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Analytical solution of the long-term service performance of tunnel considering surrounding rock rheology and lining deterioration characteristics

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Abstract: Deterioration of lining is a common phenomenon in tunnel engineering that threatens the safety of operating tunnel. For tunnel lining deterioration in rheological rock mass, an analytical model is established which the rheology effect of surrounding rock and deterioration of lining are considered. The surrounding rock displacement and support pressure of tunnel during its service life are obtained. And then the correctness of analytical model is verified by numerical simulation. Subsequently, the sensitivity analyses of deterioration coefficient of lining, thickness of lining, support time, and rheological properties of surrounding rock are carried out by the proposed analytical model. Finally, the service performance of tunnel is discussed. Research shows that under the rheological effect of surrounding rock, support pressure increases with time, while the mechanical properties of support structure decrease with time. Therefore, the time which support structure reaching its yield strength is shortened. Time history curves of support pressure and bearing capacity of lining are then obtained. Based on this, a model for predicting the service life of operating tunnel is established which the rheology effect of surrounding rock and support performance deterioration are considered. Finally, it is stated that the interaction between support and surrounding rock is the core for the long-term safety of operating tunnel.

Keywords: operating tunnel; rheology effect of surrounding rock; lining deterioration; analytical solution; service performance

1 Introduction

At present, more and more tunnels have been built in deep soft rock with high geo-stress. Meanwhile, large deformation and insufficient bearing capacity of linings occurred in some tunnels after they were put into operation, leading to lining cracking, concrete collapse. For example, significant rheological effects of surrounding rock are observed in the operation stage of the L. T. Base Tunnel in France^[1], the Uresino tunnel in Japan^[2], the Muzhailing tunnel^[3] and Xuecheng tunnel^[4] in China, which threaten the tunnel operation safety. In the 1980s, Sulem et al.^[5] and Ladanyi et al.^[6] proposed the viscoelastic analytical solutions for the excavation and installation of circular tunnels in hydrostatic stress field. Recently, researchers^[7–12] have derived viscoelastic solutions on a series of complex issues such as the three-dimensional spatial effect of tunnel excavation in non-hydrostatic stress field, excavation process of tunnel, and tunnel supported with two linings. It can be seen that the rheological phenomenon of surrounding rock has been highly concerned. Still, the long-term rheological effect of operation tunnel surrounding rock has not attracted enough attention. With the application of single-layer support in tunnel engineering^[13–14], some studies

revealed that the rheological load of surrounding rock is one of the leading causes for lining failure^[6, 15–17]. However, the existing studies have ignored the fact that the mechanical properties of support structure deteriorate over time.

Due to the shortcomings in tunnel design and construction stage, the concrete lining has a characteristic of low strength, low compactness, and high permeability. The concrete lining deteriorates after corrosive medium enters the concrete, inducing a series of tunnel damages^[18]. As shown in Fig.1, the deterioration of concrete lining widely exists in tunnel engineering^[19–23]. Zhou et al.^[24] conducted a series of accelerated corrosion tests and pointed out that carbonated water environment would cause the concrete lining structure to become loose, significantly reducing the structure's elastic modulus and uniaxial compressive strength. The bearing capacity tests of lining concrete under sulfate attack were carried out by He et al.^[25], and it was found that the bearing capacity of lining concrete decreased by 50% after 720 days of accelerated corrosion. For this reason, the time-dependent model of concrete lining mechanical properties during service period is established. For example, Niu et al.^[26–27] analyzed the concrete exposure test data and found that concrete strength changes over time. They studied the time-dependence

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law of concrete strength in general atmospheric and marine environments and proposed the time-dependence model of concrete strength. Wang et al.^[28] and Li et al.^[29] obtained the degradation law of concrete lining of a tunnel in 100 service life by indoor rapid freeze–thaw cycle tests. The results showed that the elastic modulus of concrete lining decreases exponentially with increases of freeze–thaw cycles. Aghchai et al.^[30] considered that effective thickness of concrete lining would decrease under action of groundwater, and proposed a simplified time-dependence model of lining thickness in service environment. The linear degradation model of lining elastic modulus and strength over time is presented.

The mechanical properties of concrete lining are significantly reduced after deterioration. Due to limitation of test conditions, most of the existing lining concrete deterioration tests do not consider the interaction between support and surrounding rock. In recent years, the analysis for influence of lining deterioration on tunnel service performance has been conducted by numerical simulation method. Andreotti et al.^[31] and Ding et al.^[32] used the elastic modulus reduction method to describe the deterioration of concrete lining during the service period of mountain tunnels and analyzed the vulnerability response of lining structure under earthquake action. Usman et al.^[33] established a 3D numerical simulation model, and the degradation process of concrete lining is represented by decreasing the mechanical parameters step by step. Further, the impact of concrete lining degradation on tunnel structure mechanics is analyzed. Aghchai et al.^[30] and Sandrone et al.^[34] investigated the influence of surrounding rock deterioration and the degradation of supporting performance on tunnel service safety by reducing mechanical parameters. They pointed out that the deterioration of concrete lining significantly threatens the tunnel safety.

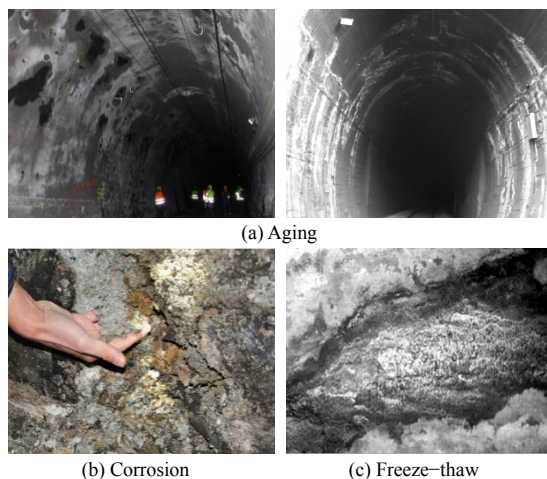


Fig. 1 Deterioration status of operating tunnel lining^[19, 21]

The load on support structures increases over time

resulting from long-term surrounding rock rheological load. Therefore, the bearing capacity of support structures can not meet the requirement of long-term tunnel stability^[35]. In addition, the lining performance gradually decreases under the adverse service environment. Under the coupled action of rheological effect of surrounding rock and degradation of concrete lining, the failure of lining structure is accelerated and the tunnel service life is shortened. At present, the study about lining degradation process under the long-term rheological effect of surrounding rock is not gaining enough attention. There is a lack of analysis of the derivation mechanism of tunnel damages in rheological rock mass.

In this study, a time-dependent analytical model considering rheology of surrounding rock and deterioration of support performance is established. A FLAC^{3D} numerical simulation is established to verify the correctness of analytical solution. Furthermore, the influence of lining degradation degree, lining thickness, installation time of concrete lining, and rheology relaxation time of surrounding rock are discussed. Finally, the service life of tunnel under the coupled effect of surrounding rock rheological effect and degradation effect of support is predicted. This study provides a reference for lining design, safety evaluation, and damage prevention of operation tunnels in rheological rock mass.

