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# A calculation method for the bearing capacity of saturated soil under undrained conditions

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**Abstract:** With regard to undrained analysis of saturated clay foundation for its ultimate bearing capacity under rapid loading, this study has proposed the calculation of the unconsolidated-undrained (UU) strength instead of employing the consolidated-undrained (CU) strength parameters directly due to overestimated results. The formula for predicting UU strength has been deducted based on the CU strength parameters referring to the reports of geological investigation, and the profile of the UU strength $c_u$ , found increases linearly with depth, has been built up. A calculation method for the bearing capacity of this foundation type is therefore proposed. The basic idea thereof is to use the value of $c_u$ at the average depth of the slip plane in the calculation, and a dimensionless parameter, which plays the key role, is introduced to determine the maximum depth of the slip surface. The accuracy and the precision of this parameter, as well as the proposed method, has been validated via a large number of comparative calculations with the finite element limit analysis method in this study.

**Keywords:** bearing capacity of foundation; saturated clayey soil; unconsolidated-undrained strength; consolidated-undrained strength parameter

## 1 Introduction

In geotechnical engineering, it is often required to calculate the bearing capacity of saturated clayey soil foundation under undrained conditions, such as to check the stability of high filled ground on saturated clay layer, or to check the basal heave resistant for deep foundation pit in soft saturated soils by examining the bearing capacity of the bottom soil[1]. In all these cases, the construction should be considered as rapid loading to the saturated soil regarding the low permeability of clayey soil. Therefore, stability analysis of the geo-technical structures should be conducted under undrained conditions, and a guidance is needed for the choice of strength parameters. In addition, a proper method is also required for the corresponding stability analysis.

For the analysis of geotechnical structures under undrained conditions, it is considered theoretically ideal to apply the effective stress method, in which the both the stiffness and strength parameters corresponding to the effective stress are employed. That requires constitutive models which can provide good description of evolution of shear dilatancy (or shear contraction) of soil, so as to give accurate calculation of both the excess pore pressure and effective stress, and consequently to predict the correct strength of the soil. However, the ideal model fulfilling those requirements is hardly available. The relatively simplified constitutive model, such as the perfectly elastic-plastic constitutive model, is not able to calculate the undrained strength accurately[2]. At present, a feasible approach for the analysis is to do total stress analysis using the UU strength of the soil that is pre-consolidated under its self-weight. For the design and analysis of common foundation on saturated clayey soils subjected to rapid loading, some Chinese codes clearly suggested that the UU strength should be employed[3]. A relevant theory of effective consolidation stress was proposed by Shen[4] in 1 960 s and has been developed continuously, the essence of which is to do total stress analysis using the UU strength of the soil. However, there are also some technical codes for certain special professions in which clear instructions are not provided with regard to this issue, and some designers just use intuitively the CU strength parameters directly for the analyses. In the field of deep excavation most of the technical codes, such as reference[1, 5], suggest employing the CU strength parameters directly to do stability analysis. From the view of this study, the latter method cannot calculate the undrained soil strength correctly, and large errors can be caused in some cases[2, 6].

The bearing capacity of shallow foundation is a classic topic in soil mechanics, and has been widely introduced in textbooks. However, when the UU strength of soil is involved, a reliable simple calculation formula for the undrained bearing capacity of the soil foundation is not yet available, since the UU strength within a soil layer is usually linearly increasing along depth. What we can find in the literature are mainly numerical solutions for this calculation using finite element method or characteristic line method[7]. Therefore, it is of significance to develop a simple and practical calculation formula for this purpose.

In this paper, a brief discussion on the deficiency of using the CU strength parameters to directly calculate the undrained soil strength is given first. Then a formula for calculating the UU strength $c_u$ from the CU strength

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parameters is derived, and finally a calculation formula for the undrained $c_u$ along depth. The key idea is to calculate the bearing capacity by using the cohesion at the mean depth of the failure slip surface, and a dimensionless parameter that can uniquely determine the depth of the slip surface is introduced. The reliability and accuracy of the calculation formula are validated through comparing the results against a large number of numerical predictions obtained using finite element limit analysis.

