hydrogeology of the tunnel cannot be well revealed. Therefore, the actual condition of surrounding rocks exposed by construction is frequently different from the designed rock mass grade, and it is necessary to implement advanced drilling to predict the geological condition in front of the tunnel face.

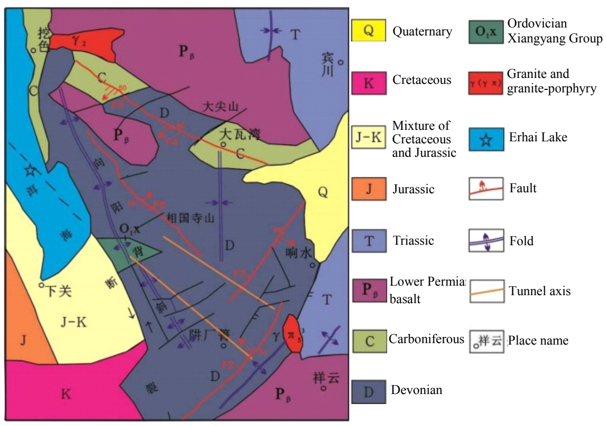


Fig. 5 Structure outline map of Jiudingshan tunnel distribution area (according to 1:200 000 regional geological survey report (Dali))[32]

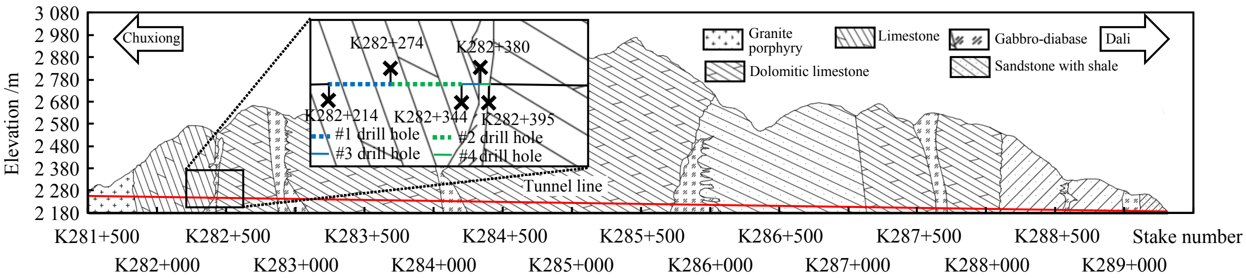


Fig. 6 Geological profile of Jiudingshan tunnel

The test section of advanced drilling is located at the mileage between K282+214 and K282+395. There are 4 groups of drilling data with a total length of 181 m. The drilling holes are numbered #1 to #4 as shown in Figs.6 and 7. According to the lithology and surrounding rock mass grade, the surrounding rocks within the test section can be divided into two types, i.e. limestone and weak interlayer, which match the overall stratigraphic background.

4.2 Collection of drilling parameters

The KOKEN RPD-180CBR drilling and injection integrated multi-functional drilling rig was used to carry out the advanced drilling. For the same tunnel face, the data of 1 to 2 drill holes are collected, and the drill holes are arranged at the lower left or upper right of the core area of the tunnel face, as shown in Fig.8.

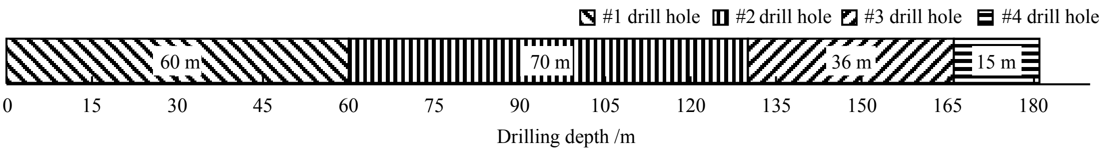


Fig. 7 Location of drill holes

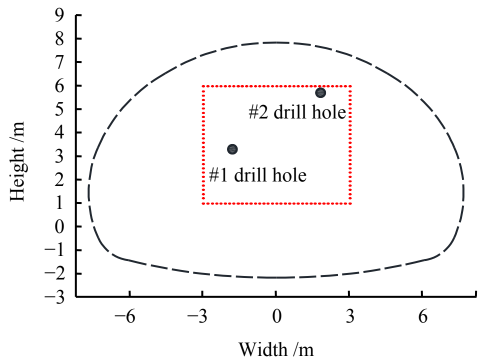


Fig. 8 Cross-section of Jiudingshan tunnel

The multi-functional drilling rig records the data including drilling speed, torque, number of revolutions, thrust, striking energy, striking number, inflow rate, inflow pressure, outflow rate, outflow pressure, and rock breaking energy, as listed in Table 4. The rock breaking energy is obtained by calculation, and other drilling parameters are directly recorded by the sensors. The data are recorded every 2 cm of drilling depth.