2 Time-dependent analytical solution

2.1 Analytical model

According to the findings in references [26–29], the degradation of concrete lining is represented by decreasing stiffness and strength over time in analytical model. The exponential attenuation function is selected to describe the degradation process. In this paper, the elastic incremental constitutive model is used to describe the mechanical properties of lining structure^[36], assuming that the deterioration rate of lining strength and stiffness is equal^[30]. The time-dependent elastic modulus $E(t)$ and compressive strength $\sigma(t)$ of concrete lining are given by as follow:

$$\frac{E(t)}{E_0} = \frac{\sigma(t)}{\sigma_0} = e^{-\beta(t-t_0)} \quad (1)$$

where t_0 is the installation time of lining; E_0 and σ_0 are the elastic modulus and strength of lining when $t = t_0$; and β is a constant, representing the lining deterioration coefficient.

Assuming that the surrounding rock is a continuous, uniform and isotropic viscoelastic body, the common viscoelastic rheological constitutive models of rock mass are shown in Fig. 2. The Kelvin and generalized Kelvin bodies are viscoelastic solids. The creep strain

first increases over time and finally tends to a specific value, which can better describe the creep attenuation stage of hard rock. Maxwell and Burgers bodies are viscoelastic fluids. The creep strain increases over time, which can well describe the stable creep stage of rock. These two models are widely used in soft rock engineering. The Maxwell and generalized Kelvin bodies are adopted in the following.

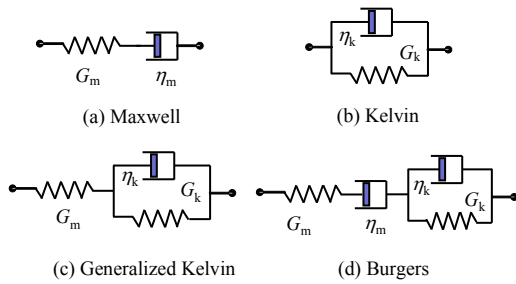


Fig.2 Viscoelastic models

As shown in Fig. 3, an analytical model considering the rheology of surrounding rock and the degradation of concrete lining is established. In the figure, p_0 is the hydrostatic pressure, r_1 is the tunnel radius, r_2 is the inner radius of concrete lining, and $E(t)$ and ν are the elastic moduli and Poisson's ratio of concrete lining. It should be pointed out that the degradation of concrete lining is related to numerous factors such as groundwater, air, temperature, and external load, which is complex and uncertain^[37]. Therefore, the interaction relationship between concrete lining and surrounding rock under different deterioration degrees will be studied next.

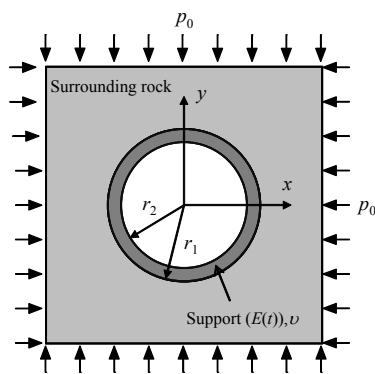


Fig.3 Analytical model

2.2 Solution in tunnel excavation stage

Assuming that the tunnel is excavated at $t = 0$, and the three-dimensional effect is considered in analytical model. The stress release coefficient of surrounding rock ($\eta(t)$) is introduced^[38]:

$$\eta(t) = 1 - \alpha e^{-mt} \tag{2}$$

where α is a parameter related to stress release of

surrounding rock, and parameter m determines the advancing speed of tunnel excavation.

The virtual support force ($\sigma_{r=r_1} = [1 - \eta(t)]p_0$) is applied on the inner boundary of tunnel $r=r_1$. Meanwhile, the other boundary condition is $\sigma_{r \rightarrow \infty} = p_0$. Therefore, the stresses of surrounding rock can be given as

$$\sigma_r(t) = \left(1 - \eta(t)\frac{r_1^2}{r^2}\right)p_0, \sigma_\theta(t) = \left(1 + \eta(t)\frac{r_1^2}{r^2}\right)p_0 \tag{3}$$

where $\sigma_r(t)$ and $\sigma_\theta(t)$ are the radial and tangential stress of surrounding rock, respectively. According to the generalized Hooke's law, we have

$$\varepsilon_z = \frac{1}{E}[\sigma_z(t) - \nu_r(\sigma_r(t) + \sigma_\theta(t))] \tag{4}$$

where $\sigma_z(t)$ is the stress of surrounding rock in z -direction, and $\varepsilon_z = 0, \nu_r = 0.5$.

Inserting Eq. (3) into Eq. (4), we have $\sigma_z(t) = p_0$. Therefore, the following expression is obtained:

$$\sigma_m = \frac{\sigma_r(t) + \sigma_\theta(t) + \sigma_z(t)}{3} = p_0 \tag{5}$$

It can be seen that the volumetric stress of surrounding rock before and after tunnel excavation is p_0 . It is considered that tunnel excavation does not cause volumetric strain change. Therefore, the increment of tangential deviator stress induced by tunnel excavation is

$$\left. \begin{aligned} s_r(t) &= \left(1 - \eta(t)\frac{r_1^2}{r^2}\right)p_0 - p_0 = -\eta(t)\frac{r_1^2}{r^2}p_0 \\ s_\theta(t) &= \left(1 + \eta(t)\frac{r_1^2}{r^2}\right)p_0 - p_0 = \eta(t)\frac{r_1^2}{r^2}p_0 \end{aligned} \right\} \tag{6}$$

where $s_r(t)$ and $s_\theta(t)$ are the radial and tangential deviatoric stresses of surrounding rock, respectively. Then, the viscoelastic integral constitutive equation is listed as

$$\left. \begin{aligned} e_{ij} &= J(t) * ds_{ij}(t) \\ \varepsilon_m &= \sigma_m / 3K \end{aligned} \right\} \tag{7}$$

where $J(t)$ is the creep shear modulus of surrounding rock; K is the bulk modulus of surrounding rock; e_{ij} and ε_{ij} are deviatoric and volumetric strain, respectively. The symbol $*$ represents convolution operation:

$$f_1(t) * df_2(t) = f_1(t)f_2(0) + \int_0^t f_1(t - \xi) \frac{\partial f_2(\xi)}{\partial \xi} d\xi \tag{8}$$

where ξ is a variable with $0 < \xi < t$.

According to the above analysis, the volumetric strain of surrounding rock before and after tunnel excavation is 0. By combining Eqs. (6)–(8), the tangential deviation strain ($\varepsilon_\theta^{dev}(t)$) of surrounding rock caused by tunnel excavation is given as

$$\varepsilon_\theta^{dev}(t) = \varepsilon_\theta(t) - \varepsilon_m(t) \tag{9}$$

where $\varepsilon_\theta(t)$ is the tangential stress. Considering Eqs.