## 2 Strength parameters and undrained strength

Figure 1 has once been used by the author to explain the deficiencies of directly using the CU strength parameters to calculate the soil strength[2, 6]. In test 1, the confining pressure corresponding to point *a* is applied to the soil sample for consolidation, and then the vertical stress is increased under undrained conditions to shear the soil. The total stress path is the line ab with a slope of 1, and point *b* is the apex of the total stress Mohr circle at shear failure of the soil sample. For clarity, the effective stress path of this test is also plotted in the figure. Due to the generation of excess pore pressure, the effective stress path bends upwards to the left, but the shear stress corresponding to point *b*' is the same as point *b*. Similarly, the consolidation confining pressure in test 2 corresponds to point *c*, and the total stress path and effective stress path are *cd* and *cd*' respectively, of which the endpoint corresponds to failure. The failure lines $K_f'$ and $K_f$ are defined with the points *b*', *d*' for effective stress and points *b*, *d* for total stress respectively. After the determination of the failure line $K_f$ corresponding to the total stress, given the total stress path of a point in the geotechnical structure *ce*, assuming the soil properties and consolidation stress are the same as those in the triaxial test *cd*, but with the total stress path flatter than that of the triaxial test, the shear strength corresponding to point *e*, the cross point of the total stress path with the line $K_f$, will be obtained if it is calculated from the CU strength parameters directly using a formula of the same form as the Coulomb strength. However, the actual shear strength should be the same as the test cd because the density of the soil is the same. From the contradiction one can understand that the soil strength calculated from CU strength parameters directly using the Coulomb strength formula is correct only when the total stress path is

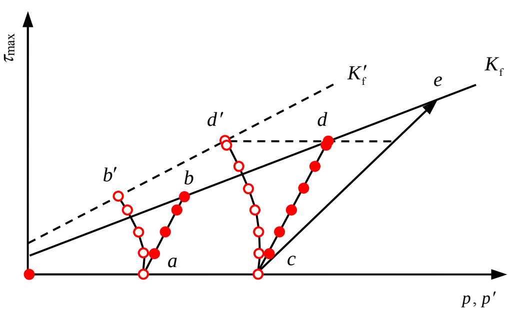



**Fig. 1 Stress path and failure line of CU triaxial tests[2, 6]**

parallel to that in the triaxial test. Otherwise, it will lead to an overestimation of the strength for a path with gentle slope and an underestimation of the strength on the contrary. Therefore, for the undrained analysis of saturated soil, the UU strength $c_u$ of undisturbed soil samples pre-consolidated under the in-situ self-weight stress should be used.

The UU strength $c_u$ of soil can be directly determined by laboratory tests or in-situ tests, and the vane shear test at different depths is considered suitable for saturated soft clay layers. However, the CU strength parameter $c_{cu}$ and $\varphi_{cu}$ are usually provided in the foundation survey report in China. Hence, it is appropriate to find a formula for calculating the correct UU strength of the soil from the corresponding CU parameters, so that the data in the geological survey report could be utilized. Such a formula can be derived through examining the total stress path in Fig.1. The stress path is the locus of the top point of the Mohr circle; the horizontal axis is marked as *p* or *p*', indicating the mean principal stress or the mean effective principal stress; $\tau_{max}$ is the maximum shear stress, read as half of the difference between the major and minor principal stresses; the $K_f$ line gives the ultimate shear stresses correspond to various consolidation stress. From the limit state conditions of soil, the intercept and the slope of the $K_f$ line can be obtained as $c_{cu}\cos\varphi_{cu}$ and $\sin\varphi_{cu}$ respectively. Taking the total stress path of any triaxial test, such as *ab*, denoting the consolidation stress at point *a* as $\sigma_{mc}'$, the shear stress at point *b* is the corresponding UU strength $c_u$. Noting the slope of the total stress path *ab* of the triaxial test is 1, its projection length on the horizontal axis is equal to $c_u$. Then, the following equation can be written as

$$c_u = c_{cu}\cos\varphi_{cu} + (\sigma_{mc}' + c_u)\sin\varphi_{cu} \tag{1}$$

Thus $c_u$ is solved as

$$c_u = c_{cu}\frac{\cos\varphi_{cu}}{1-\sin\varphi_{cu}} + \sigma_{mc}'\frac{\sin\varphi_{cu}}{1-\sin\varphi_{cu}} \tag{2}$$

