Rock and Soil Mechanics

Volume 42 | Issue 9 Article 1

12-14-2021

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Custom Citation

ZHANG Chao, YANG Chu-qing, BAI Yun. Investigation of damage evolution and its model of rock-like brittle materials[J]. Rock and Soil Mechanics, 2021, 42(9): 2344-2354.

Rock and Soil Mechanics 2021 42(9): 2344–2354 ISSN 1000−7598
https: //doi.org/10.16285/j.rsm.2021.5278 rocksoilmech.researchcommons.org/journal

Investigation of damage evolution and its model of rock-like brittle materials

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Abstract: To investigate the description and evolution of the damage state of rock-like brittle materials, the physical meaning of each elastic modulus method parameter based on the strain equivalence hypothesis and the limitations of the model application are discussed. The modulus change during the triaxial cyclic loading and unloading test of limestone is studied. Moreover, the defects of the unloading modulus substitution method and the statistical damage evolution model in damage evolution analysis are discussed. The results show the existing elastic modulus method can only be used to reflect the damage evolution process of rock under uniaxial compression, and the unloading modulus substitution method cannot correctly describe the damage state and its evolution law. In addition, the statistical damage constitutive model can only be regarded as a theoretical self-consistent solution under the numerical range [0, 1] of the statistical damage evolution model. Based on the above research, a damage characterization variable and its evolution model considering the effects of damage strain threshold are proposed. Additionally, the constitutive model below the damage strain threshold and the damage constitutive model above the damage strain threshold are established, respectively. The sensitivity of model parameters is also analyzed in this study. The final results show that the proposed model and method can not only reasonably explain the damage mechanism of rocks under triaxial compression, but also accurately simulate the full stress-strain process, which is rationable and feasible.
Keywords: rock; elastic modulus method; damage evolution; constitutive model; sensitivity

1 Introduction

As one kind of natural geological material, rock mass contains a large number of randomly distributed micro-defects, such as micro-cracks, micro-cavities[1]. These micro-defects can be regarded as a substantial representation of damage. Therefore, using the damage mechanics theory to explain the entire process of the evolution of the deformation and mechanical properties of brittle materials such as rock has become one of the hot topics in the research field of rock mechanics, which includes the rock deformation and failure mechanism[2−3], the damage model[4−5], and the determination method of model parameters[6−7], etc. The primary and the most basic content of this topic is how to describe the rock damage state and its evolution, and then establish a simulation method for rock deformation and failure on this basis.

Under the action of external load, the micro-defects in the rock masses gradually propagate and coalesce, forming a dynamic evolution process of damage accumulation, which cause irrecoverable damage to the macroscopic deformation mechanical characteristics of the rock masses. To describe this damage state and evolution process, it is necessary to choose an appropriate damage definition method. Since the damage changes the physical properties of the internal structure of the rock, the damage definition method for the description of the rock damage state can be selected from three types: macroscopic, mesoscopic, and microscopic. The first type is based on the rock macroscopic mechanics test and some macroscopic measurable parameters, e.g., elastic modulus, wave velocity, crack volumetric strain, and AE characteristic parameters[8], are selected to define damage variables. In the second type, the meso-statistical damage mechanics method is used, where the proportion of the representative volume element (RVE) of the damaged rock[9] is defined as a damage variable. The damage evolution model is established on the assumptions that the physical properties of the RVE obey a certain probability distribution, such as the Weibull distribution[10], log-normal distribution[11], and normal distribution[12]. In the third type, parameters of rock microstructures such as the number, length, area, and volume of micro-defects[13] are defined as damage variables. However, since the internal structure of the rock material is generally complicated and there are also certain limitations in the damage detection technique, this method is inconvenient for practical applications. Therefore, it is the basic principle to select the damage definition methods according to the difficulty of obtaining the parameters describing the damage state. The strain equivalence hypothesis[14] is one of the basic hypotheses for the investigation of rock damage theory. The elastic modulus method based on this hypothesis is a macro definition method of damage variables, which is also the

Received: 22 February 2021 Revised: 6 May 2021
This work was supported by the Hunan Provincial Natural Science Foundation(2018JJ3163) and the Scientific Research Project of the Hunan Provincial Education Department(18C0356).
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theoretical basis for the analysis of rock damage evolution. Basically, this method is a description method for elastic damage. However, in most research, the unloading modulus of the rock is taken as the instantaneous elastic modulus of the damaged rock material[15−16] when this method is used to measure the damage state of brittle materials e.g. rock. Strictly speaking, this processing method has disadvantages in significantly simplifying or concealing the real damage mechanical behavior of rock materials. In fact, some scholars have already found that using this processing method to investigate the rock damage evolution could produce wrong results that deviate from reality, and they have also drawn the conclusions that the elastic modulus method is not always an effective method to measure rock damage[17−18]. However, these studies did not fundamentally investigate the real reason for this problem, and the explanation or the suggestion to find other macro- and meso-parameters to redefine the damage variable concealed the key issues.

In this regard, the physical meaning of the elastic modulus of damaged rock and its changes in the elastic modulus method is investigated in this study. Based on the results of the rock triaxial cyclic loading and unloading test, the problem of using the unloading modulus rather than the rock elastic modulus in the transient damage state to describe the damage evolution of rock materials is discussed. The shortcomings of the damage variable and its evolution model using the statistical damage method are also pointed out. Moreover, a characterization method for damage variables and a damage evolution model considering the influence of the damage strain threshold are proposed. The rationality and feasibility of the proposed method are verified, and a damage constitutive model that can better simulate the triaxial stress-strain process of the rock is established.

2 Elastic modulus method based on the strain equivalence hypothesis

The evolution state of micro-defects in rock materials is generally described by the damage variable D. To consider some factors of damage without making the damage constitutive model too complicated, Lemaitre[14] proposed the strain equivalence hypothesis, where the strain ε caused by the nominal stress σ acting on the damaged rock material is equivalent to the strain ε' caused by the effective stress σ' acting on the undamaged rock material, as shown in Fig.1. In the figure, D is the damage variable, and A_0 is the acting area of the nominal stress σ. The strain equivalence hypothesis[14] can be expressed as:

$$\varepsilon = \sigma / E = \varepsilon' = \sigma' / E' \quad (1)$$

where E and E' are the elastic modulus of the rock material in the damaged state and the undamaged state, respectively.

