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Abstract: To unify the source power of water migration, a pressure-suction element model of film water is constructed on the basis of the film water hydraulic driving force model and the surface adsorption force model. Model analysis shows that under the dual action of net suction and actual liquid pressure (or theoretical suction and actual ice pressure), the surface adsorption force can be generated, which drives the tangential migration of water along the surface of the substrate. In view of the fact that the surface adsorption force has nothing to do with the boundary conditions, it is suitable for any form of unfrozen water, and it is the unified source power of water migration. Based on this, the pressure–suction element model is introduced into the frozen fringe theory, and it is found that the actual ice pressure determines the temperature and position of the segregated ice formation, the theoretical suction determines the direction of water migration, and the surface adsorption force determines the velocity of water migration. Finally, substituting the main parameters from the Konrad (1980) test into the surface adsorption force equation, it is found that even if the temperature gradient increases from 0.1 °C /cm to 0.67 °C /cm, the sample height increases from 6.4 cm to 28 cm, as long as the segregation freezing temperature and the overburden pressure keep unchanged, the surface adsorption force is always constant at –23 kPa, which verifies the correctness of the surface adsorption force equation. In short, the development of this model has important theoretical value and practical significance for improving the existing frost heave theory and guiding engineering practice.

Keywords: film water; pressure–suction element model; surface adsorption force; segregated ice; water migration velocity

1 Introduction

For the studies of frost heave and ice formation mechanism of soil/rock, the analysis of water migration driving force the key is. In terms of the driving force of water migration, fourteen hypotheses^[1] such as capillary suction, hydrostatic pressure, crystallization pressure, and osmotic pressure have been put forward. However, only two migration mechanisms of capillary water and film water are widely accepted.

Capillary water migration mechanism mainly studies the migration of unfrozen water in capillaries. Everett^[2] and Hopke^[3] believed that the migration driving force was capillary suction. The mechanism of film water migration mainly studies the water migration of unfrozen water film on the particle surface^[4]. At present, there are three main discussions on the driving force of film water migration. Harlan^[5] and Konrad et al.^[6], which did not consider the role of the ice pressure term, raised that the driving force of water migration was equivalent to the theoretical suction in the paper. Gilpin^[7], O’Neill and Miller^[8], Nixon^[9], and Sheng et al.^[10] proposed that the driving force of water migration was equivalent to the surface adsorption force in the paper by transforming the generalized Clapeyron equation^[11–12]. Considering

the comprehensive influence of the above factors, Thomas et al.^[13] thought that the driving force of water migration was equivalent to the resultant force of surface adsorption force, low-temperature suction (capillary suction), and gravity.

In conclusion, the driving force of water migration, whether it is capillary water migration mechanism or film water migration mechanism, has not been unified yet^[14]. The main reason is that the existing frost heave model is difficult to explain how the negative suction of water migration to the ice lens coexists with the positive pressure of soil particles squeezed by the ice lens^[15]. For example, Hopke^[3] and Gilpin^[7] believed that film water was only subjected to compression in the normal direction but suction in the tangential direction, which obviously violated the basic principle of isotropic isobaric in hydrodynamics. O’Neill and Miller^[8], Sheng et al.^[10], Zhou and Li^[16] held that the pore water in the capillary was negative pressure while the pore ice was positive pressure and used the concept of ‘neutral pressure’ to forcibly combine the positive and negative pressures, which obviously could not explain the pressure–suction equilibrium between the ice and water phases in the capillary. In addition, Thomas et al.^[13] stated that there

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was a negative suction at the freezing front for the film water, which could drive the water migration, but there was a positive pressure at the warm end of the ice lens, resulting in the generation of segregated ice. Obviously, this could not account for the phenomenon of water migration and phase transition to the warm end of the ice lens. Therefore, the construction of the film water pressure–suction equilibrium element model has important theoretical value and significance to improve the existing frost heave theory.

Cheng et al.^[17–18] established the pressure–suction equilibrium mechanical model of film water, but he only analyzed the pressure and suction variables from a one-dimensional perspective and did not introduce the pressure–suction model into the frost heave theory. In this study, the pressure–suction element model of film water is constructed from a two-dimensional perspective for the first time based on the hydraulic driving force model and surface adsorption force model of film water. Therefore, the source power of water migration is unified, and the pressure–suction element model is introduced into the frozen fringe theory model to reveal the water migration and ice formation mechanism of freezing soil. Finally, the correctness of the surface adsorption force equation is verified according to the test results of Konrad and Morgenstern^[19].