(7) and (8), the following expression is obtained:

$$\varepsilon_{\theta}^{\text{dev}}(t) = J(t)s_{\theta}(0) + \int_0^t J(t-\xi)[s_{\theta}(\xi)]' d\xi \quad (10)$$

According to the geometric relation of surrounding rock, the relationship between tangential deviation strain ($\varepsilon_{\theta}^{\text{dev}}(t)$) and radial displacement ($u_R^1(t)$) of surrounding rock is written as

$$\varepsilon_{\theta}^{\text{dev}}(r, t) = \frac{u_R^1(t)}{r} \quad (11)$$

Substituting Eqs. (6) and (11) into Eq. (10), the radial displacement induced by tunnel excavation can be calculated by

$$\frac{u_R^1(t)}{r} = p_0 J(t)\eta(0) + p_0 \int_0^t J(t-\xi)\eta'(\xi)d\xi \quad (12)$$

2.3 Solution for supported tunnel

The installation of concrete lining is at $t = t_0$ and $t = t_0 + \tau$ is defined for ease of analysis. The contact pressure between concrete lining and surrounding rock is $q(\tau)$, and radial displacement of concrete lining is $u_L(\tau)$. It can be seen that $u_L(0) = 0$. The time sequence of tunnel excavation and lining installation is shown in Fig.4.

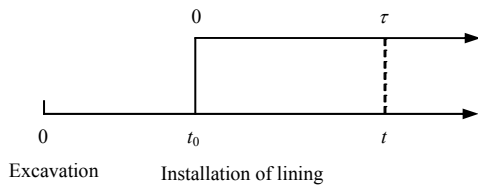


Fig. 4 Time of tunnel excavation and support

2.3.1 Rock displacement after lining installation ($t \geq t_0$)

Same as the derivation process in Section 2.2, the radial displacement of surrounding rock ($u_R^2(\tau)$) under the action of concrete lining is listed as

$$u_R^2(\tau) = r \left[q(0)J(\tau) + \int_0^{\tau} J(\tau-\xi) \frac{dq(\xi)}{d\xi} d\xi \right] = r \left[q(\tau)J(0) + \int_0^{\tau} q(\xi) \frac{dJ(\tau-\xi)}{d(\tau-\xi)} d\xi \right] \quad (13)$$

To reduce the time variables in the analysis, t is replaced by $t_0 + \tau$, the radial displacement of surrounding rock induced by tunnel excavation during $t_0 - t$ is

$$u_R^1(t_0 + \tau) - u_R^1(t_0) = rp_0 [J(t_0 + \tau) - J(t_0)]\eta(0) + rp_0 \left[\int_0^{t_0 + \tau} J(t_0 + \tau - \xi)\eta'(\xi)d\xi - \int_0^{t_0} J(t_0 - \xi)\eta'(\xi)d\xi \right] \quad (14)$$

2.3.2 Displacement and stress of concrete lining

The time-dependent degradation model of support structure is introduced here, and the exponential degradation model is adopted for lining: $E(t) = E_0 e^{-\beta t}$. In the derivation process, lining degradation effect is realized by reducing elastic modulus. It is assumed that in each time increment $\Delta\tau$, the support stress and

displacement on the contact surface between support and surrounding rock are $\Delta q(\tau)$ and $\Delta u_L(\tau)$, respectively. The following expression could be obtained:

$$\Delta q(\tau) = \frac{\Delta u_L(\tau)}{r_1} K_S(\tau) \quad (15)$$

where $K_S(\tau) = K_L e^{-\beta\tau}$; and $K_L = \frac{(1-r_2^2/r_1^2)E_0}{(1-2\nu+r_2^2/r_1^2)(1+\nu)}$.

By integrating both sides of Eq. (15), the following equation can be obtained after simplification:

$$q(\tau) = \frac{K_L}{r_1} e^{-\beta\tau} u_L(\tau) + \frac{K_L \beta}{r_1} \int_0^{\tau} u_L(\tau) e^{-\beta\tau} d\tau \quad (16)$$

According to the elastic theory, the stress component of support structure can be expressed by

$$\left. \begin{aligned} \sigma_r(r, \tau) &= \frac{r_1^2 r_2^2 q(\tau)}{r^2 (r_2^2 - r_1^2)} - \frac{r_1^2 q(\tau)}{(r_2^2 - r_1^2)} \\ \sigma_{\theta}(r, \tau) &= -\frac{r_1^2 r_2^2 q(\tau)}{r^2 (r_2^2 - r_1^2)} - \frac{r_1^2 q(\tau)}{(r_2^2 - r_1^2)} \end{aligned} \right\} \quad (17)$$

2.3.3 Compatibility condition of displacement at the interface between surrounding rock and support

After the installation of lining, the displacement at interface between surrounding rock and support should satisfy the compatibility condition when $t \geq t_0$. Therefore, the continuity of displacement between surrounding rock and support imposes:

$$u_R^1(\tau + t_0) - u_R^1(t_0) - u_R^2(\tau) = u_L(\tau) \quad (18)$$

For convenience, the undetermined function $h(\tau)$ is introduced here:

$$u_R^1(\tau + t_0) - u_R^1(t_0) = h(\tau)r_1 \quad (19)$$

Considering Eq. (14), the expression of $h(\tau)$ can be listed as

$$h(\tau) = p_0 [J(t_0 + \tau) - J(t_0)]\eta(0) + p_0 \left[\int_0^{t_0 + \tau} J(t_0 + \tau - \xi)\eta'(\xi)d\xi - \int_0^{t_0} J(t_0 - \xi)\eta'(\xi)d\xi \right] \quad (20)$$

Substituting Eqs. (13) and (19) into Eq. (18), the following expression is obtained

$$r_1 h(\tau) - r_1 \int_0^{\tau} J(\tau - \xi) \frac{dq(\xi)}{d\xi} d\xi = u_L(\tau) \quad (21)$$

Combining Eq. (16), we have

$$\frac{dq(\tau)}{d\tau} = \frac{K_L}{r_1} e^{-\beta\tau} \frac{du_L(\tau)}{d\tau} \quad (22)$$

Substituting Eq. (22) into Eq. (21), the equation of $u_L(\tau)$ can be given as follows:

$$r_1 h(\tau) - K_L \int_0^{\tau} J(\tau - \xi) e^{-\beta\xi} du_L(\xi) = u_L(\tau) \quad (23)$$

2.4 Determination of the coupled solution

The generalized Kelvin model is composed of the Maxwell model and Hooke body in series. The generalized Kelvin model degenerates into the Maxwell model when $G_k \rightarrow 0$. Therefore, the detailed derivation

process could be presented when surrounding rock obeys the generalized Kelvin model. The threedimensional creep modulus of the Generalized Kelvin model is given by the following expression:

$$J(t) = \frac{1}{2G_m} + \frac{1}{2G_k} (1 - e^{-\frac{G_k t}{\eta_k}}) \quad (24)$$