This is the formula needed for calculating the corresponding UU strength from the CU strength parameters and the isotropic consolidation stress before undrained shearing. In the above derivation no approximation is conducted, and consequently relatively accurate UU strength can be reflected when the quality of the soil sampling is satisfactory, regardless over-consolidated or normally consolidated state. The same formula has been given in reference[4], though with a little bit more complex derivation. There are also some similar derivations according to the publications in recent years[8], and relatively speaking, the derivation from Fig.1 is the most concise.

For a soil that is $K_0$ consolidated rather than isotropically consolidated, it can be deduced from the critical state soil mechanics theory that the identical UU strength will be reached with the isotropically consolidated sample as long as the mean consolidation stress is the same[9]. Therefore, the CU strength parameters measured by isotropic consolidated undrained shear can be used to calculate the UU strength $c_u$ of $K_0$ consolidated soil by taking the consolidation stress $\sigma'_{mc}$ in formula (3) as $(1+K_0)\gamma' z/2$. Thus, the calculation formula adopted in this paper is obtained:

$$c_u = c_0 + c_{inc} z \tag{3}$$

where

$$\left.\begin{aligned} c_0 &= c_{cu}\frac{\cos\varphi_{cu}}{1-\sin\varphi_{cu}} \\ c_{inc} &= \frac{1+K_0}{2(1-\sin\varphi_{cu})}\gamma'\sin\varphi_{cu} \end{aligned}\right\} \tag{4}$$

where $c_0$ is the cohesion value at the top surface of the soil foundation; $c_{inc}$ is the increase rate of the cohesion with depth; $z$ is the depth measured from the top surface downwards; $\gamma'$ is the effective weight of soil. The $K_0$ can be estimated using the effective internal friction angle and degree of over consolidation with empirical formula, and $\varphi_{cu}$ can also be used for approximate calculation when the effective internal friction angle is unknown.

## 3 Ultimate bearing capacity calculation by using $c_u$

### 3.1 Calculation scheme

This section presents the calculation method for the bearing capacity of saturated cohesive soil foundation under rapid loading. As mentioned above, the UU strength $c_u$ should be employed, and it increases approximately linearly with the depth, as can be determined using Eqs. (3) and (4). The corresponding total stress friction angle $\varphi_u$ is 0.

In order to calculate the ultimate bearing capacity of this type of foundations, the following calculation formula is given via theoretical analysis as well as numerical simulation:

$$p_u = (c_0 + 0.5\beta c_{inc} Z_{max})N_c + qN_q \tag{5}$$

where $p_u$ is the ultimate bearing capacity of the foundation; $q$ is the overload around the footing corresponding to the buried depth of the footing. Both $N_c$ and $N_q$ are the bearing capacity coefficients in Prandtl-Reissner formula. Since the friction angle corresponding to the total stress under unconsolidated and undrained conditions is 0, so these two coefficients should be taken as $2+\pi$ and 1, respectively; $Z_{max}$ is the maximum depth of the sliding surface in case of foundation instability and failure; and $\beta$ is the correction factor considering the shape of the slip surface.

$Z_{max}$ is estimated by multiplying the maximum depth of sliding surface given by Prandtl for the foundation soil of constant cohesion and zero friction angle[10] with the correction coefficient $\alpha$ , i.e.

$$Z_{max} = \alpha\sqrt{2}B/2 \tag{6}$$

where the coefficient $\alpha$ can be called the depth coefficient of sliding surface. The determination of $\alpha$ as well as the shape coefficient $\beta$ will be given in Section 3.2.

The shape coefficient of the slip surface in Eq. (5) is approximately 1 when the cohesion increases relatively mildly with depth, and thus $c_0 + 0.5\beta c_{inc} Z_{max}$ therein can be regarded as the cohesion at the average depth of the slip surface. However, calculation results show that when the cohesion increases steeply with depth, the slip surface becomes flat, and consequently the average depth is greater than the half depth. More importantly, the foundation bearing capacity coefficient in that case is also different from the $N_c$ given by Prandtl. Therefore, the coefficient $\beta$ plays the role of correcting the errors in both aspects.