2 Development of pressure–suction element model of film water

2.1 Analysis of hydraulic driving force of film water

According to O’Neil and Miller^[8], the substrate surface was negatively charged, while the water molecules were dipoles. Under the action of electric field force, the water molecules were aligned directionally to form a diffusion electric double layer, which was defined as the film adsorbed water. On the basis of the research of Cheng et al.^[17], the hydraulic driving force model of film water could be obtained, as shown in Fig.1. In the figure, y is the distance between film water and particle surface; P_L is the hydraulic pressure of film water; P_{Lh} is the theoretical hydraulic pressure of film water at $h=y$ under equilibrium state; P_S is the actual ice pressure; γ_{SL} is the ice–water interfacial tension, which can be taken as 29 g/s^2 ; R is the effective radius of ice–water interface; γ_{SL}/R is the interface pressure; P_{Ly} is the actual hydraulic pressure of ice pressure P_S acting on film water after removing the influence of interface pressure; P_{Ld} is the hydraulic driving force for film water migration; P_{S0} is the theoretical ice pressure in equilibrium; P_{OB} is the overburden pressure; L is the phase change latent heat of water, which can be taken as $334.88 \times 10^7 \text{ cm}^2/\text{s}^2$; v_S

is the specific volume of ice; T is the ice–water interface temperature ($^\circ\text{C}$); and T_A is the absolute freezing temperature of pure water, taken as 273.15 K.

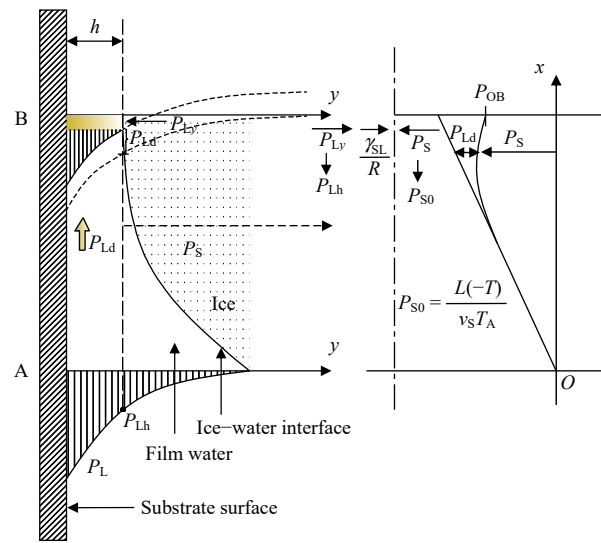


Fig. 1 Model of hydraulic driving force due to film water

According to Gilpin^[20], the theoretical hydraulic pressure of water molecules at $y = h$ of the thick film A in the equilibrium state is

$$P_{Lh} = \frac{a}{v_L} h^{-\alpha} \quad (1)$$

where a is a coefficient; v_L is the specific volume of water; and α is power. Based on the film water theory^[4], the thickness of film water is reduced due to freezing, which results in the water molecules at film B being affected by ice pressure, interface pressure, and hydraulic pressure at the ice–water interface simultaneously^[20]. So there is

$$P_S = P_{Ly} + \frac{\gamma_{SL}}{R} \quad (2)$$

Through the two-dimensional force analysis of the film water in Fig.1, it is found that at the same plane of $y = h$, the hydraulic pressure of the water molecules at thick film A is P_{Lh} , and the hydraulic pressure of the water molecules at film B is P_{Ly} . Therefore, there is a normal pressure difference in water molecules between A and B, which drives the water molecules to migrate tangentially along the substrate surface, and the corresponding hydraulic driving force is

$$P_{Ld} = P_{Lh} - P_{Ly} \quad (3)$$

According to Eq. (3), the hydraulic driving force tends to zero when the actual hydraulic pressure approaches the theoretical hydraulic pressure. At this time, the system reaches the equilibrium state, and the water migration stops, which means

$$P_{Ly} = P_{Lh} \quad (4)$$

In addition, based on the research of Gilpin^[7], the theoretical ice pressure equation in the equilibrium state is

$$P_{S0} = \frac{L(-T)}{v_s T_A} \quad (5)$$

Combined with Eqs. (2), (3), and (5), the stress state equation of water molecule at B on the ice-water interface in equilibrium state could be obtained,

$$P_{S0} = P_{Lh} + \frac{\gamma_{SL}}{R} \quad (6)$$

Substituting Eqs. (2), (5), and (6) into Eq. (3), we can get

$$P_{Ld} = P_{Lh} - P_{Ly} = \frac{L(-T)}{v_s T_A} - P_S \quad (7)$$

It can be seen from Eq. (7) that the hydraulic driving force of film water can be expressed as either normal hydraulic pressure difference or normal ice pressure difference.

2.2 Adsorption force analysis of the film water surface

Based on the theoretical model^[18] of pressure-suction balance, the conversion relationship between pressure-suction variables of film water could be expressed as

$$P_{Lb} = -\frac{v_s}{v_L} \left[\frac{L(-T)}{v_s T_A} - P_S \right] = -\frac{v_s}{v_L} P_{Ld} = \lambda P_{Ld} \quad (8)$$

where P_{Lb} is the surface adsorption force^[21]; and λ is the thermodynamic conversion coefficient of pressure-suction variables, which could be taken as -1.09 . By transforming Eq. (8), we can obtain

$$P_{Lb} = \lambda P_{Ld} = \frac{LT}{v_L T_A} - \lambda P_S \quad (9)$$

In the light of Konrad and Morgenstern et al.^[19], the equation of theoretical suction P_{SU} in the equilibrium state is

$$P_{SU} = \frac{LT}{v_L T_A} \quad (10)$$

Therefore, substituting Eq. (10) into Eq. (9) can yield

$$P_{Lb} = P_{SU} - \lambda P_S \quad (11)$$

where λP_S is the actual ice pressure offset factor. It can be seen from Eq. (11) that the surface adsorption force comes from the difference between the theoretical suction and the actual ice pressure offset factor. In addition, in combination with Eqs. (5) and (10), we can get Eq. (12).