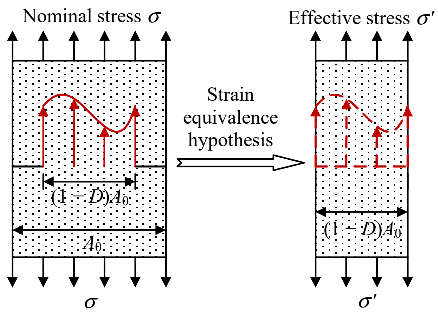


Fig. 1 Schematic of the strain equivalence hypothesis

According to the geometric description of the damage state of the rock material[15], the relationship between the nominal stress σ, and the effective stress σ', can be written as:

$$\sigma = (1 - D)\sigma' \quad (2)$$

Therefore, the elastic modulus method based on the strain equivalence hypothesis can be obtained by combining Eqs. (1) and (2):

$$D = 1 - E / E' \quad (3)$$

It is obvious that the elastic modulus in this elastic modulus method is actually the deformation modulus, which is the slope of the secant line of the point representing the transient damage state in the stress−strain curve of rock materials. The deterioration process of the elastic modulus is the continuously decreasing process of the slope of the secant line. The initial compaction deformation stage doesn't exist in the entire stress−strain process of the tight rock under uniaxial compression and the residual strength is 0, as shown in Fig.2. σ_1 and ε_1 are the axial stress and the axial strain, respectively; ε_d and ε_e are the irrecoverable strain and the recoverable strain, separately; E_u is the unloading modulus; σ_d and σ_f are the damage stress threshold and the peak stress, respectively. The change law of the normalized elastic modulus E/E' with the axial strain generally conforms to the Logistic model[19] and D gradually increases from 0 to 1, which can reflect the damage evolution behavior of rock materials under uniaxial compression. Therefore, the damage constitutive model based on this method can well simulate the entire deformation and failure process of the tight rock under uniaxial compression. As shown in Fig.3, the entire stress−strain process of porous rocks under triaxial compression includes both the initial compaction deformation stage (OA) and the residual strength deformation stage (DG), and E is expressed as

$$E = \sigma_1 / (\varepsilon_1 - \varepsilon_c) \quad (4)$$

where ε_c is the crack closure strain. According to the damage mechanics theory, D no longer changes with the increase of the axial strain[15] when the rock enters the residual strength deformation stage (i.e., the axial strain is greater than the residual strain). However, D defined in the elastic modulus method gradually decreases at this stage, which makes it impossible to accurately describe the damage evolution behavior of rock materials under triaxial compression. Therefore, the damage constitutive model established based on this method cannot well simulate the entire deformation and failure process of the porous rock under triaxial compression. The existing elastic modulus method and its damage constitutive model have significant limitations in describing the evolution of rock damage and simulating the entire deformation and failure process of rocks.

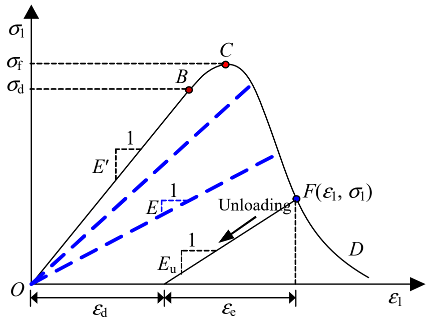


Fig. 2 Deformation of tight rocks under uniaxial compression

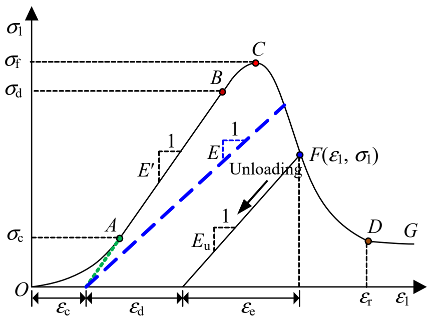


Fig. 3 Deformation of porous rocks under triaxial compression

According to Figs.2 and 3, the loading and unloading paths of the rock material in the elastic deformation stage coincide with each other. The unloading modulus E_u is equal to the elastic modulus E, and the damage variable D is 0, indicating that the rock is undamaged. However, the irrecoverable strain ε_d of the rock material in the damaged deformation stage happens after unloading, and E_u is different to E. There is no doubt that large errors and even wrong conclusions will be produced if E_u is still used as E to calculate the damage degree of rock materials and describe the damage evolution. This is the fundamental reason why many studies have concluded that the elastic modulus method is not always an effective method for measuring damage.

3 Rock damage evolution

To further study the problems of the unloading modulus substitution method in the rock damage evolution, the triaxial cyclic loading and unloading tests of limestone are carried out in this study. On this basis, the statistical damage model that has achieved good results in the simulation of the entire deformation and failure process of rocks is further investigated and its shortcomings are also pointed out. Moreover, a new method for characterizing rock damage variables and a damage constitutive model are proposed.

3.1 Triaxial cyclic loading and unloading test

The fresh limestone with good integrity and homogeneity is collected on-site, and then it is drilled, cut and polished following the testing requirements of rock mechanics tests. A standard cylinder of ϕ50 mm (diameter)× 100 mm (height) is made, and the diameter deviation of the end face of the rock sample is ensured within 0.3 mm, and the axial deviation is within 0.25º. Therefore, the influence of the rock sample preparation process on the analysis of test results can be minimized.

Triaxial cyclic loading and unloading tests under three confining pressures (5, 10, and 15 MPa) are conducted on the MTS 815 rock mechanics test system. An unloading method using the constant confining pressure loaded with the equal axial displacement u_s (the single-cycle axial displacement value is 0.1 mm) is used in this study. The axial loading and the axial unloading are controlled by displacement and load, respectively, with the loading rate of 0.003 mm /s, and the unloading rate of 2.5 kN /s. This cyclic process stops until the end of the residual strength deformation stage. The stress paths of the triaxial cyclic loading and unloading tests are shown in Fig.4. d_0 (the initial axial displacement is only under the confining pressure σ_3, which generally approximates to 0; load the rock sample to the designed value of the confining pressure at 0.5 MPa /s; the test starts) → d_0 + 0.1 mm (displacement control, the axial loading is set as the designed value of u_s) → confining pressure σ_3 (loading control, the axial unloading is set as the designed value of vertical stress) → d_0 + 0.2 mm (loading) → σ_3 (unloading) → d_0 + 0.3 mm

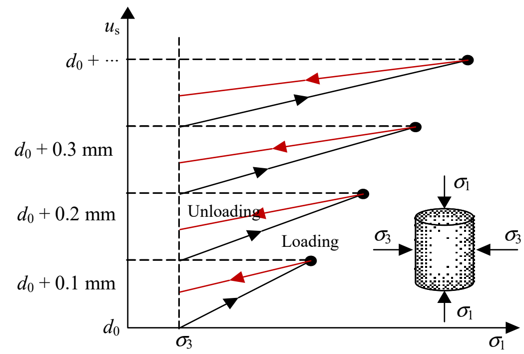
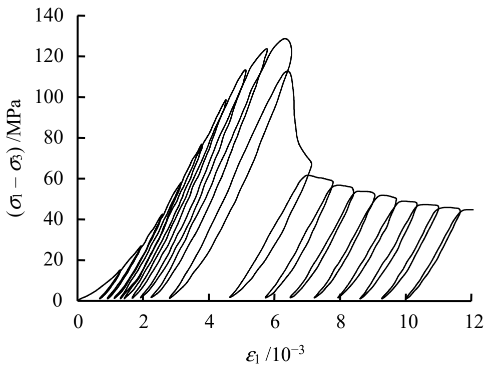
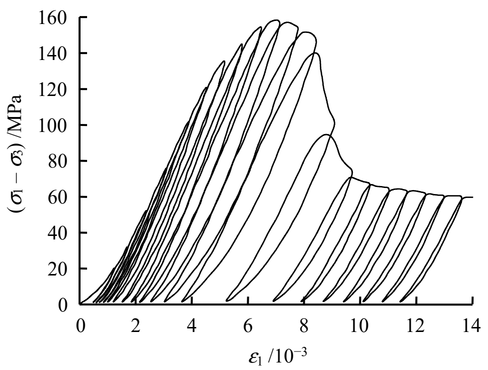