Substituting Eq. (24) into Eq. (23), we can obtain the following equation:

$$r_1 h(\tau) - K_L \left(\frac{1}{2G_m} + \frac{1}{2G_k} \right) \int_0^\tau e^{-\beta \xi} du_L(\xi) + \frac{K_L}{2G_k} e^{-\frac{G_k \tau}{\eta_k}} \int_0^\tau e^{\left(\frac{G_k}{\eta_k} - \beta\right)\xi} du_L(\xi) = u_L(\tau) \quad (25)$$

Taking the derivative of both sides of Eq. (25) with respect to τ , we have

$$u_L(\tau) e^{-\beta \tau} - \left(\frac{G_k}{\eta_k} - \beta \right) e^{-\frac{G_k \tau}{\eta_k}} \int_0^\tau u_L(\xi) e^{\left(\frac{G_k}{\eta_k} - \beta\right)\xi} d\xi = \frac{2\eta_k}{K_L} \left[r_1 h'(\tau) - \left(\frac{K_L}{2G_m} e^{-\beta \tau} + 1 \right) u_L'(\tau) \right] \quad (26)$$

Considering $u_L(0) = 0$ at $\tau=0$, the following initial condition could be given

$$u_L'(0) = r_1 h'(0) / \left(1 + \frac{K_L}{2G_m} \right) \quad (27)$$

Both sides of Eq.(26) is multiplied by $e^{G_k \tau / \eta_k}$, and then, taking the derivative of both sides of Eq. (26), the following second-order differential equation about $u_L(\tau)$ is obtained as

$$C_1 u_L''(\tau) + C_2 u_L'(\tau) + C_3 = 0 \quad (28)$$

where

$$\left. \begin{aligned} C_1 &= 1 + \frac{K_L e^{-\beta \tau}}{2G_m} \\ C_2 &= \frac{G_k}{\eta_k} + \frac{K_L}{2G_m} \left(\frac{G_k}{\eta_k} - \beta \right) e^{-\beta \tau} + \frac{K_L}{2\eta_k} e^{-\beta \tau} \\ C_3 &= -r_1 h''(\tau) - r_1 \frac{G_k}{\eta_k} h'(\tau) \end{aligned} \right\} \quad (29)$$

Combining Eqs. (2), (14), and (20), the expressions of $h'(\tau)$ and $h''(\tau)$ are given as

$$\left. \begin{aligned} h'(\tau) &= \frac{p_0(1-\alpha)}{2\eta_k} e^{-\frac{G_k(\tau+t_0)}{\eta_k}} + \frac{p_0 m \alpha}{2G_m} e^{-m(\tau+t_0)} + \frac{p_0}{2} \frac{m \alpha}{(G_k - m\eta_k)} \left[e^{-m(\tau+t_0)} - e^{-\frac{G_k(\tau+t_0)}{\eta_k}} \right] \\ h''(\tau) &= -\frac{p_0 m^2 \alpha}{2} \left(\frac{1}{G_m} + \frac{1}{G_k - m\eta_k} \right) e^{-m(\tau+t_0)} + \frac{p_0 G_k}{2\eta_k} \left(\frac{m \alpha}{G_k - m\eta_k} - \frac{1-\alpha}{\eta_k} \right) e^{-\frac{G_k(\tau+t_0)}{\eta_k}} \end{aligned} \right\} \quad (30)$$

As mentioned above, combining $u_L(0) = 0$ and

Eq. (27), the initial condition of Eq. (28) can be listed as

$$u_L(0) = 0, u_L'(0) = r_1 h'(0) / \left(1 + \frac{K_L}{2G_m} \right) \quad (31)$$

By combining Eqs. (28)–(31), the solution of $u_L(\tau)$ can be derived. Then, considering Eq. (16), the solution of support pressure $q(\tau)$ can be obtained. given the difficulty in calculating the explicit solution, the numerical method is proposed in this paper.

3 Validation of proposed solution

Ladanyi et al.^[6] and Chu et al.^[38] derived the analytical solutions for the excavation of a supported tunnel in the Generalized Kelvin body and Maxwell rock. The parameters employed in this study are the same as those in references [6] and [38]. Meanwhile, the degradation rate β is chosen as 0.02 a^{-1} . The correctness of proposed solutions is verified by comparison with solutions calculated by FLAC^{3D}. The parameters of surrounding rock and concrete lining are listed in Table 1 and Table 2^[6, 38].

Table 1 Maxwell model parameters of surrounding rock and lining parameters

		Lining			
E_0 /GPa	σ_0 /MPa	ν	β/a^{-1}	r_1 /m	r_2 /m
35.5	36	0.2	0.02	4.6	4
Surrounding rock		Other parameters			
G_m /GPa	η_m /(GPa · a)	p_0 /MPa	t_0 /a		
3.45	28.73	4.5	0.96		

Table 2 Generalized Kelvin model parameters of surrounding rock and lining parameters

		Lining			
E_0 /GPa	σ_0 /MPa	ν	β/a^{-1}	r_1 /m	r_2 /m
70.9	50	0.2	0.02	4	3.65
Surrounding rock		Other parameters			
G_m /GPa	G_k /GPa	η_k /(GPa · a)	p_0 /MPa	t_0 /a	
2.07	2.07	20.69	7.6	0.96	

The comparisons between analytical solutions and those calculated by FLAC^{3D} are presented in Fig. 5. The proposed analytical solutions are consistent with the existing solutions when the degradation of concrete lining is not considered ($\beta = 0$). Meanwhile, a good agreement between analytical solutions and numerical results can be noted. Therefore, the correctness of solutions is verified. As shown in Fig.5 (a), due to the stable creep stage of viscoelastic Maxwell rock, the surrounding rock deformation increases over time when the lining degradation is considered. At the same time, the support pressure is smaller than the results when lining degradation is not considered. What's more, a more significant rock displacement and a smaller support pressure are observed when a generalized

Kelvin viscoelastic model is adopted when considering the lining degradation (Fig.5 (b)). The generalized Kelvin model can only describe the attenuation creep stage of surrounding rock. Therefore, the interaction between surrounding rock and lining reaches an equilibrium condition when rock load is stabilized. As long as the bearing capacity of lining is greater than effective stress, the reduction of lining stiffness will not break the existing equilibrium condition. Therefore, the rock displacement and support pressure will not change after the rheological load is stable.