The second term from Eq.(5) stands for the contribution to the bearing capacity from the weight of soil within foundation buried depth, treated as an overload. This term follows the calculation configuration of the existing formula as proved through both theoretical analyses and numerical calculations, and thus it needs no modification.

Then the coefficients $\alpha$ and $\beta$ is to be determined, which requires some theoretical analyses and a lot of refined numerical calculations. To that end the finite element limit analysis software OptumG2 [11], has been applied. The software calculates the limit load or safety factor of the structure according to the upper and lower limit theorem of limit analysis, and gives the upper and lower limits respectively. The adaptive technique is also used to gradually refine the finite element mesh where the failure slip surface is located, so as to improve the calculation accuracy and make the upper and lower limits approaching each other. The bisection value of the upper and lower solutions is expected to achieve high accuracy when the magnitudes of both are close[11–12].

In the analyses, the ground under rigid foundation is discretized into finite elements. As central loads are considered at present, a half mesh has been set up due to the symmetry of the project. The corresponding soil weight $q$ within the buried depth shall be applied to the top boundary of the mesh around the foundation as overload when it comes to a buried form. The constitutive model of soil adopts the ideal elastic-plastic Mohr Coulomb model, and because the friction angle is 0 herein, it performs actually the same as Tresca model. Under such conditions, the ultimate load is independent of the modulus of soil, and any reasonable value can be adopted in the calculations.

### 3.2 The depth growth rate of cohesion and the determination of $\alpha$ and $\beta$

The coefficient $\alpha$ is the depth correction coefficient of slip surface, and its physical significance is the difference between the depth of sliding surface and the corresponding Prandtl solution when the cohesion increases with depth. It is not difficult to understand that the greater the relative growth rate of cohesion with depth, the smaller the depth of the slip surface will be. This relative growth rate of cohesion with depth should be understood as the ratio of incremental cohesion to its stable value at certain increment of depth. Through theoretical analyses, this relative growth rate is defined as the dimensionless parameter $k$ [13] calculated by the following formula:

$$k = \frac{c_{inc}B}{c_0} \tag{7}$$

where $B$ is the foundation width.

A large number of calculations by finite element limit analysis show that regardless of the variation of the three parameters $B$, $c_0$ and $c_{inc}$ at the right end of Eq. (7), as long as the dimensionless parameter $k$ calculated by the formula retains the same value, the correction coefficient $\alpha$ for determining the maximum depth of sliding surface is consistent[13]. Therefore, the following regression formula for determining $\alpha$ from $k$ is given after fitting a large number of calculation results:

$$\alpha = 1 - e^{-1/k^{0.5}} \tag{8}$$

Apparently, the coefficient $\alpha$ is 1 at $k = 0$ and tends to 0 when $k$ tends to infinity, which looks reasonable at least from the theoretical aspects.

A large dataset of calculations also shows that the value of the coefficient $\beta$ is also uniquely determined by the value of $k$. Since the coefficient $\beta$ has the function of involving the shape and average depth of slip surface into calculation and correcting the corresponding bearing capacity coefficient $N_c$, its value needs to be determined by fitting the bearing capacity. The approach employed for that is to directly fit the specific calculation formula of bearing capacity without buried depth, and then sort out the $\beta$ calculation formula with clear physical significance in the form of the first term at the right end of Eq. (5). The calculation formula given by a large number of calculations and fitting is

$$\beta = 1 + \frac{k^{0.5}}{\sqrt{2}(2+\pi)} \tag{9}$$

This coefficient is 1 at $k = 0$ and tends to infinity when $k$ becomes infinity. The constant $\sqrt{2}$ comes in fact from the calculation formula of the slip surface depth, and $(2+\pi)$ is actually $N_c$ when soil friction angle is zero.