4.3 Identification of strata in front of tunnel face

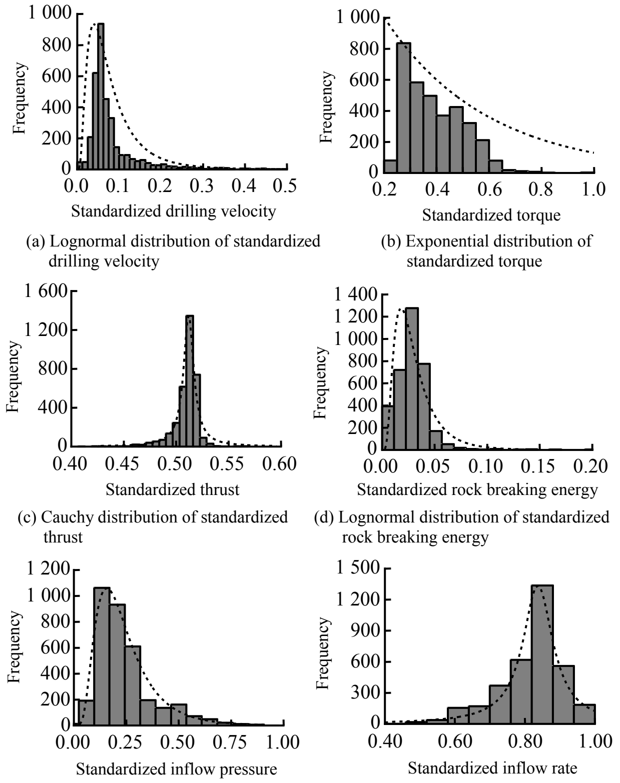
4.3.1 Drilling parameter sensitivity and stratum identification model

Firstly, the collected data such as drilling speed, torque, number of revolutions, thrust, striking energy, striking number, inflow rate, inflow pressure, outflow rate, outflow pressure, and rock breaking energy are preprocessed by standardization.

Table 4 Drilling parameters

Drilling parameter	Range	Meaning
Torque/(kN • m)	0–8.0	Torque during rock breaking by the drill bit
Number of revolutions /(r • min^{-1})	0–72	Drill revolution per minute
Thrust /kN	0–60	Reactive force on the drill rod
Striking energy /J	0–750	Energy imposed on rock mass by impact drill
Striking number /(strike • min^{-1})	0–2200	Number of strikes per minute
Inflow rate /L	0–240	Water flow to drill bit
Inflow pressure /MPa	0–10	Water pressure delivered to drill bit
Outflow rate /L	0–240	Water flow out of the drill hole
Outflow pressure /MPa	0–10	Water pressure delivered out of drill hole

The frequency analysis of the standardized drilling parameters is carried out, and the frequency distribution diagram of each parameter and its distribution characteristics are shown in Fig.9. According to the frequency distribution of each parameter, the following results are obtained: drilling speed, rock breaking energy and inflow pressure roughly conform to log-normal distribution; thrust and inflow rate roughly conform to Cauchy distribution; and torque conforms to the exponential distribution. Lognormal, Cauchy and exponential distributions are analyzed by converting them to normal distributions by equivalent normalization. The dotted line in the figure is the probability distribution curve.



(a) Lognormal distribution of standardized drilling velocity
(b) Exponential distribution of standardized torque
(c) Cauchy distribution of standardized thrust
(d) Lognormal distribution of standardized rock breaking energy
(e) Lognormal distribution of standardized inflow pressure
(f) Cauchy distribution of standardized inflow rate

Fig. 9 Frequency distribution diagram

In the "bivariate correlation" module of the SPSS (statistical product and service solutions), the Kendall method is used to perform sensitivity analysis on the drilling parameters without equivalent normalization, and the correlation coefficient and the concomitant probability are calculated and listed in Table 5. The four parameters most sensitive to the degree of stratum fragmentation are torque, drilling speed, rock breaking energy and thrust. Since the DTH impact rock breaking drilling method is implemented, the effect of torque on the degree of stratum fragmentation actually is small, but the Kendall analysis obtains the opposite result. If the equivalent normalization treatment is not performed, the obtained sensitivity analysis result is inconsistent with reality.