Fig. 4 Schematic of stress path under triaxial cyclic loading and unloading tests

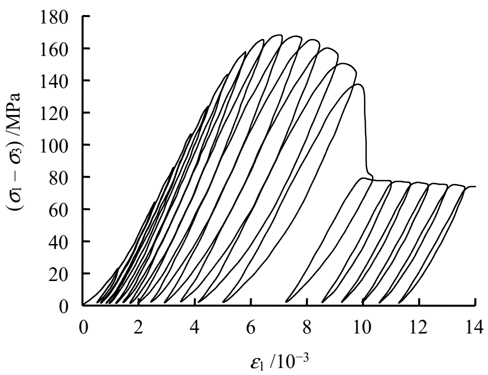
(loading) →σ_3 (unloading) → ··· → residual strength deformation stage (the test ends). Triaxial cyclic loading and unloading tests of five rock samples are carried out at each confining pressure level, and only part of the rock sample test data are listed in Fig.5 due to the space limilations.



(a) σ_3 = 5 MPa

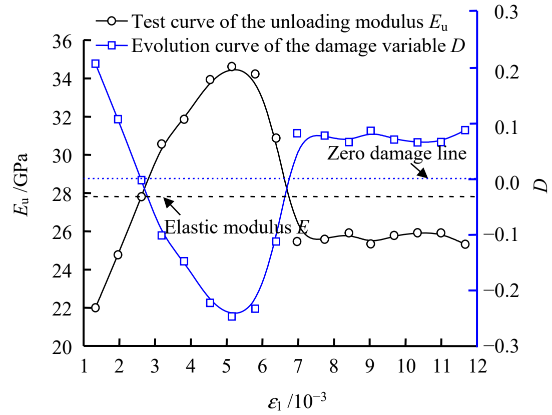


(b) σ_3 = 10 MPa

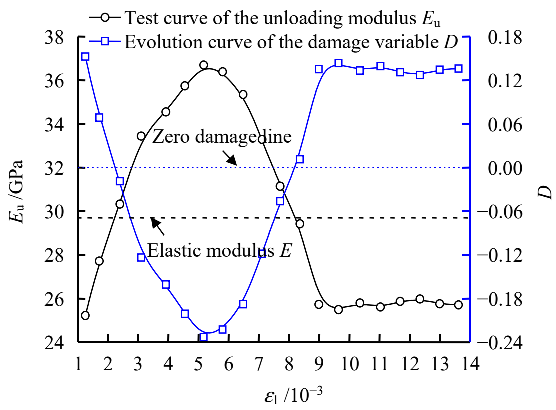


(c) σ_3 = 15 MPa

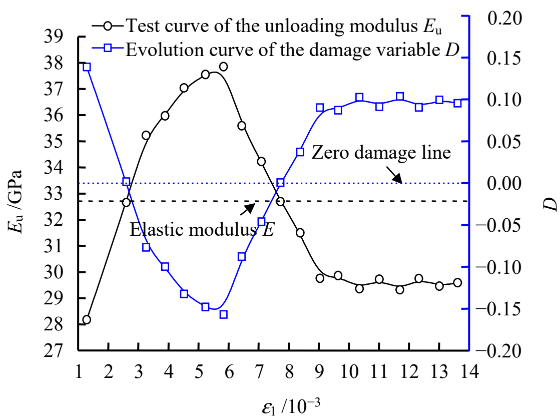
Fig. 5 Test curves of the limestone under triaxial cyclic loading and unloading conditions



(a) σ_3 = 5 MPa



(b) σ_3 = 10 MPa



(c) σ_3 = 15 MPa

Fig. 6 Unloading modulus and damage variable as a function of deformation

3.2 Analysis of damage evolution based on the unloading modulus substitution method

According to the triaxial cyclic loading and unloading test results, the relationship between the unloading modulus E_u and the axial strain ε_1, as well as the damage evolution curve of limestone which is calculated by substituting E_u as E into the elastic modulus method are shown in Fig.6. It can be observed that:

(1) Under different confining pressures, E_u first increases, then decreases and finally remains unchanged with the increase of the axial strain. The unloading modulus of limestone is relatively low at the initial stage of loading deformation, and it gradually increases with the increases of the axial stress. When the axial stress increases to about 50% of the peak stress σ_f, E_u is equal to E'. When the axial stress increases to about 95% of the peak stress σ_f, E_u reaches its maximum value, and then gradually decreases with a decrease of the stress in the post-peak zone. During the residual strength deformation stage,

DOI: 10.16285/j.rsm.2021.5278

E_u is about 90% of E' and remains unchanged.

(2) The minimum unloading modulus is obtained at the initial compaction deformation stage, where the initial fissures close and the limestone is continuously compacted but not damaged. The maximum unloading modulus is witnessed at the yield hardening deformation stage, where damage starts to exist in the limestone, i.e., new micro-voids are generated inside and continue to propagate. As the confining pressure increases, the ratio of the maximum unloading modulus to the minimum unloading modulus gradually decreases. When the confining pressures are 5, 10, and 15 MPa, the corresponding ratios are 1.57, 1.45, and 1.34, respectively.

(3) Under different confining pressures, the damage variable D first increases, then decreases and finally remains unchanged with the increase of the axial strain. At the initial stage of loading, the initial internal fissures in limestone continue to close. Therefore, the effective bearing area gradually increases, and the damage degree of the limestone also decreases. However, as the axial stress increases to 50% of the peak stress σ_f, the damage degree is below the zero damage line and the damage degree of limestone is only about 10% in the residual strength deformation stage, which is obviously wrong. Therefore, although the unloading modulus substitution method is simple for calculating the damage degree, it ignores the irrecoverable strain ε_d generated by the unloading of the rock material in the damage and deformation stage, which introduces a significant mistake to describe the evolution of rock damage using this method.

The closure of the initial fissures in the limestone will lead to the increase of E_u. However, E_u begins to attenuate only when the internal damage of limestone accumulates to a certain degree. Before this, E_u may be still larger than E in a certain damage state. At this time, if E_u is substituted as E into the elastic modulus method to calculate the damage degree of limestone and describe the damage evolution, the wrong conclusion that D is negative will be drawn. Otherwise, if the elastic modulus method is strictly used to describe the damage behavior of rock materials, D increases with the increase of the axial strain and eventually increases to 1. However, D in the residual strength deformation stage should remain unchanged according to the damage mechanics theory. Therefore, there are still some shortcomings when using the existing elastic modulus method to calculate the damage degree and describe the entire process of the damage evolution.