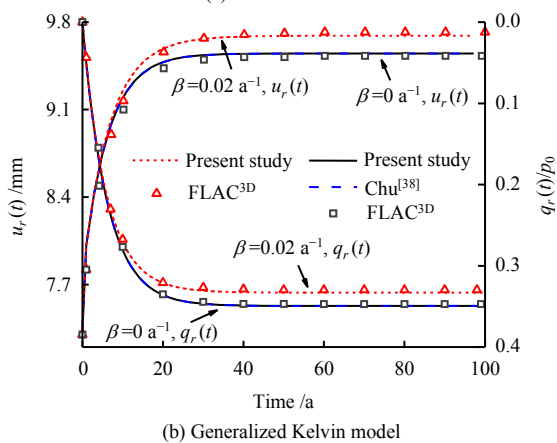
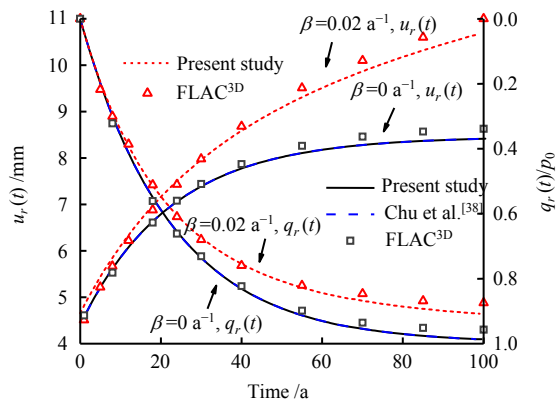


Fig. 5 Comparison with the existing calculation results

4 Parameter sensitivity investigation

In this section, the influence of degradation coefficient, thickness and installation time of concrete lining, and relaxation time of surrounding rock are investigated. The rock displacement and support pressure are analyzed in different operation stages. For comparative purposes, the support pressure is normalized by p_0 .

4.1 Influence of lining deterioration coefficient

The service environment, rock load, and interaction between support and surrounding rock all affect the degradation rate of concrete lining. According to references [26–29], the deterioration coefficient β of 0.005, 0.01, and 0.02 (a^{-1}) are analyzed. The other parameters are the same as those in Section 3. The rock displacement and support pressure under different deterioration coefficients are shown in Fig. 6. As depicted in Fig. 6(a),

the degradation of the lining stiffness varies the interaction relationship between surrounding rock and support when the Maxwell viscoelastic model is adopted. In the stable creep stage of surrounding rock, the greater the lining deterioration coefficient, the smaller the lining stiffness, resulting in more significant deformation of surrounding rock and smaller support pressure. This trend is more prominent with the increase of time. As shown in Fig.6 (b), the greater the lining deterioration coefficient, the more significant the reduction of structural stiffness when the generalized Kelvin model characterizes the surrounding rock. Therefore, the greater the deformation rate of surrounding rock, the greater the final value of rock deformation and the smaller the final value of support pressure. It has been mentioned above that the reduction of lining stiffness will not break the existing tunnel equilibrium condition when the rheological load of surrounding rock is stable. A new unbalanced force will be generated when the bearing capacity of lining structure is less than the support load, resulting in a deterioration of relationship between support and surrounding rock. The relationship between the surrounding rock and support structures will transfer from relatively stable to unstable.

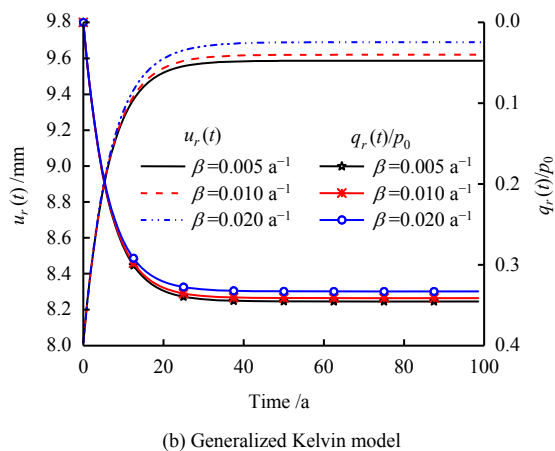
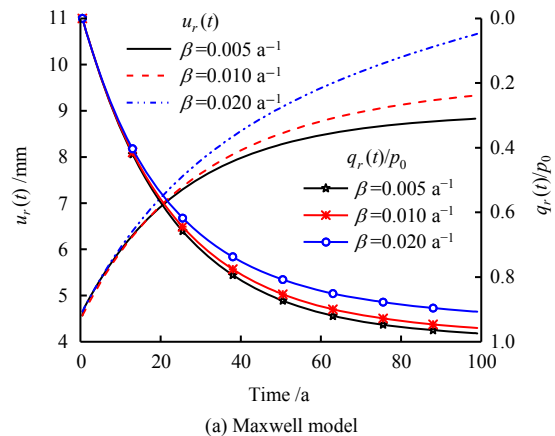
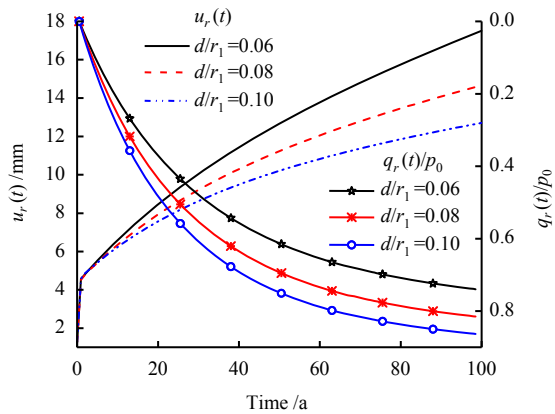


Fig. 6 Displacement and support pressure of lining with different deterioration coefficients

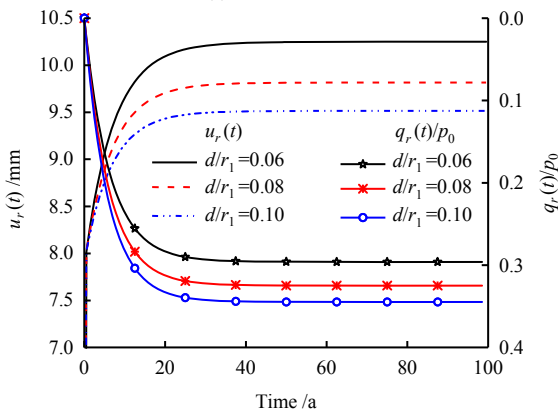
4.2 Influence of lining thickness

The lining thickness d is a crucial design parameter

for the lining. Only the outer diameter r_2 is varied when analyzing the influence of lining thickness. The rock displacement and support pressure under different lining thicknesses are presented in Fig. 7. It can be seen that the lining thickness has a significant impact on the rock displacement and support pressure. The structural stiffness decreases with the decrease of lining thickness, resulting in the increase of rock displacement and the reduction of support pressure. For Maxwell surrounding rock, a smaller thickness of lining leads to a greater growth rate of rock displacement and a greater value of rock displacement. On the contrary, a slower growth rate of support pressure and a smaller final value of support pressure. For the generalized Kelvin model, the smaller lining thickness results in a more significant early surrounding rock displacement and smaller support pressure. The rock displacement and support pressure gradually tend to be stable when the rheological load is stable. The change of lining thickness has a more significant impact on rock displacement and support pressure when the Maxwell viscoelastic model is adopted than that of the generalized Kelvin model.