Table 5 Sensitivity of original drilling parameters to degree of stratum fragmentation

Parameter	Correlation coefficient	Concomitant probability
Drilling speed	0.307	<0.005
Torque	0.318	<0.005
Thrust	−0.206	<0.005
Inflow rate	−0.133	<0.005
Inflow pressure	0.139	<0.005
Rock breaking energy	−0.265	<0.005

The equivalent normalization is performed according to the following steps: clarify the distribution function and density function of the variable; find the distribution function and density function of the equivalent normal variable; obtain the relationship between the two distribution parameters of the equivalent normal variable according to the inverse function of the distribution function; the parameter relationship is substituted into the variable density function and the equivalent normal density function, and the two distribution parameters of the equivalent normal variable are obtained by solving the equations, and the equivalent normalization is completed. The equivalent normalizations of the three probability distributions in this paper are described as follows: logarithmic transformation of lognormal distribution data, square root sine transformation of exponential distribution data, and multiplication of Cauchy distribution data and standard normal distribution data.

The sensitivity analysis is performed on the equivalent normalized data with the Kendall method, and the results are listed in Table 6. It can be seen from the table that the four parameters including rock breaking energy, drilling speed, torque and thrust are the most sensitive to stratum change, and the absolute value of the correlation coefficient exceeds 0.3. Drilling speed and torque have a positive correlation with the fragmentation degree of the surrounding rocks, while rock breaking energy and thrust hold the opposite trend. This shows that in general, the faster the drilling speed is, the larger the torque, the smaller the thrust and the rock breaking energy, the weaker and more fragmentary the stratum will be; the slower the drilling speed is, the smaller the torque, the larger the thrust and the rock-breaking energy, the more integral and hard the stratum will be. This result is consistent with that obtained by the stratum identification with rock breaking energy and penetration rate.

The four drilling parameters which are the most sensitive to stratum change, i.e. rock breaking energy, drilling speed, torque and thrust, are selected to calculate the three comprehensive indices of rock breaking energy, logistic regression probability and stratum hardness, and the probability classification method based stratum identification model is established.

Table 6 Sensitivity of equivalent normalized drilling parameters to degree of formation fragmentation

Parameter	Correlation coefficient	Concomitant probability
Drilling speed	0.339	<0.005
Torque	0.316	<0.005
Thrust	−0.335	<0.005
Inflow rate	−0.222	<0.005
Inflow pressure	0.148	<0.005
Rock breaking energy	−0.361	<0.005

4.3.2 Stratum identification based on rock breaking energy index

Taking the fragmented stratum and weak interlayer

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with low hardness as the positive category, the ROC analysis of the stratum identification model based on rock breaking energy index is shown in Fig.10, and the analysis results are listed in Table 7.

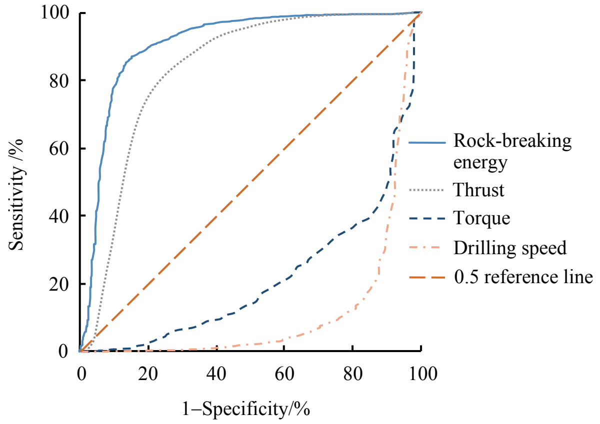


Fig. 10 ROC curves of drilling parameters of #1 drill hole

Table 7 ROC analysis results of drilling parameters

Variable	#1 drill hole				#2 drill hole			
	AUC	Critical value	Sensitivity /%	Specificity /%	AUC	Critical value	Sensitivity /%	Specificity /%
Drilling speed	0.119	0.708 3	82.60	93.21	0.210	0.101 0	90.9	69.7
Torque	0.187	0.500 0	90.10	66.60	0.228	0.226 0	74.0	44.4
Thrust	0.827	0.377 0	90.10	60.80	0.593	0.341 0	80.8	62.9
Rock breaking energy	0.905	0.018 8	87.10	84.10	0.806	0.018 4	90.0	72.0