3.3 Analysis of damage evolution based on the statistical damage theory

The statistical damage theory has been widely used to simulate the entire deformation and failure process of rocks, and has achieved good results[20−21]. Based on the triaxial cyclic loading and unloading test data of the limestone, the entire process of the rock damage evolution is analyzed by the rock damage evolution model established on the statistical damage theory, and its defects are also pointed out in this study.

The representative volume element (RVE) is the basic unit of the statistical damage theory of rock materials. To characterize the heterogeneity of rock materials at the meso-scale, the Weibull distribution function is often used to describe the spatial distribution of certain mechanical properties of RVE, such as elastic modulus, strength, and Poisson's ratio:

$$\varphi(\alpha)=\frac{m}{\alpha_0}\left(\frac{\alpha}{\alpha_0}\right)^{m-1}\exp\left[-\left(\frac{\alpha}{\alpha_0}\right)^{m}\right] \qquad (5)$$

where $\varphi(\alpha)$ is the probability density function of the distribution parameter α; α_0 is the scale parameter that defines the eigenvalue of the distribution parameter α; m is a shape parameter describing the degree of spatial concentration and dispersion of α, which is an index of homogeneity. The strain strength theory is used to describe the strength characteristics of RVE, and the statistical damage variable D_s can be expressed as the integral of $\varphi(\alpha)$:

$$D_s=\int_0^{\varepsilon_1}\varphi(\varepsilon_1)\mathrm{d}\varepsilon_1 \qquad (6)$$

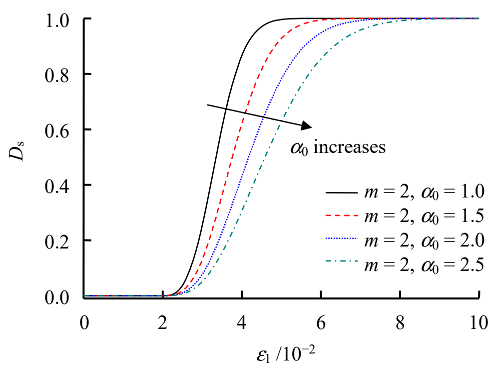
When the axial strain is smaller than the damage strain threshold ε_h, the rock material is in the elastic deformation stage (i.e., D_s= 0). When the axial strain is larger than the damage strain threshold ε_h, damage in the rock material starts. Therefore, the statistical damage evolution model for rock materials can be expressed as

$$D_s=\begin{cases}0,\ \varepsilon_1\leqslant\varepsilon_h\\ 1-\exp\left[-\left(\dfrac{\varepsilon_1-\varepsilon_h}{\alpha_0}\right)^{m}\right],\ \varepsilon_1>\varepsilon_h\end{cases} \qquad (7)$$

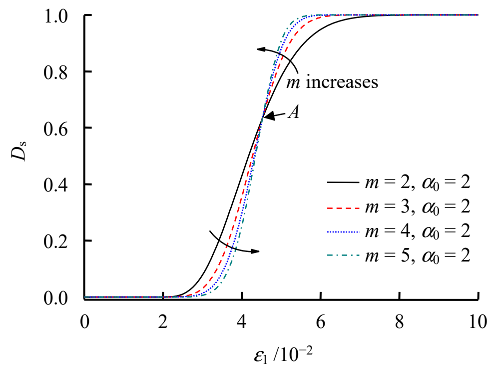
Therefore, the evolution of the statistical damage variable D_s with the axial strain ε_1 and the analysis of the parameters of the statistical damage evolution model are shown in Fig.7. Some findings are summarized:

(1) The change of D_s with the increase of axial strain is in an 'S-shape', and its change range is not related to the rock stress level and damage characteristics, ranging from 0 to 1. The change rate of the rock material damage (i.e., $\partial D/\partial\varepsilon_1$) depends on the model parameters, namely the scale parameter α_0 and the shape parameter m. In fact, the change rate of damage change rate is an important factor that can directly reflect the brittleness of the rock.

(2) When m is constant, the damage change rate decreases as α_0 increases. The damage evolution curve is approximately horizontally stretched along the direction of the increasing axial strain. Meanwhile, the required deformation for the rock starting from the damage to failure increases. When α_0 is constant, the damage evolution



(a) Effect of the scale parameter α_0 on the statistical damage variable D_s



(b) Effect of the shape parameter m on the statistical damage variable D_s

Fig. 7 Evolution of statistical damage variable D_s as a function of model parameters

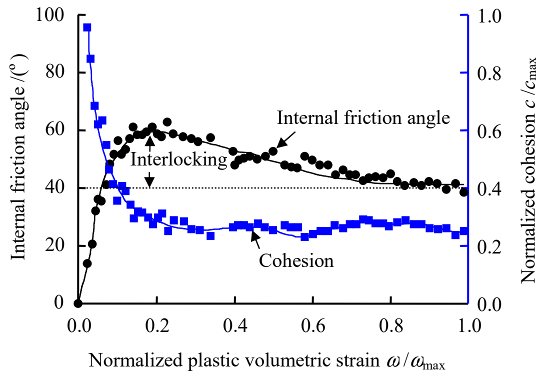


Fig. 8 Variation of internal friction angle and cohesion with normalized plastic volumetric strain[22]

curve rotates counterclockwise around the fixed point A, and the damage change rate increases with the increase of m. In addition, the required deformation for the rock starting from the damage to failure decreases.

It shows that the deformation and failure process of rock materials can be described by the statistical damage evolution model to a certain extent, and the heterogeneity of rock materials can also be characterized by the model parameters α_0 and m. However, this kind of model can only describe the damage evolution behavior of rock materials from the perspective of RVE strength statistics, but cannot investigate the cause of rock strength variation and its influence mechanism. Therefore, based on the triaxial cyclic loading and unloading test of granite, Martin et al.[22] obtained the changes of the internal friction angle and the cohesion with the normalized plastic volumetric strain using the constraint relationship between the strength parameters and the plastic parameters in the Mohr-Coulomb theoretical system, which is shown in Fig.8. Combined with the deformation and failure process of brittle rocks, the failure mode can be analyzed:

(1) The deformation and failure process has typical characteristics at different stages. At the initial stage of loading, no damage occurs inside the rock, and the frictional strength and cohesive strength are not fully utilized. As the axial stress increases, the cohesive strength is fully utilized firstly. Then the micro-cracks begin to intersect and coalesce with each other, and the potential slip surface is gradually generated.

(2) When the axial stress reaches the damage stress threshold, the friction strength starts to be fully utilized, while the cohesive strength decreases continuously. The macroscopic fissures gradually penetrate, and the roughness of the rock fracture surface increases due to the slippage, resulting in an interlocking effect and the friction strength reaches the peak value.

(3) As the crack scale increases, the slip surface is gradually formed and the interlocking effect also gradually disappears. The frictional strength decreases and tends to a stable value, i.e., the residual frictional strength. Meanwhile, the cohesive strength continues to decrease, and the rock generally loses its cohesive strength when entering the residual strength deformation stage.