It can be seen from the above calculation formula of $\alpha$ and $\beta$ that when $c_0 = 0$, $c_{inc} > 0$, $k$ is infinite, and the first term $0.5\beta c_{inc} Z_{max} N_c$ at the right side of Eq. (5) is indefinite. Nevertheless, the foundation bearing capacity in this case can still be deducted by limit calculation:

$$p_u = \frac{1}{4} C_{inc} B + q \tag{10}$$

which is consistent with the solution given by Davis et al. [7] using the slip line method

In addition, for the case of smooth footing base, the above two coefficients are obtained by fitting with the results of finite element analysis as follows:

$$\left.\begin{aligned} \alpha &= 1 - e^{-0.6/k^{0.5}} \\ \beta &= 1 + \frac{k^{0.5}}{0.6\sqrt{2}(2+\pi)} \end{aligned}\right\} \tag{11}$$

The foundation bearing capacity now is smaller than that when the base is rough, but it is not difficult to verify that both bearing capacity values tend to be the same when $k = 0$ and $k$ tends to infinity.

## 4 Verification of the calculation method

After the maximum depth and shape coefficient $\beta$ of the sliding surface being determined, the foundation bearing capacity can be calculated using Eqs. (5)–(9) for the rough footing base condition, whilst the Eqs. (5)–(7) and (11) for the smooth footing base condition.

Table 1 and 2 list the comparison of calculated bearing capacity $p_u$ and slip surface depth $Z_{max}$ against the finite element limit analysis results for the conditions of smooth and rough foundation, respectively, with varying foundation width $B$, foundation soil strength parameters $c_0$, $c_{inc}$, and corresponding $k$.

**Table 1 Verification of the calculation method for rough strip footing**

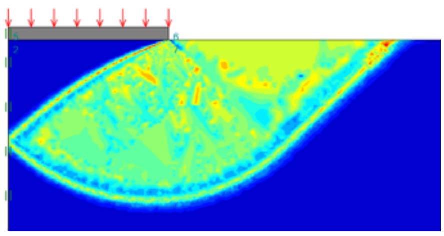
| Case No. | $B$ /m | $c_0$ /kPa | $c_{inc}$ /(kPa • m$^{-1}$) | $q$ /kPa | Dimensionless results $k$ | Limit analysis results $Z_{max}$ /m | Limit analysis results $\alpha$ | Limit analysis results $p_u$ /kPa | Results of the formula $Z_{max}$ /m | Results of the formula $\alpha$ | Results of the formula $p_u$ /kPa | Deviation of the predicted bearing capacity /% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 10.0 | 0 | 0 | 0.0 | 4.24 | 1.00 | 51.4 | 4.24 | 1.00 | 51.4 | 0.0 |
| 2 | 12 | 10.0 | 0 | 0 | 0.0 | 8.49 | 1.00 | 51.4 | 8.49 | 1.00 | 51.4 | 0.0 |
| 3 | 3 | 10.0 | 1 | 0 | 0.3 | 1.77 | 0.83 | 56.7 | 1.78 | 0.84 | 56.3 | –0.7 |
| 4 | 6 | 20.0 | 1 | 0 | 0.3 | 3.54 | 0.83 | 113.4 | 3.56 | 0.84 | 112.7 | –0.7 |
| 5 | 6 | 20.0 | 1 | 50 | 0.3 | 3.54 | 0.83 | 163.4 | 3.56 | 0.84 | 162.7 | –0.5 |
| 6 | 3 | 10.0 | 2 | 0 | 0.6 | 1.57 | 0.74 | 61.1 | 1.54 | 0.72 | 60.2 | –1.6 |
| 7 | 6 | 20.0 | 2 | 0 | 0.6 | 3.13 | 0.74 | 122.2 | 3.08 | 0.72 | 120.3 | –1.5 |
| 8 | 12 | 20.0 | 1 | 0 | 0.6 | 6.27 | 0.74 | 122.2 | 6.15 | 0.72 | 120.3 | –1.5 |
| 9 | 3 | 10.0 | 4 | 0 | 1.2 | 1.28 | 0.60 | 68.3 | 1.27 | 0.60 | 66.4 | –2.7 |
| 10 | 12 | 10.0 | 1 | 0 | 1.2 | 5.12 | 0.60 | 68.3 | 5.08 | 0.60 | 66.4 | –2.7 |
| 11 | 3 | 5.0 | 4 | 0 | 2.4 | 1.01 | 0.48 | 39.7 | 1.01 | 0.48 | 38.3 | –3.5 |
| 12 | 6 | 10.0 | 4 | 0 | 2.4 | 2.04 | 0.48 | 79.3 | 2.02 | 0.48 | 76.6 | –3.5 |
| 13 | 12 | 5.0 | 3 | 0 | 7.2 | 2.65 | 0.31 | 55.6 | 2.64 | 0.31 | 53.6 | –3.7 |
| 14 | 12 | 10.0 | 6 | 0 | 7.2 | 2.65 | 0.31 | 111.3 | 2.64 | 0.31 | 107.2 | –3.7 |
| 15 | 6 | 1.0 | 4 | 0 | 24.0 | 0.82 | 0.19 | 19.1 | 0.78 | 0.18 | 18.6 | –2.6 |
| 16 | 12 | 1.0 | 2 | 0 | 24.0 | 1.60 | 0.19 | 19.1 | 1.57 | 0.18 | 18.6 | –2.5 |
| 17 | 6 | 0.2 | 4 | 0 | 120.0 | 0.45 | 0.11 | 10.5 | 0.37 | 0.09 | 10.6 | 0.5 |
| 18 | 12 | 0.4 | 4 | 0 | 120.0 | 0.91 | 0.11 | 21.0 | 0.74 | 0.09 | 21.1 | 0.5 |
| 19 | 6 | 0.0 | 2 | 0 | +∞ | 0.05 | 0.01 | 3.1 | 0.00 | 0.00 | 3.0 | –2.5 |
| 20 | 12 | 0.0 | 2 | 0 | +∞ | 0.10 | 0.01 | 6.1 | 0.00 | 0.00 | 6.0 | –1.5 |