It can be seen from Fig.10 that the ROC curves of thrust, rock breaking energy, drilling speed and torque are all far away from the 0.5 reference line, which indicates that the stratum classification effects of the four parameters are satisfactory. By comparing the ROC analysis results of #1 and #2 drill holes in Table 7, the AUC value, sensitivity and specificity of drilling speed and rock breaking energy are all at high levels, but the critical values of drilling speed are 0.708 3 and 0.101 0, respectively, which indicates a difference of several times between the two drill holes; the critical values for the two drill holes are 0.500 0 and 0.226 0, respectively, about two times difference; the critical values of the thrust are relatively close, but the specificity is too low to be selected as a stratum identification basis. The critical values of rock breaking energy are close, and the sensitivity and specificity are both high. Therefore, the stratum identification effect based on the rock breaking energy index is better than that of other single drilling parameters.

The ideal operation in actual engineering is to use the drill holes with known stratum information as calibration holes to obtain benchmark parameters, and then use the benchmark parameters to perform the stratum identification for the drill holes with unknown stratum distribution, which are also called test holes. It can be seen from the data in Table 7 that the critical values of rock breaking energy between adjacent drill holes are relatively close, indicating that for the probability classification method, the rock breaking energy index has the potential ability in cross-hole stratum identification.

Applying the critical value of rock breaking energy of #1 drill hole to #2 drill hole, the accuracy of limestone stratum identification is 86.8%; the accuracy of weak interlayer identification is 84.6%, and the overall accuracy is 86.3%. It can be seen from the above analysis that the rock breaking energy, as a comprehensive index, has a good effect on stratum identification between adjacent drill holes.

4.3.3 Stratum identification based on logistic regression probability index

Considering the drilling speed, torque, thrust and rock breaking energy comprehensively, the multivariate logistic regression probability index is calculated, and the logistic regression equations and stratum identification models of different drilling holes are established. When taking the weak interlayer as the positive category, the multivariate logistic regression equations for #1 and #2 drill holes are respectively written as follows:

$$z_1 = 4.850 - 21.100x_v - 6.478x_n + 7.004x_t + 13.336x_e \tag{15}$$

$$z_2 = 6.522 - 11.678x_v - 13.117x_n + 3.172x_t + 43.958x_e \tag{16}$$

where x_v is the drilling speed; x_n is the torque; x_t is the thrust; and x_e is the rock breaking energy. The ROC analysis results are listed in Table 8. The AUC values of the logistic regression probability index based classification models for the two drill holes are close to 1, and the sensitivity and specificity are both at high level, thus the stratum identification effect is good.

Table 8 ROC analysis results of stratum identification based on logistic regression probability

Drill hole No.	AUC	Critical value	Sensitivity /%	Specificity /%
1	0.945	0.7083	82.60	93.21
2	0.938	0.7427	91.85	83.36

Applying the critical value of #1 drill hole to the stratum identification model of #2 drill hole, the accuracy of limestone stratum identification is 81.0%; the accuracy of weak interlayer identification is 94.2%, and the overall accuracy rate is 84.1%. This shows that the logistic regression probability index can be used for cross-hole stratum identification, especially a better prediction result of the weak interlayer with fewer data samples can be obtained.

The critical values of the logistic regression probability for two adjacent drill holes are close, and the sensitivity and specificity are both high. Therefore, the stratum identification model based on the logistic regression probability index also outperforms that with other single drilling parameters.