(a) Maxwell model



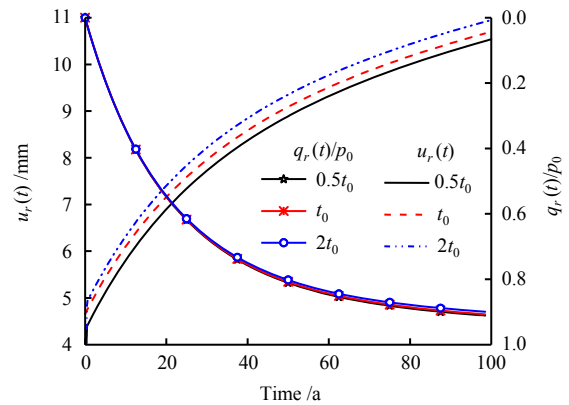
(b) Generalized Kelvin model

Fig. 7 Displacement and support pressure of lining with different thicknesses

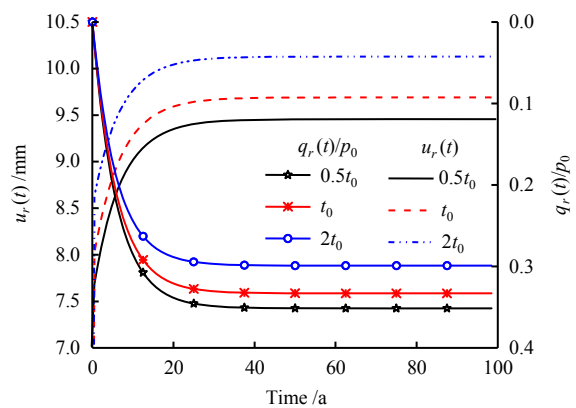
4.3 Influence of lining installation time

The influence of lining installation time on the

long-term mechanical response of tunnel is analysed. Fig. 8 shows the time history curves of surrounding rock displacement and support pressure when the installation time is $0.5t_0$, t_0 , and $2t_0$, where $t_0=0.96$ a. Other parameters are the same as those in section 3. The later the lining installation corresponds to the greater the stress release of surrounding rock, leading to the bigger rock displacement before the installation of lining and the slower growth rate of support pressure after installation of lining. The lining installation time has little influence on rock displacement and support pressure when surrounding rock is defined by the Maxwell viscoelastic model. The lining installation time significantly impacts the rock displacement and support pressure in the early stage when surrounding rock is described by the generalized Kelvin viscoelastic model. Meanwhile, the later the lining installation, the greater the rock displacement is and the smaller the final value of support pressure is. However, the variation laws of rock displacement and support pressure under different installation times are similar in the later stage of operation.



(a) Maxwell model



(b) Generalized Kelvin model

Fig. 8 Displacement and support pressure for lining with different support time

4.4 Influence of rock relaxation time

The rock displacement and support pressure under different rheological relaxation time T_d of surrounding

rock is shown in Fig. 9. For the Maxwell and generalized Kelvin models, the relaxation time is defined by $T_d = \eta_m / G_m$ and $T_d = \eta_k / G_k$, respectively. The change of relaxation time is realized by changing parameter η_m of Maxwell model and parameter η_k of generalized Kelvin model, respectively. Other parameters are the same as those in section 3. As shown in Fig. 9, with increases of rock relaxation time, the growth rate of rock displacement and support pressure slows down in the early stage of tunnel operation, but it takes longer for the interaction between support and surrounding rock to stabilize. The greater the relaxation time T_d , the slower the rheological load release of surrounding rock. Due to the lining stiffness decreases with time, a greater T_d leads to a longer time for the rock displacement to reach stability. For the Maxwell viscoelastic rock, rheological load increases with time, while stiffness of lining decreases with time. Under the coupled effect, the rock displacement is more significant later, while the support pressure's growth rate gradually slows down. For the generalized Kelvin viscoelastic rock, a greater T_d results in a smaller final rock displacement and a larger final support pressure. It can be seen that the long-term tunnel mechanical response is significantly influenced by rock relaxation time when surrounding rock is characterized by Maxwell viscoelastic rock.

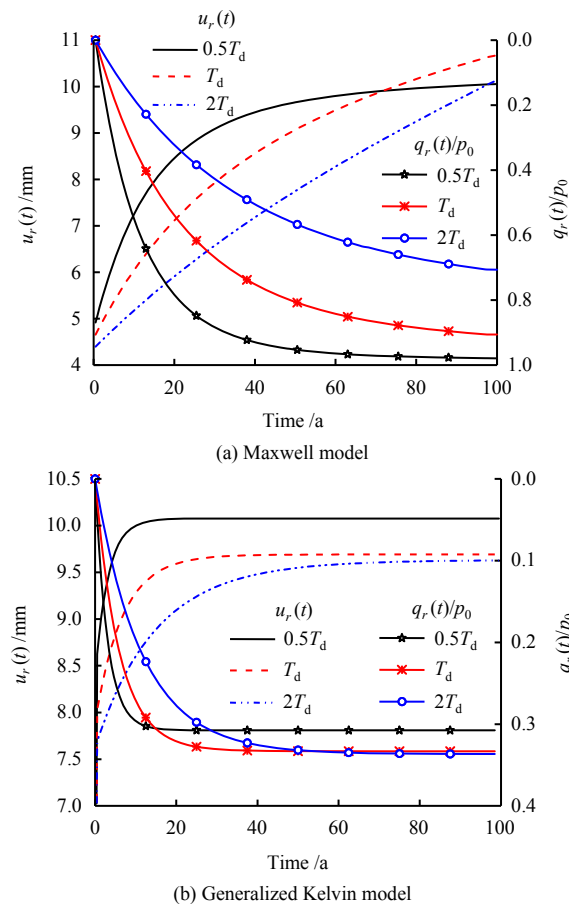


Fig. 9 Displacement and support pressure of rocks with different relaxation time

5 Analyse of tunnel long-term stability

5.1 Long-term mechanical response of lining

The long-term safety of operation tunnels is closely related to the stress state of support structure and deformation characteristics. Therefore, the rock displacement rate and support pressure rate is analyzed, respectively. Fig. 10 shows the rock displacement rate and support pressure rate under different lining deterioration coefficients. As the rheological effect of surrounding rock weakens, the growth rate of rock displacement and support pressure gradually decrease with time. As shown in Fig.10(a), for the Maxwell viscoelastic rock, a greater lining deterioration coefficient β leads to a greater growth rate of rock displacement ($d[u_r(t)]/dt$) and a lower growth rate of support pressure ($d[q_r(t)]/dt$). Meanwhile, the larger β is, the longer it takes for the deformation of surrounding rock to stabilize, and the longer for the rock displacement to reach stability. For the generalized Kelvin viscoelastic rock, the growth rates of rock displacement and support pressure decreases until they reach zero. The lining deterioration coefficient β will not influence the growth rate of rock displacement and support pressure. It can be concluded that the deterioration coefficient of lining affects the interaction between rock and lining more significantly when the tunnel is surrounded by Maxwell viscoelastic rock mass.