**Table 2 Verification of the calculation method for smooth strip footing**

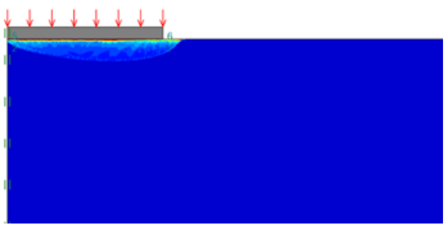
| Case No. | $B$ /m | $c_0$ /kPa | $c_{inc}$ /(kPa • m$^{-1}$) | $q$ /kPa | Dimensionless parameter $k$ | Limit analysis results $Z_{max}$ /m | Limit analysis results $\alpha$ | Limit analysis results $p_u$ /kPa | Results of the formula $Z_{max}$ /m | Results of the formula $\alpha$ | Results of the formula $p_u$ /kPa | Deviation of the predicted bearing capacity /% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 10.0 | 0 | 0 | 0.0 | 4.24 | 1.00 | 51.4 | 4.24 | 1.00 | 51.4 | 0.0 |
| 2 | 12 | 10.0 | 0 | 0 | 0.0 | 8.49 | 1.00 | 51.4 | 8.49 | 1.00 | 51.4 | 0.0 |
| 3 | 3 | 10.0 | 1 | 0 | 0.3 | 1.25 | 0.59 | 54.2 | 1.41 | 0.67 | 55.5 | 2.4 |
| 4 | 6 | 20.0 | 1 | 0 | 0.3 | 2.50 | 0.59 | 108.4 | 2.82 | 0.67 | 111.0 | 2.4 |
| 5 | 6 | 20.0 | 1 | 50 | 0.3 | 2.50 | 0.59 | 158.4 | 2.82 | 0.67 | 161.0 | 1.6 |
| 6 | 3 | 10.0 | 2 | 0 | 0.6 | 1.08 | 0.51 | 56.7 | 1.14 | 0.54 | 58.3 | 2.8 |
| 7 | 6 | 20.0 | 2 | 0 | 0.6 | 2.15 | 0.51 | 113.5 | 2.29 | 0.54 | 116.7 | 2.8 |
| 8 | 12 | 20.0 | 1 | 0 | 0.6 | 4.35 | 0.51 | 113.5 | 4.57 | 0.54 | 116.7 | 2.8 |
| 9 | 3 | 10.0 | 4 | 0 | 1.2 | 0.91 | 0.43 | 61.3 | 0.89 | 0.42 | 62.9 | 2.7 |
| 10 | 12 | 10.0 | 1 | 0 | 1.2 | 3.63 | 0.43 | 61.2 | 3.58 | 0.42 | 62.9 | 2.8 |
| 11 | 3 | 5.0 | 4 | 0 | 2.4 | 0.77 | 0.36 | 34.5 | 0.68 | 0.32 | 35.2 | 1.9 |
| 12 | 6 | 10.0 | 4 | 0 | 2.4 | 1.54 | 0.36 | 69.1 | 1.36 | 0.32 | 70.4 | 1.9 |
| 13 | 12 | 5.0 | 3 | 0 | 7.2 | 1.87 | 0.22 | 47.0 | 1.70 | 0.20 | 46.9 | –0.3 |
| 14 | 12 | 10.0 | 6 | 0 | 7.2 | 1.87 | 0.22 | 94.1 | 1.70 | 0.20 | 93.8 | –0.3 |
| 15 | 6 | 1.0 | 4 | 0 | 24.0 | 0.70 | 0.16 | 16.2 | 0.49 | 0.12 | 15.8 | –2.1 |
| 16 | 12 | 1.0 | 2 | 0 | 24.0 | 1.40 | 0.16 | 16.2 | 0.98 | 0.12 | 15.8 | –2.1 |
| 17 | 6 | 0.2 | 4 | 0 | 120.0 | 0.38 | 0.09 | 9.3 | 0.23 | 0.05 | 9.2 | –1.2 |
| 18 | 12 | 0.4 | 4 | 0 | 120.0 | 0.76 | 0.09 | 18.6 | 0.45 | 0.05 | 18.4 | –1.1 |
| 19 | 6 | 0.0 | 2 | 0 | +∞ | 0.05 | 0.01 | 3.1 | 0.00 | 0.00 | 3.0 | –2.5 |
| 20 | 12 | 0.0 | 2 | 0 | +∞ | 0.10 | 0.01 | 6.1 | 0.00 | 0.00 | 6.0 | –1.6 |