4.3.4 Stratum identification based on stratum hardness index

Considering the drilling speed, torque, thrust and rock breaking energy comprehensively, the stratum hardness indices of #1 and #2 drill holes can be obtained by Eqs.(5) to (7):

Stratum hardness index D of #1 drill hole:

$$D_1 = x_v^{0.407} x_n^{0.124} x_t^{0.048} x_e^{0.612} \tag{17}$$

Stratum hardness index D of #2 drill hole:

$$D_2 = x_v^{0.452} x_n^{0.137} x_t^{0.066} x_e^{0.528} \tag{18}$$

The ROC analysis results of the stratum identification model are listed in Table 9. Taking the weak interlayer as the positive category and performing the ROC analysis, the results are shown in Fig.11.

Table 9 ROC analysis results of stratum identification based on stratum hardness index

Drill hole No.	AUC	Critical value	Sensitivity /%	Specificity /%
1	0.909	1.49	85.93	82.99
2	0.940	1.18	93.88	82.46

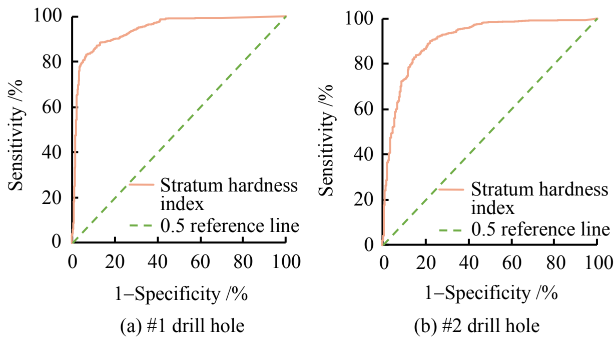


Fig. 11 ROC curve of stratum identification based on stratum hardness index

Applying the stratum identification model of #1 drill hole to the adjacent #2 drill hole, the accuracy of the limestone stratum identification is 79.0%, and that of the weak interlayer identification is 90.6%. It shows that the stratum hardness index D has a strong ability in adjacent cross-hole stratum identification. The accuracy of the stratum identification model of #1 drill hole is comparable to that of the stratum identification model of #2 drill hole itself. Therefore, the stratum hardness index D is suitable for short-distance cross-hole stratum identification.

The critical values of stratum hardness indices of the two adjacent drill holes are also close, and the sensitivity and specificity are both high. Therefore, the stratum identification model based on the stratum hardness index is also outperforms that with other single drilling parameters.

Employing the weighted average value of the weights of each factor of #1 and #2 drill holes, the combined stratum hardness index of #1 and #2 drill holes is obtained as follows:

$$D_{1\text{-}2} = x_v^{0.424} x_n^{0.129} x_t^{0.055} x_e^{0.581} \tag{19}$$

According to the drilling results, the study section consists of four drill holes with a total length of 181 m. The stratum lithology includes limestone and weak interlayer, and it meets the condition for long-distance cross-hole analysis. The strata of #3 and #4 drill holes, which are far away from each other, are predicted, and the results are listed in Table 10. The ROC curve is shown in Fig.12. The AUC values of #3 and #4 drill holes show high sensitivity and specificity, indicating that the stratum identification model based on the combined stratum hardness index of #1 and #2 drill holes has the ability in long-distance cross-hole stratum identification.

Taking the #3 drill hole as an example, the stratum identification is based on the combined stratum hardness index of #1 and #2 drill holes, the results indicate that the accuracy of limestone stratum identification is 86.7%, and that of weak interlayer identification is 76.5%. Although the accuracy of the prediction results is slightly lower than that of the stratum identification model based on the hardness index of #3 drill hole, the accuracy of 85.2% can be achieved only with the strata information of #1 and #2 drill holes. It can be seen from the data in Table 10 that a similar conclusion can be drawn for the #4 drill hole. Therefore, the stratum hardness index D is suitable for long-distance cross-hole stratum identification, which is of great significance for reducing the number of coring holes and realizing real non-coring stratum prediction.

Table 10 ROC analysis results of long-distance stratum identification based on the combined hardness index of #1 and #2 drill holes

Drill hole No.	AUC	Sensitivity /%	Specificity /%
3	0.847	76.5	86.7
4	0.918	75.9	93.2