Since the strength of the rock material is composed of frictional strength and cohesive strength, the strength is continuously changing during the entire deformation and failure process of rocks. However, the strength will not be completely lost during this process. Even if the rock is at the residual strength deformation stage, its strength still exists, and the value is equal to the residual friction strength. The residual friction strength is related to the confining stress of the rock. When the confining stress is equal to 0, the residual friction strength is completely lost. Therefore, the rock does not have the residual bearing capacity, and the rock is completely damaged or destroyed. In this case, the damage variable is equal to 1. When the confining pressure is not equal to 0, the strength is not completely lost. Although the rock is destroyed, it still has the residual bearing capacity, indicating that the damage variable of the rock at the residual strength deformation stage is smaller than 1. However, the final evolution value of the statistical damage variable D_s is not related to the confining pressure level of the rock, and its value is always equal to 1. On this basis, the established statistical damage constitutive model that simulates the entire deformation and failure process of rocks under different confining pressures still has its drawbacks. Many scholars choose to ignore this fact or tacitly agree that the evolution law of statistical damage variable D_s can be established, so that some constitutive models of rock damage that can simulate the residual strength deformation stage are established on this basis. One of the most representative models is based on the strain equivalence hypothesis and the characteristics of residual strength σ_r, which assumes the rocks are considered to be composed of two parts of non-damaged material and damaged material[23]:

$$\sigma = \sigma'(1 - D_s) + \sigma_r D_s \quad (8)$$

According to this equation, the theoretical bearing capacity of the rock material is the residual strength σ_r when $D_s = 1$. Actually, the rock damage model considering the characteristics of residual strength (i.e., Eq.(8)) weights the theoretical bearing capacity of the undamaged material and the damaged material, and the weights are $1-D_s$ and D_s, respectively. This type of method can only be regarded as a theoretical self-consistent solution under the range of the evolution value of the statistical damage variable D_s [0, 1], so that the established statistical damage constitutive model can be used to simulate the deformation and failure process. However, the strength and failure mechanism is not fundamentally reflected during the evolution of rock damage.

4 Characterization method and evolution model of rock damage

4.1 Characterization variables of rock damage based on the elastic modulus method

According to Figs.2 and 3, there is a conversion relationship between E and E_u of the rock material in the transient damage state, which can be used to describe and measure the damage evolution behavior based on the definition of the elastic modulus method. Taking point F as an example, its unloading modulus E_u can be expressed as

$$E_u = \frac{\sigma_1}{\varepsilon_1 - \varepsilon_{cd}} \quad (9)$$

where ε_{cd} is the sum of the crack closure strain ε_c and the irrecoverable strain ε_d:

$$\varepsilon_{cd} = \varepsilon_c + \varepsilon_d \quad (10)$$

Combining Eqs. (4) and (9), the following equation can be obtained:

$$E = \frac{\varepsilon_1 - \varepsilon_{cd}}{\varepsilon_1 - \varepsilon_c} E_u \quad (11)$$

Then a damage characterization variable D_n, which takes into account the influence of the damage strain threshold, can be determined based on the elastic modulus method:

$$D_n = \begin{cases} 0, \ \varepsilon_1 < \varepsilon_h \\ 1 - \dfrac{\varepsilon_1 - \varepsilon_{cd}}{\varepsilon_1 - \varepsilon_c}\left(\dfrac{E_u}{E'}\right), \ \varepsilon_h \leqslant \varepsilon_1 < \varepsilon_r \\ 1 - \dfrac{\varepsilon_r - \varepsilon_{cd}}{\varepsilon_r - \varepsilon_c}\left(\dfrac{E_u}{E'}\right), \ \varepsilon_1 \geqslant \varepsilon_r \end{cases} \quad (12)$$

According to the above equation, if the rock deformation fully recovers after unloading ($\varepsilon_d = 0$), E_u will degenerate to E, and the second part in Eq.(12) will be the same as Eq.(3). This indicates that Eq.(3) is a special expression of Eq.(12). Therefore, E_u cannot be taken as E when using the existing elastic modulus method to describe and measure the damage evolution behavior of rock materials.

Therefore, the variation of the damage characterization variable D_n of limestone with the increase of axial strain ε_1 can be obtained based on the results of the entire deformation and failure process of the triaxial compression under different confining pressures, as shown in Fig.9. The variation of D_n with the increase of axial strain is ‘S-shape’, and its change range is related to the rock stress level. The value of D_n generally increases from 0 to a certain value smaller than 1 and then remains unchanged. Under low confining pressure, the damage degree of limestone in the residual strength deformation stage is about 75%, which makes sense. Therefore, the damage degree (only 10%) calculated based on the unloading modulus substitution method is wrong.

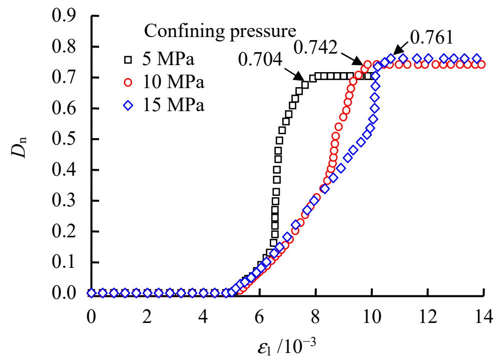


Fig. 9 Evolution of the damage characterization variable D_n

4.2 Evolution model of rock damage characterization variables

Since the rock damage characterization variable D_n is related to the elastic modulus E', the crack closure strain ε_c, the unloading modulus E_u, the irrecoverable strain ε_d and the residual strain ε_r, it is difficult to directly apply the characterization method of D_n in the rock damage constitutive model to simulate the entire deformation and failure process of rocks. Therefore, this paper prospoosed a D_n evolution model based on the statistical damage evolution model:

$$D_n=\begin{cases}\lambda_1, & \varepsilon_1\leqslant\varepsilon_h\\ \dfrac{\lambda_1-\lambda_2}{\exp\left[\left(\dfrac{\varepsilon_1-\varepsilon_h}{\varphi_0}\right)^p\right]}+\lambda_2, & \varepsilon_1>\varepsilon_h\end{cases} \tag{13}$$

where λ_1, λ_2, p, and φ_0 are model parameters. Therefore, the multi-parameter fitting analysis method is used to fit the entire evolution process of the damage characterization variable D_n of limestone based on the D_n evolution model (Eq.(13)), as shown in Fig.10. The fitting results are displayed in Table 1. It shows that the D_n evolution model can well simulate the whole process of the axial damage evolution of limestone under different confining pressures, and the fitting correlation coefficient R^2 is above 0.97. Moreover, the values of model parameters λ_1 and λ_2 are the same as the initial and final damage value of limestone, respectively.