It can be seen from the tables that the maximum depth of the sliding surface is the same as long as the corresponding $k$ value unchanged, regardless of the varying combination of $B$、 $c_0$ and $c_{inc}$. The bearing capacity calculated by the formula is in good agreement with the results of the finite element limit analysis, and the relative error is less than 5%.

For demonstrating the shape of the slip surface when the foundation reaches the limit state, Fig. 2 gives the plastic strain contour given by finite element limit analysis in case 8 and 18 from Table 1 as examples. The bright part in the figure is the plastic area. It is found that when the dimensionless parameter $k$ is small (Fig. 2 (a), $k = 0.6$), the depth of the sliding surface is large, whereas a large $k$ value (Fig. 2 (b), $k = 20.0$) leads to the shallow and flat slip surface. The elastic core beneath the foundation can be found from Fig.2 (a) in this case, due to the existence of cohesion and the rough base assumption although the friction angle is zero.



(a) Case 8 from Table 1



(b) Case 18 from Table 1

**Fig. 2 Diagrams of typical slip surfaces**

## 5 Discussion of some influence factors

### 5.1 Influence of the foundation width

For foundation soil with constant cohesion and zero friction angle, the bearing capacity is independent of the foundation width. When the friction angle of the soil is non-zero, the self-weight of the foundation soil enhances the bearing capacity with the foundation width, and according to the classical theory the relevant term increases linearly with the footing width.

For the foundation soil studied here, although the friction angle is 0, the cohesion increases with the depth, and consequently the bearing capacity of the foundation soil increases with the foundation width, since the depth of the failure slip surface increases with the foundation width. Figure 3 shows this feature for a case with rough footing base, and the soil strength parameters are $c_0 = 20$ kPa and $c_{inc} = 2.88$ kPa/m (calculated from $\varphi_{cu} = 15°$ and $\gamma' = 10$ kN/m$^3$). It can be seen that the gradient of bearing capacity increasing with the foundation width is not constant, but develops from a steeper value to a stabilized one.