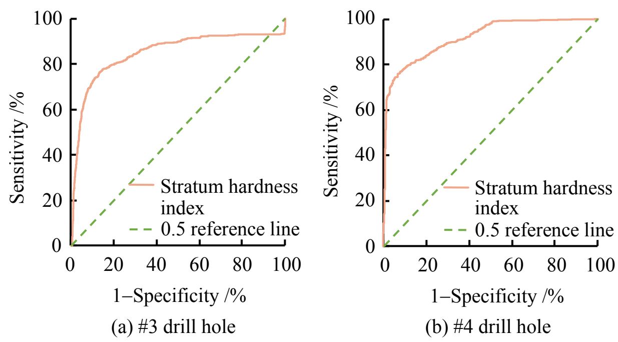


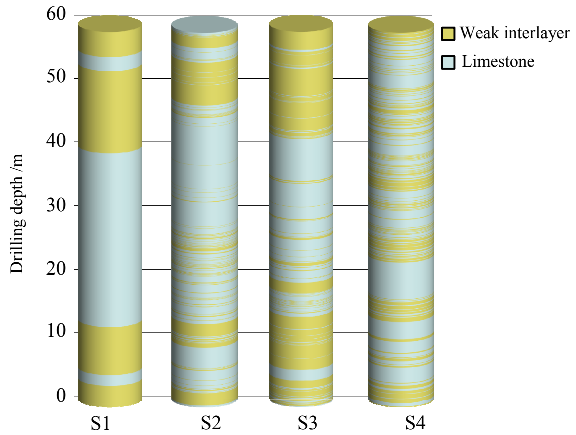
Fig. 12 ROC curve of long-distance stratum identification based on the combined hardness index of #3 and #4 drill holes

4.3.5 Comparison and analysis of strata identification effects

Comparing the results of the stratum identification models based on the three comprehensive indices of drilling parameters, the accuracies of stratum identification are listed in Table 11. Figs.13 and 14 show the comparison between the predicted results based on various methods and the actual stratum conditions.

Table 11 Identification accuracies of the three stratum identification models with respect to different comprehensive indices

Stratum	Rock breaking energy /%	Logistic regression probability /%	Stratum hardness index /%		
	Close	Close	Close	#3 drill hole	#4 drill hole
Limestone	86.8	81.0	79.0	86.7	93.2
Weak interlayer	84.6	94.2	90.6	76.5	75.9
Average accuracy	86.3	84.1	81.8	85.2	87.0



S1 Actual strata S2 Predicted strata based on rock breaking energy
S3 Predicted strata based on logistic regression probability
S4 Predicted strata based on strata hardness index

Fig. 13 Short-distance prediction results of #2 drill hole based on rock breaking energy, logistic regression probability and stratum hardness index

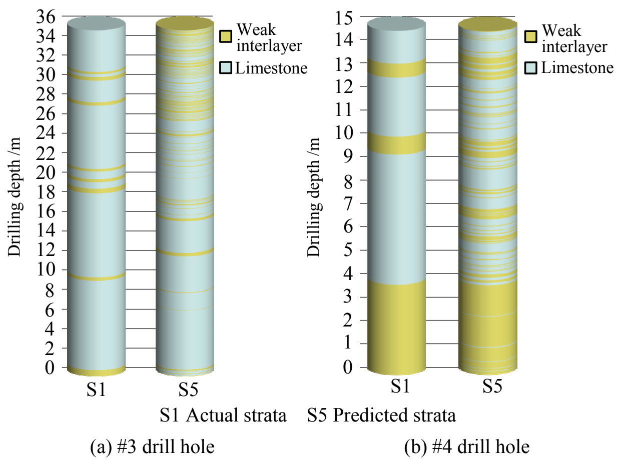


Fig. 14 Long-distance prediction results of #3 and #4 drill holes based on the combined hardness index of #1 and #2 drill holes

From the prediction accuracies of the stratum identification models based on different indices, the accuracies of the rock breaking energy index in the identification of soft and hard strata are basically the same, which is 84%−87%. The accuracies of the logistic regression probability index in the identification of the two strata are both high, and the accuracy for the weak interlayer identification exceeds 94%. When the stratum hardness index is used for short-distance prediction, the accuracy of limestone identification is 79%, and the accuracy of weak interlayer identification exceeds 90%. When the stratum hardness index is used for long-distance prediction, the accuracy of limestone identification exceeds 86%.

The rock breaking energy index incorporates the effect of three drilling parameters including striking energy, striking number and drilling speed. The energy required by the drill bit to break different rocks significantly varies. The rock is intact and hard, and the required rock breaking energy is large. The rock is soft and fragmentary, and the required rock breaking energy is small. Therefore, the rock breaking energy index is suitable for the identification of strata with different hardness values.