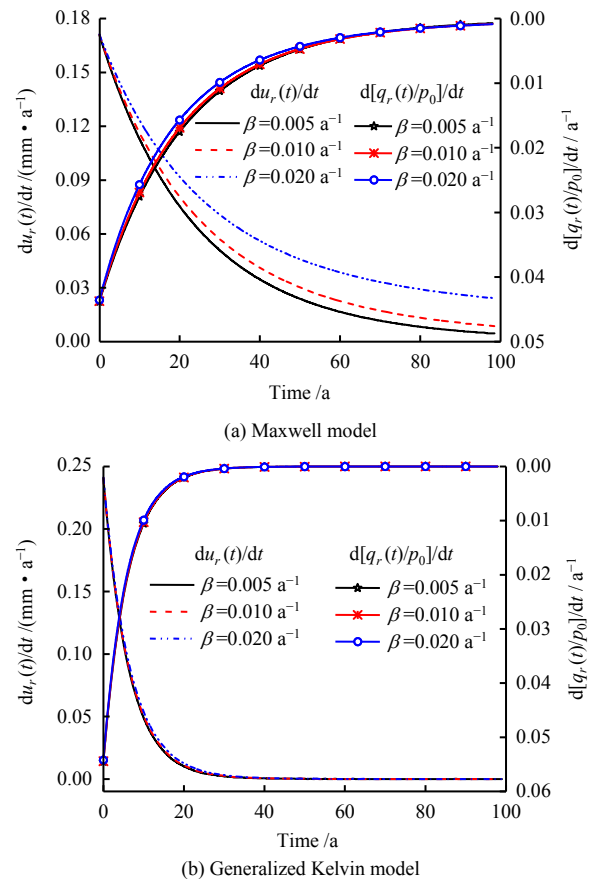


Fig. 10 Displacement rate and support pressure rate of lining with different deterioration coefficients

The growth rates of rock displacement and support pressure under different rheological relaxation times are presented in Fig.11. The growth rates of rock displacement and support pressure decrease with time when rock is described by the Maxwell and the generalized Kelvin models. The slower release process of rock load is consistent with the larger rock rheological relaxation time T_d . At the same time, the lower growth rate of rock displacement and supporting pressure are observed with larger T_d . The larger the rheological relaxation time T_d is, the more significant the long-term load effect of surrounding rock is. Therefore, the longer stability time of surrounding rock displacement and support pressure rate are found when the operation time of tunnel increases. It also has a significant impact on the long-term stability of tunnels.

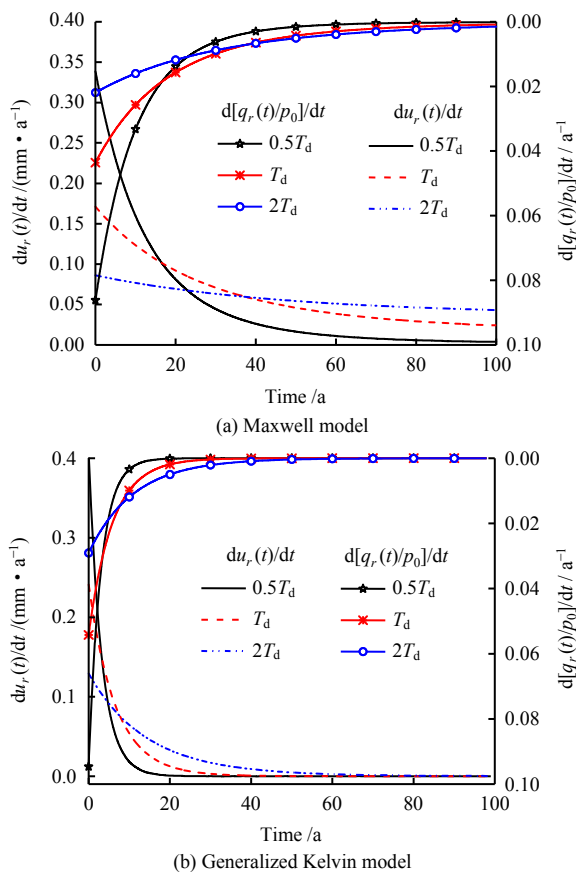


Fig. 11 Displacement rate and support pressure rate of rocks with different relaxation time

5.2 Prediction of service-life of lining

Under the coupled effect of surrounding rock rheological effect and lining degradation, the support pressure changes continuously. Due to the degradation of lining, the compressive strength of concrete lining reduces over time. Hence, the lining structure will fail when the stress exceeds its bearing capacity. The failure criterion of the concrete lining is given as^[39–40]:

$$\sigma_{\theta}(r, t) - k\sigma_r(r, t) \leq \sigma_0 \quad (32)$$

The stress of lining could be obtained from Eq.(17). The radial stress on the inner boundary of lining is zero ($\sigma_r(r_2, t) = 0$). Combining Eq. (32), it can be seen that the inner side of lining will fail firstly. Meanwhile, the time-dependent strength of lining could be obtained by Eq.(1). Based on these assumptions, the comparison between stress and strength of lining structures is presented in Fig.12. Given the length limitation of this paper, only the conditions of $\beta = 0.005 \text{ a}^{-1}$ and $\beta = 0.02 \text{ a}^{-1}$ are considered, and the other parameters are the same as those in section 3. It can be seen that the rheological load of surrounding rock increases over time in the early operation stage of tunnel. As a result, the degradation process reduces the bearing capacity of lining structure, accelerating the failure of structure and shortening the service life of tunnel.

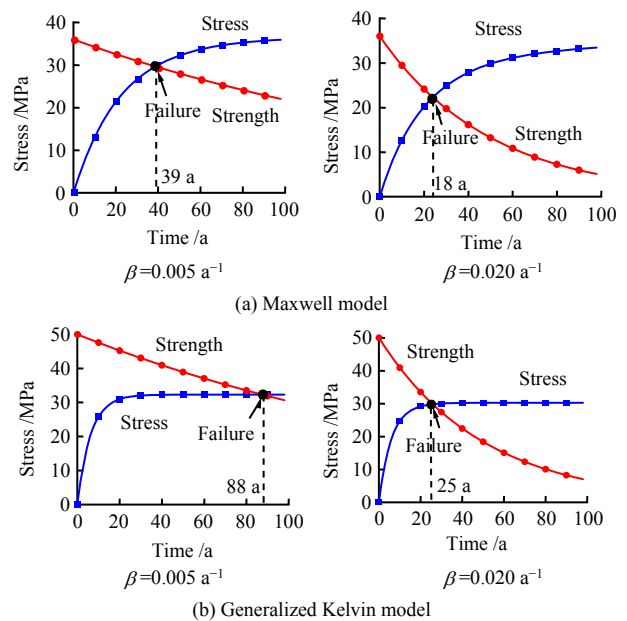


Fig. 12 Comparison of support pressure and bearing capacity under different deterioration coefficients of lining

Figure 13 shows the comparison of support pressure and bearing capacity under different rock rheological relaxation times. In order to highlight the result, the deterioration coefficient is chosen as 0.01 a^{-1} in analysis, and other parameters are the same as section 3. For the Maxwell viscoelastic rock, the greater the rock relaxation time T_d results in slower growth rate of support pressure and the longer it takes the structure to fail. Since the Maxwell model can characterize the stable creep stage of surrounding rock, the change of rock rheological parameters has a significant impact on the service life

of tunnel. According to the previous analysis, the larger T_d leads to lower growth rate of support pressure in the early stage when the surrounding rock is characterized by the generalized Kelvin model. In contrast, the more significant final value of support pressure is observed, and the failure of lining structure can be discovered first.