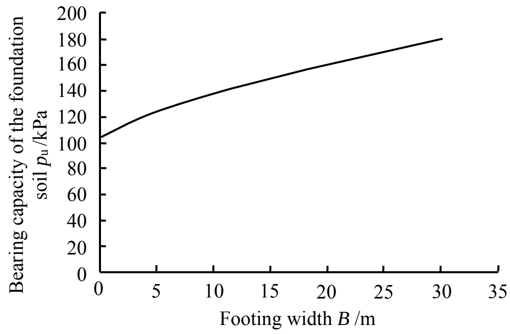



**Fig. 3 Relation between bearing capacity and footing width**

### 5.2 Influence of the soil parameters $c_0$ and $c_{inc}$

Figure 4 shows that the bearing capacity of foundation soil increases with $c_0$. The foundation width is 6 m,

$c_{\mathrm{inc}} = 2.88$ kPa/m (calculated from $\varphi_{\mathrm{cu}} = 15°$ and $\gamma' = 10$ kN/m$^3$) in this calculation. It can be seen from the figure that the slope is positive starting from a higher value and then tending to constant.

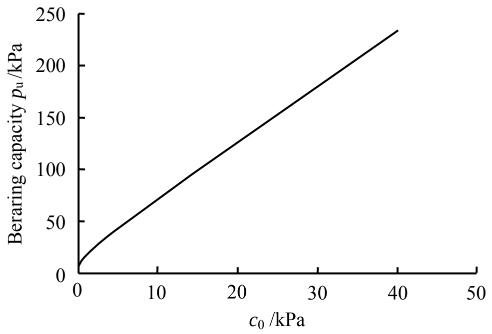



**Fig. 4 Relation between bearing capacity and $c_0$**

Figure 5 shows that the foundation soil bearing capacity increases with $c_{\mathrm{inc}}$. The foundation width of 6 m and soil parameter $c_0 = 20$ kPa are also taken here. Similarly, the positive trend of bearing capacity with $c_{\mathrm{inc}}$ performs higher gradient at the beginning and then converges to a certain constant value.

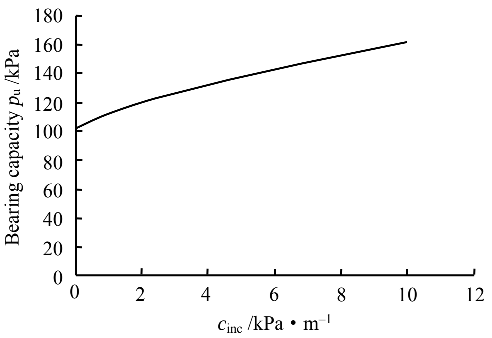



**Fig. 5 Relation between bearing capacity and $c_{\mathrm{inc}}$**

## 6 Summary

In this paper, the calculation of bearing capacity of saturated cohesive soil foundation under relatively rapid loading is studied. Firstly, it is clarified that the unconsolidated undrained (UU) strength should be used for this calculation, and the formula for deriving the UU strength from the consolidated undrained strength parameters, usually provided in the field investigation reports in China, is introduced. Secondly, the linearly increasing UU strength with the depth has been considered, and the corresponding calculation method is established for the foundation bearing capacity. The key philosophy is to construct a dimensionless parameter $k$ uniquely determining the depth of the sliding surface as well as the correction of the foundation bearing capacity coefficient considering the change of the shape of the sliding surface. A large number of comparisons against the numerical results using finite element limit analysis show good agreement of the results, validating the precision and correctness of the dimensionless parameters and the calculation method for bearing capacity. Finally, the influence of foundation width and two soil strength parameters on foundation bearing capacity have been calculated and analyzed, which shows that the foundation bearing capacity increases with these three quantities, but the slope shows converging shape and stabilizes to a constant value. Despite the zero-friction angle for the soil studied here, the bearing capacity of foundation increases with the foundation width, which is different from that when the cohesion is constant.

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