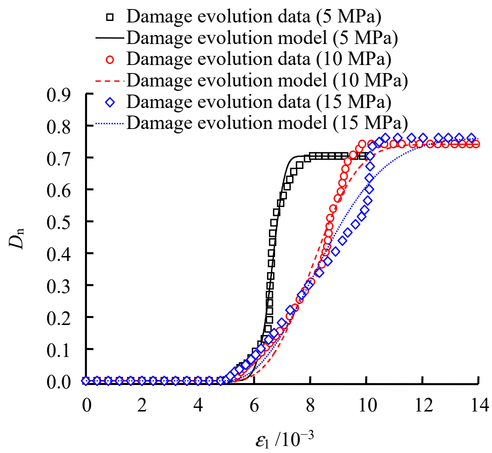


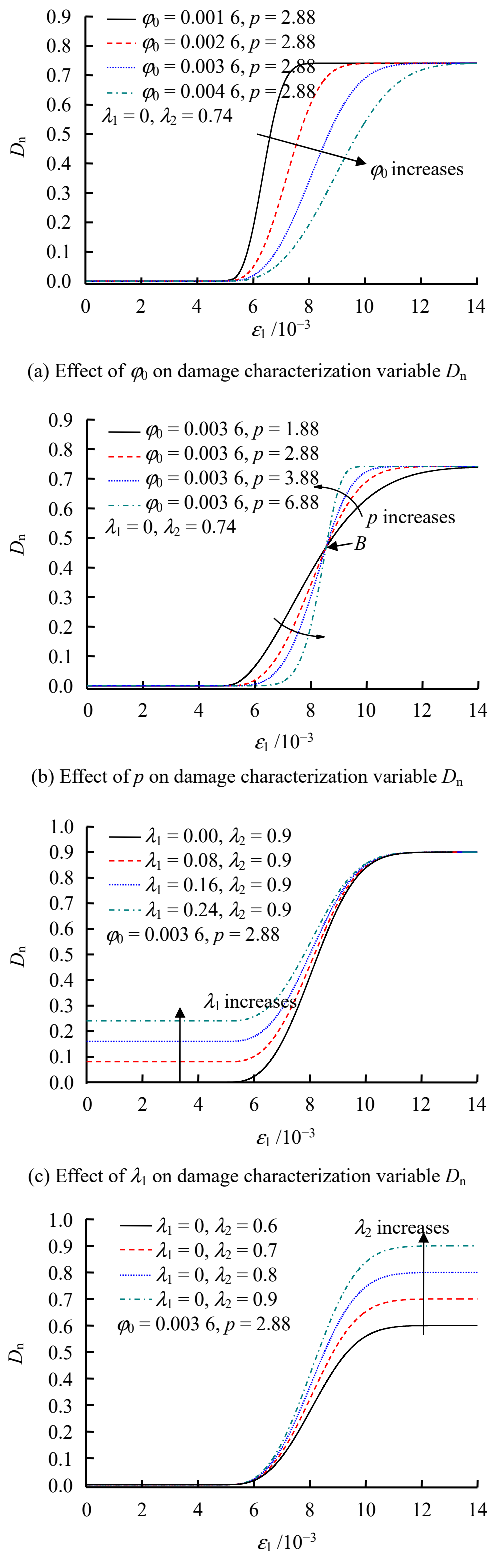
Fig. 10 Evolution model of the damage characterization variable D_n

Table 1 Parameters of the damage evolution model D_n

Confining pressure σ_3 /MPa	φ_0	p	λ_1	λ_2	Correlation coefficient R^2
5	0.001 7	5.14	0	0.704	0.983
10	0.003 6	2.88	0	0.741	0.978
15	0.004 0	2.20	0	0.761	0.976

In order to verify the rationality and feasibility of the proposed D_n evolution model, and obtain the parameters determination methods by analyzing their physical meanings, the sensitivity of the model parameters is analyzed, as shown in Fig.11:

(1) When λ_1 = 0, λ_2 = 0.74, and the parameter p remains unchanged, the damage change rate D_n decreases as φ_0 increases, and the damage evolution curve is approximately horizontally stretched along the direction of the increasing axial strain. Meanwhile, the amount of deformation required for the rock during the process from the beginning of the damage to failure increases. When φ_0 is constant, the damage evolution curve rotates counterclockwise around the fixed point B, and the damage change rate D_n increases with the increase of p. Moreover, the required deformation for the rock starting from the damage to failure decreases. Therefore, parameters φ_0 and p are equivalent to the scale parameter α_0 and the shape parameter m in the statistical damage evolution



(a) Effect of φ_0 on damage characterization variable D_n

(b) Effect of p on damage characterization variable D_n

(c) Effect of λ_1 on damage characterization variable D_n

(d) Effect of λ_2 on damage characterization variable D_n

Fig. 11 Evolution of damage characterization variable D_n with model parameters

model, which can characterize the heterogeneity of rock materials from the perspective of RVE strength statistics.

(2) When φ_0 = 0.003 6, p = 2.88, and the parameter λ_2 remains unchanged, the damage change rate decreases as λ_1 increases and the change of D_n with the increase of axial strain is 'S-shaped'. When the axial strain is below the damage strain threshold ε_h, D_n is λ_1. As the axial strain increases, D_n increases and the peak value is λ_2, which remains unchanged. When λ_1 is constant and λ_2 increases, the damage rate increases and the change of D_n with the increase of axial strain is still 'S-shaped'. D_n is λ_1 when the axial strain is below the damage strain threshold ε_h. As the axial strain increases, D_n increases and the peak value is λ_2, which is constant.

5 Simulation method for the entire deformation and failure process of rocks

Generally, the entire deformation and failure process of rocks is often divided into five deformation stages: initial compaction, linear elastic, yield hardening, strain softening, and residual strength. However, the entire deformation and failure process of rocks can be divided into the undamaged deformation stage and the damaged deformation stage considering the influence of the damage strain threshold. In the following sections, the corresponding constitutive models are established based on these two deformation stages to obtain the simulation method of the entire deformation and failure process of rocks.

5.1 Rock constitutive model before reaching the damage strain threshold

The existence of the initial compaction deformation stage is closely related to the number of micro-cracks in the rock[24]. If there are only a few micro-cracks, this deformation stage is not obvious or even missing, as shown in Fig.2. When there is a large number of micro-cracks, the obvious nonlinearity can be observed during this deformation stage, which cannot be ignored, as displayed in Fig.3. Therefore, two traditional deformation stages including the linear elastic deformation stage and the initial compaction deformation stage should be simulated in the rock constitutive model before reaching the damage strain threshold. Since the irrecoverable deformation of the rock during the initial compaction deformation stage is mainly caused by the crack closure, the rock material can be regarded as the rock matrix when the crack closure deformation is completed, and both elastic deformation and irrecoverable deformation caused by yielding can be generated. As shown in Fig.12, a random point A is taken on the stress−strain curve of the initial compaction deformation stage. The axial strain ε_{cm} at point A is composed of the axial strain of the rock matrix ε_m and the axial strain of the crack closure ε_{cc}:

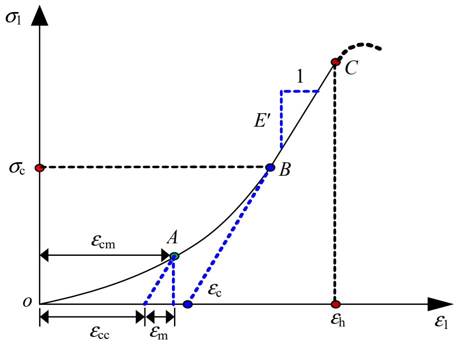


Fig. 12 Analysis of axial deformation before reaching the damage threshold

$$\varepsilon_{cm} = \varepsilon_m + \varepsilon_{cc} \tag{14}$$

where

$$\varepsilon_m = (\sigma_1 - \sigma_3) / E' \tag{15}$$

Then the relationship between the axial strain of the crack closure (ε_{cc}) and the axial stress can be obtained by subtracting the axial strain of the rock matrix (ε_m) from the axial strain of point A (ε_{cm}). The triaxial compression test data[25−26] show that the axial strain of the crack closure (ε_{cc}) gradually increases with the increase of the axial stress and tends to be stable when the axial stress reaches the crack closure stress. This change law can be described by a negative exponential model:

$$\varepsilon_{cc} = \xi\left[1 - \exp\left(-\frac{\sigma_1 - \sigma_3}{n}\right)\right] \tag{16}$$

where ξ and n are model parameters.