The logistic regression probability index considers four parameters including drilling speed, torque, thrust and rock breaking energy. Because more drilling parameters are considered, the logistic regression probability index has higher accuracies in identifying hard and soft strata. The accuracy of weak interlayer identification exceeds 90%, mainly because the rock breaking energy in the logistic regression probability function has a large weight, and its absolute value is greater than the other three drilling parameters, thus it has a greater impact on the identification result. Meanwhile, combining the influences of drilling speed, torque and thrust, the total effect makes the prediction result tends to be a weak interlayer when the rock breaking energy is small, the drilling speed and torque are large, and the logistic regression probability value is small.

The stratum hardness index also considers four parameters of drilling speed, torque, thrust and rock breaking energy. The weights of each drilling parameter are rock breaking energy, drilling speed, torque and thrust in descending order. Since the stratum hardness index is obtained by multiplying the exponents of the four parameters, the effect of each parameter is reinforced, and the change rate of the dependent variable is fast. For weak interlayer, such effect is greater, thus its identification accuracy is higher. Due to the spatial variability of surrounding rocks, with the increase in distance, the probability of the change in surrounding rock property increases. The stratum identification results show that the accuracy of limestone identification gradually increases, and that of weak interlayer identification gradually decreases. The reason for this phenomenon probably is that the spatial variability makes the model threshold to be inconsistent with the real threshold. The farther the distance is, the difference between the model threshold and the real threshold will be more obvious, and the accuracy of weak interlayer identification reduces. In the practical application, the threshold value θ should be corrected. It can be seen from Fig.2 that when the threshold value θ moves toward the positive direction of the distribution domain, the identification accuracy of the negative category increases, and that of the positive category decreases. The correction should take into account both safety and economy. A good stratum identification model should be safe, but not too conservative, and will not result in a waste of resources.

The proposed stratum identification method maintains high accuracy in stratum prediction, generally, the accuracy exceeds 80%. The surrounding rock information is obtained by coring in the conventional advanced geological drilling and prospecting method, which is time-consuming and costly. Meanwhile, it is difficult to obtain intact cores in the fragmented strata, and the result is usually qualitative. Besides, the geophysical prediction has the shortcoming of non-unique solution. The proposed method is based on multiple drilling parameters and the probability classification method, which can significantly improve the identification efficiency of surrounding rocks, save cost, and reduce construction interference.

5 Conclusions

In this paper, based on multiple drilling parameters collected in the drilling process, the Bayesian principle based probability classification method is employed to establish the stratum identification model, the ROC analysis method is used to obtain the model parameters, and the stratum identification based on multiple drilling parameters and probability classification method is realized. Taking the advanced drilling in the Jiudingshan tunnel of Chu−Da Expressway as an example, the application of the proposed method in the advanced geological prediction is introduced. The main conclusions are drawn as follows:

(1) The parameters including drilling speed, rock breaking energy, torque and thrust are most sensitive to stratum change, and the non-normal distribution can be transformed by equivalent normalization, which is suitable for the probability classification method.

(2) Three comprehensive indices for stratum identification, including rock breaking energy, logistic regression probability and stratum hardness, are established. It is the first time that the probability classification method is employed in stratum identification. Meanwhile, a probability classification model for stratum identification based on the Bayesian principle is established. The results indicate that the comprehensive index of multiple drilling parameters is better than that of a single drilling parameter for stratum identification.

(3) The effects of cross-hole stratum identification based on the three comprehensive indices, i.e. rock breaking energy, logistic regression probability and stratum hardness, are satisfactory. The ROC analysis results show that the sensitivity and specificity of the three methods are all high, and the stratum identification accuracies all exceed 80%. The rock breaking energy and logistic regression probability indices are suitable for short-distance cross-hole stratum identification, and the average identification accuracies of limestone and weak interlayer are 85.8% and 87.2%, respectively. The identification accuracy of weak interlayer based on the logistic regression probability index reaches up to 94.2%. The stratum hardness index is suitable for long-distance cross-hole stratum identification, and the maximum accuracy of limestone identification is 93.2%.

(4) Different indices have different applicability. For long-distance stratum identification, the model threshold should be corrected comprehensively by considering the safety, economy, and spatial variability of surrounding rocks.

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