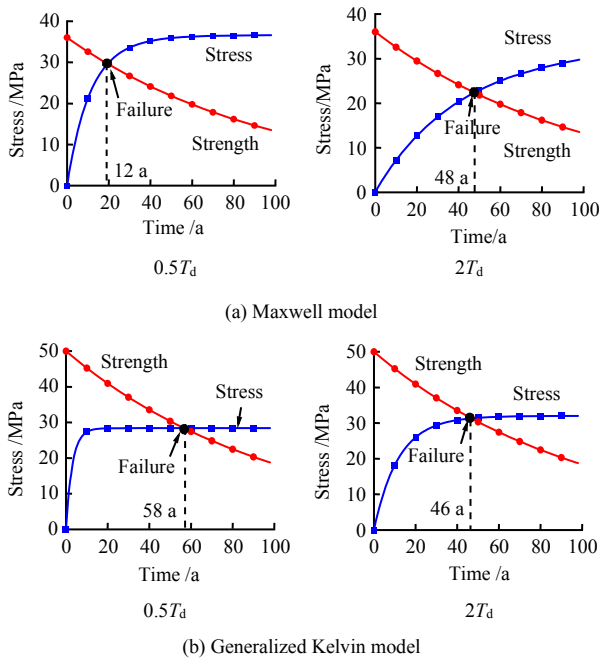


Fig. 13 Comparison of support pressure and bearing capacity under different relaxation time of surrounding rock

The lining degradation effect and rheological characteristics of surrounding rock both impact on the service performance of tunnel. The service life of a tunnel under the coupled effect is predicted, and the results are shown in Fig.14. Since the Maxwell model can characterize the stable creep stage of surrounding rock, the rock has more significant rheological properties due to a larger value of T_d . Therefore, the service life of tunnel is much shortened when the degradation of lining is considered. The generalized Kelvin model can only describe the attenuation creep stage of surrounding rock. The parameter T_d has little effect on the support pressure when lining has a specific deterioration coefficient. Therefore, the influence of lining deterioration coefficient on the service performance of tunnel is more significant than that of surrounding rock rheology. The current design concept has not paid enough attention to the long-term rheological effect of surrounding rock, which is also one of the main reasons for the frequent occurrence of structural defects in some operation tunnels. In addition, the mechanical properties of concrete lining reduce in

hostile service environment. Under the coupled effect of long-term rheological effect of surrounding rock and degradation of support performance, the failure process of lining structure is accelerated and the service life of tunnel is shortened.

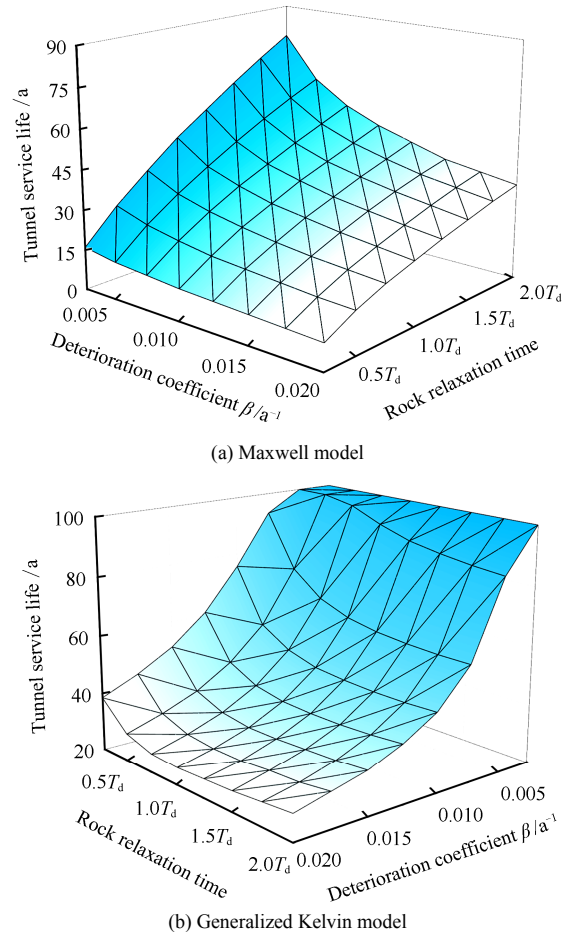


Fig. 14 Service life of supporting structure under coupling action

At present, the research on lining damage of operation tunnels mainly focuses on detection and evaluation methods. The research on the damage mechanism of lining structures seriously lags behind the tunnel development in China, resulting in unreasonable and uneconomical phenomena in the treatment of tunnel damages. Therefore, the long-term healthy and green development of tunnel engineering is restricted. It should be pointed out that the interaction between support and surrounding rock is the key point of long-term tunnel safety. In this paper, the service performance of tunnel structure is preliminarily discussed from the perspective of support and surrounding rock action. The research on the mechanism of tunnel disaster has a long way to go, and there are still many problems to be solved.

6 Conclusion

In this study, an analytical model considering the rheological effect of surrounding rock and degradation of lining is established. The correctness of analytical solutions is verified by comparison with existing solutions and numerical solutions. The influence of deterioration coefficient, lining thickness, rheological properties of rock mass is investigated. The main conclusions are drawn as follows:

(1) For the viscoelastic rock with a stable creep stage, the deformation of surrounding rock increases over time when lining degradation is considered. For the viscoelastic rock with only the attenuation creep stage, surrounding rock and support will reach a mechanical equilibrium condition when the rheological load is stabilized. The reduction of structural stiffness will not break the existing equilibrium state when the support pressure is less than the bearing capacity of structure.

(2) More considerable final value of rock displacement and smaller final value of support pressure can be observed when the lining has a more significant deterioration coefficient, a smaller thickness, or the later lining installation. Greater rheological relaxation time of surrounding rock leads to slower release of rheological load. In addition, lining stiffness decreases over time. Therefore, the greater the relaxation time, the longer the time for surrounding rock deformation to reach stability.

(3) The support pressure increasing over time results from the rheological effect of surrounding rock. Meanwhile, the mechanical properties of concrete lining reduce over time in hostile service environment. Therefore, under the coupled effect of long-term rheological effect of surrounding rock and degradation of support performance, the failure process of lining structure is accelerated and the service life of tunnel is shortened. The current design concept has not paid enough attention to the long-term rheological effect of surrounding rock, which is also one of the main reasons for the frequent occurrence of structural defects in some operation tunnels.

(4) The analytical solution that considers the surrounding rock rheology and lining deterioration is proposed in this paper. The solution method of elastic-viscoelastic correspondence principle is no longer applicable. Therefore, this paper only gives the solution when surrounding rock meets Maxwell and generalized Kelvin viscoelastic models under hydrostatic pressure. Subsequent study can be carried out for other complex

viscoelastic rock models and tunnels with composite linings under non-hydrostatic pressure.

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