According to the theoretical model of crack closure, the stress−strain curve of the undamaged deformation stage of limestone under different confining pressures is fitted as shown in Fig.13. The parameters for the theoretical model of crack closure are displayed in Table 2. The theoretical model of crack closure can well simulate the entire process of the undamaged deformation stage of limestone under different confining pressures, and the fitting correlation coefficient R^2 is over 0.99. The physical meaning of the parameter ξ is the crack closure strain ε_c, which indicates the rationality and feasibility of this model. Therefore, the constitutive model before reaching the damage strain threshold ε_h can be obtained by combining Eqs. (14) to (16):

$$\varepsilon_1 = \frac{\sigma_1 - \sigma_3}{E'} + \xi\left[1 - \exp\left(-\frac{\sigma_1 - \sigma_3}{n}\right)\right] \tag{17}$$

5.2 Rock constitutive model after reaching the damage strain threshold

Based on the classical damage mechanics theory, the constitutive model of rock damage under triaxial compression[27] can be written as

$$\sigma_1 = [1 - D_n(\varepsilon_1)]E'\varepsilon_1 + 2\mu\sigma_3 \tag{18}$$

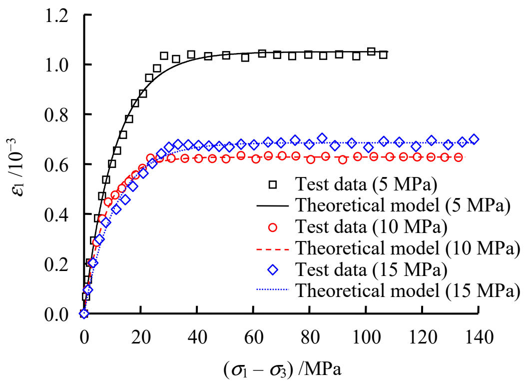


Fig. 13 Comparison of the theoretical model and test data of the crack closure

Table 2 Parameters of the theoretical model of the crack closure

Confining pressure σ_3 /MPa	E' /GPa	ε_c /10^{-3}	ξ/10^{-3}	n /MPa	Correlation coefficient R^2
5	27.76	1.03	1.05	11.06	0.996
10	29.76	0.63	0.63	7.59	0.996
15	32.71	0.68	0.69	11.17	0.991

where μ is the Poisson's ratio. Considering the influence of the damage strain threshold on the mechanical properties of rock, the constitutive model after reaching the damage strain threshold can be obtained by combining Eqs. (13) and (18):

$$\sigma_1 = \begin{cases} [1 - D_n(\varepsilon_1)]E'(\varepsilon_1 - \xi) + 2\mu\sigma_3,\ \varepsilon_h \leqslant \varepsilon_1 < \varepsilon_r \\ (1 - \lambda_2)E'(\varepsilon_r - \xi) + 2\mu\sigma_3,\ \varepsilon_1 \geqslant \varepsilon_r \end{cases} \tag{19}$$

To apply this model to simulating the entire process of damage and deformation, the damage strain threshold ε_h and residual strain ε_r should be determined. For the rock in the undamaged deformation stage, $D_n(\varepsilon_1) = 0$, and its deformation characteristics can be described by the generalized Hooke's law. According to Eq.(18), it can be obtained that:

$$\varepsilon_1 = (\sigma_1 - 2\mu\sigma_3) / E' \tag{20}$$

When the rock enters the yielding state from the elastic state, the irrecoverable deformation is produced and the damage occurs and accumulates continuously. The yield failure criterion for rocks can be introduced to determine the damage strain threshold ε_r. Assuming that the yield of the rock material obeys the Mohr-Coulomb criterion, the following equation can be obtained:

$$\sigma_1 - \frac{1 + \sin\varphi_y}{1 - \sin\varphi_y}\sigma_3 - \frac{2c_y\cos\varphi_y}{1 - \sin\varphi_y} = 0 \tag{21}$$

where c_y and φ_y are the cohesion and internal friction angle when rock material yields. The damage strain threshold ε_h related to the confining pressure σ_3 can be determined by substituting Eq.(21) into Eq.(20),

$$\varepsilon_h = \left(\frac{1 + \sin\varphi_y}{1 - \sin\varphi_y}\sigma_3 + \frac{2c_y\cos\varphi_y}{1 - \sin\varphi_y} - 2\mu\sigma_3\right) / E' \tag{22}$$

Meanwhile, as shown in Fig.3, the residual strain ε_r of the rock material can be obtained by the elastic modulus method:

$$\varepsilon_r = \sigma_r / [(1 - \lambda_2)E'] + \xi \tag{23}$$

where

$$\sigma_r = \frac{1 + \sin\varphi_r}{1 - \sin\varphi_r}\sigma_3 + \frac{2c_r\cos\varphi_r}{1 - \sin\varphi_r} \tag{24}$$

where c_r and φ_r are the cohesion and the internal friction angle of the rock at the residual strength deformation stage. According to the constitutive model after reaching the damage strain threshold ε_h, the stress−strain curves of limestone during the damage and deformation stage under different confining pressures are simulated. Then the theoretical model of the entire deformation and failure process of limestone can be acquired by combining it with the theoretical constitutive model before reaching the damage strain threshold, which is displayed in Fig.14. It shows that this model can well simulate the entire deformation and failure process of limestone, indicating that the rock damage characterization variables and its evolution model can reflect the rock damage evolution and failure mechanism. Moreover, the relationship between the crack closure deformation and the rock matrix deformation can be described by the analysis model of the rock axial deformation before reaching the damage strain threshold, which verifies the rationality and feasibility of the proposed model and method in this paper.

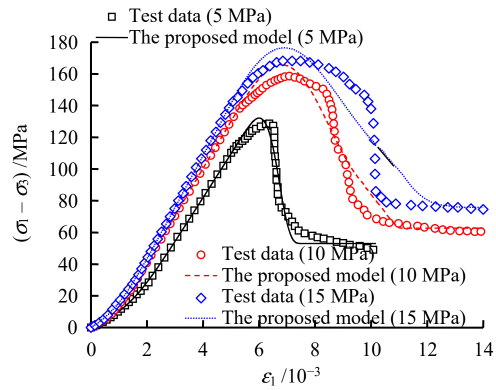


Fig. 14 Comparison of the theoretical model and test data for the whole deformation and failure process of the limestone

6 Conclusions

(1) The elastic modulus method and its damage constitutive model can only simulate the deformation and failure process of rocks under uniaxial compression. The damage evolution based on the unloading modulus substitution method is not able to correctly demonstrate the entire deformation and failure process of rocks under

compression.

(2) The statistical damage evolution model is limited to describing the entire deformation and failure process under triaxial compression. And on this basis the statistical damage constitutive model can only be regarded as a theoretical self-consistent solution in the numerical range [0, 1] of the statistical damage evolution model.

(3) The rock damage characterization variables and its evolution model based on the elastic modulus method can reflect the rock damage evolution and failure mechanism. Parameters φ_0 and p are equivalent to the scale parameter α_0 and the shape parameter m, respectively. Parameters λ_1 and λ_2 can be regarded as the initial and the final damage degree.

(4) The constitutive model before reaching the damage strain threshold can well simulate the initial compaction and linear elastic deformation stages, and the constitutive model after reaching the damage strain threshold can well simulate the yield hardening, strain softening, and residual strength stages. Therefore, the rationality and feasibility of the proposed model and method in this paper are verified.

References

[1] HU Jing, YAO Yang-ping, ZHANG Xue-dong, et al. Dynamic strength criterion for rock-like materials[J]. Chinese Journal of Geotechnical Engineering, 2020, 42(3): 495−502.

[2] CHENG Jian-long, LUO Song, LI Jia-bao, et al. Triaxial loading test of strength behavior and failure mechanism of composite rock[J]. Journal of Mining & Safety Engineering, 2020, 37(6): 1246−1254.

[3] LI Cheng-jie, XU Ying, FENG Ming-ming, et al. Deformation law and failure mechanism of coal-rock-like combined body under uniaxial loading[J]. Journal of China Coal Society, 2020, 45(5): 1773−1782.

[4] ZHANG Chao, BAI Yun. Establishment and application of parametric geometrical damage model of rocks[J]. Rock and Soil Mechanics, 2020, 41(12): 3899−3909.

[5] WANG Hui, ZHOU Shi-chen, ZHOU Bo, et al. Statistical damage models for hydrate-bearing sediments based on different failure criteria[J]. Rock and Soil Mechanics, 2020, 41(12): 4015−4026.

[6] CAO Wen-gui, ZHAO Heng, ZHANG Yong-jie, et al. Strain softening and hardening damage constitutive model for rock considering effect of volume change and its parameters determination method[J]. Rock and Soil Mechanics, 2011, 32(3): 647−654.

[7] LI Yan-long, LIU Chang-ling, LIU Le-le. Damage statistic constitutive model of hydrate-bearing sediments and the determination method of parameters[J]. Acta Petrolei Sinica, 2016, 37(10): 1273−1279.

[8] ZHANG Guo-kai, LI Hai-bo, WANG Ming-yang, et al. Comparative study on damage characterization and damage evolution of rock under uniaxial compression[J]. Chinese Journal of Geotechnical Engineering, 2019, 41(6): 1074−1082.

[9] ZHANG Chao, YANG Qi-jun, CAO Wen-gui. Study of damage constitutive model of brittle rock considering post-peak stress dropping rate[J]. Rock and Soil Mechanics, 2019, 40(8): 3099−3106.

[10] CHEN Hong-ran, QIN Si-qing, XUE Lei, et al. Characterization of brittle failure of rock and limitation of Weibull distribution[J]. Progress in Geophysics, 2017, 32(5): 2200−2206.

[11] ZHANG Ming, WANG Fei, YANG Qiang. Statistical damage constitutive model for rocks based on triaxial compression tests[J]. Chinese Journal of Geotechnical Engineering, 2013, 35(11): 1965−1971.

[12] CAO Wen-gui, MO Rui, LI Xiang. Study on statistical constitutive model and determination of parameters of rock based on normal distribution[J]. Chinese Journal of Geotechnical Engineering, 2007, 29(5): 671−675.

[13] XUE Hua-qing, XU Rui-na, JIANG Pei-xue, et al. Characterization of rock microstructure using 3D X-ray computed tomography[J]. Chinese Journal of Theoretical and Applied Mechanics, 2015, 47(6): 1073−1078.

[14] LEMAITRE J. How to use damage mechanics[J]. Nuclear Engineering and Design, 1984, 80(2): 233−245.

[15] TANG Chun-an. Catastrophe in rock unstable failure[M]. Beijing: China Coal Industry Publishing House, 1993.

[16] LOU Zhi-wen. Damage mechanics foundation[M]. Xi'an: Xi'an Jiaotong University Press, 1991.

[17] CHENG Guang-xu, LOU Zhi-wen, KUANG Zhen-bang. A research approach based on the exhaustion model of material ductility for fatigue damage[J]. Chinese Journal of Theoretical and Applied Mechanics, 1993, 25(4): 496−499.

[18] HOU R B, ZHANG K, TAO J, et al. A nonlinear creep damage coupled model for rock considering the effect of initial damage[J]. Rock Mechanics and Rock Engineering, 2018, 52(2): 1−11.

[19] LIU Dong-qiao. Study on rock damage constitutive model and chaos characteristics in the process of deformation and failure[D]. Beijing: China University of Mining and Technology(Beijing), 2014.

[20] CAO W G, TAN X, ZHANG C, et al. Constitutive model to simulate the full deformation and failure process for rocks considering initial compression and residual strength behaviors[J]. Canadian Geotechnical Journal, 2019, 56(5): 649−661.

[21] CAO W G, ZHAO H, LI X, et al. Statistical damage model with strain softening and hardening for rocks under the influence of voids and volume changes[J]. Canadian Geotechnical Journal, 2010, 47(8): 857−871.

[22] MARTIN C D, CHANDLER N A. The progressive fracture of Lac du Bonnet granite[J]. International Journal of Rock Mechanics and Mining Science and Geomechanics Abstracts, 1994, 31(6): 643−659.

[23] CAO Wen-gui, ZHAO Heng, LI Xiang, et al. A statistical damage simulation method for rock full deformation process with consideration of the deformation characteristics of residual strength phase[J]. China Civil Engineering Journal, 2012, 45(6): 139−145.

[24] ZHANG Chao, CHEN Qiu-nan, YANG Qi-jun, et al. Whole process simulation method of brittle rocks deformation and failure considering initial voids closure and its influence[J]. Journal of China Coal Society, 2020, 45(3): 1044−1052.

[25] ZUO Jian-ping, CHEN Yan, SONG Hong-qiang, et al. Pre-peak axial crack strain evolution and nonlinear model for coal-rock combined body[J]. Chinese Journal of Geotechnical Engineering, 2017, 39(9): 1609−1615.

[26] PENG J, RONG G, CAI M, et al. A model for characterizing crack closure effect of rocks[J]. Engineering Geology, 2015, 189: 48−57.

[27] LI Hai-chao, ZHANG Sheng. A constitutive damage model of rock based on the assumption of modified Lemaitre strain equivalence hypothesis[J]. Rock and Soil Mechanics, 2017, 38(5): 1321−1326.s