$$P_{SU} = \frac{LT}{v_L T_A} = -\frac{v_s}{v_L} \frac{L(-T)}{v_s T_A} = \lambda P_{S0} \quad (12)$$

Then, substituting Eq. (6) into Eq. (12) can lead to Eq. (13).

$$P_{SU} = \lambda P_{Lh} + \lambda \frac{\gamma_{SL}}{R} = P_{Suh} + \lambda \frac{\gamma_{SL}}{R} \quad (13)$$

where P_{Suh} is the net suction of the substrate surface to the film water, removing the influence of interface pressure. According to Eq. (13), the theoretical suction is the total suction of the particle surface on the film water, and the net suction is the net value acting on the film water removing the influence of interface pressure. Accordingly, by substituting Eqs. (2) and (13) into Eq. (11), the second expression of surface adsorption force can be obtained,

$$P_{Lb} = P_{Suh} + \lambda \frac{\gamma_{SL}}{R} - \lambda \left(P_{Ly} + \frac{\gamma_{SL}}{R} \right) = P_{Suh} - \lambda P_{Ly} \quad (14)$$

where λP_{Ly} is the actual hydraulic offset factor. To sum up, there are two expressions of surface adsorption force, which can be expressed by the difference between theoretical suction and actual ice pressure offset factor or between net suction and actual hydraulic offset factor. The specific distribution and physical significance of the above parameters are shown in Fig.2.

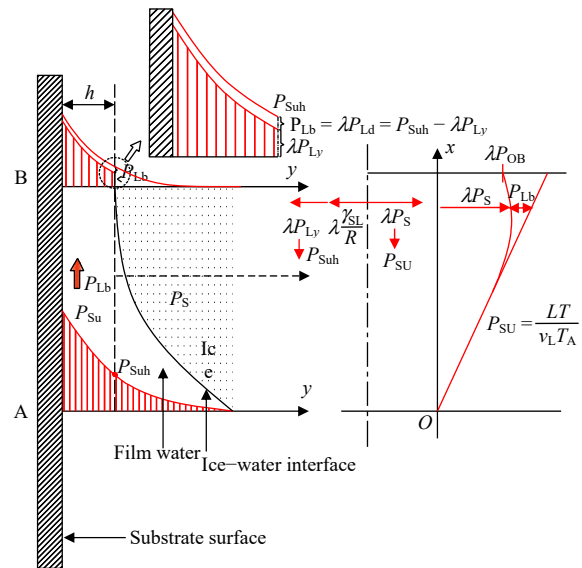


Fig.2 Model of surface adsorption force due to film water

Through the two-dimensional force analysis of the film water in Fig.2, the surface adsorption force is generated to drive the water to migrate tangentially along the substrate surface under the dual action of theoretical suction and actual ice pressure. In addition, in order to eliminate the influence of interface pressure, it is found that the surface adsorption force is also produced under the dual action of net suction and actual hydraulic pressure. Therefore, the interfacial pressure does not affect the surface adsorption force, i.e., the surface adsorption force has nothing to do with the boundary conditions, which is valid for any form of unfrozen water^[7].

In addition, in order to verify the correctness of the derivation process of Eq. (14), substituting Eq. (7) into Eq. (8), we can obtain

$$P_{Lb} = \lambda \left[\frac{L(-T)}{v_s T_A} - P_S \right] = \lambda (P_{Lh} - P_{Ly}) = P_{Suh} - \lambda P_{Ly} \quad (15)$$

It can be seen from Eqs. (7) and (15) that the derivation process of the above surface adsorption force is reasonable and correct from both the hydrodynamic analysis of film water and the thermodynamic analysis of water molecules.

In fact, both the hydraulic driving force and surface adsorption force can drive film water to migrate from the thick film A to film B, but their study perspectives are different. The former mainly studies the hydraulic driving mechanism of film water so that the force analysis conforms to the basic principle of hydrodynamics; the latter mainly investigates the low-temperature adsorption mechanism of film water so that the force analysis conforms to the basic principle of thermodynamics.

For the convenience of analysis, the hydraulic driving mechanism can be likened to gravity water under the

action of liquid layer pressure. Therefore, water fluid flows from the high potential energy (thick film A) to the low potential energy (thin film B) under the action of the gravitational potential. The adsorption mechanism is analogous to the adsorption of the earth’s gravitational field so that water molecules migrate from the low gravitational field strength (thick film A) to the high gravitational field strength (film B) under the action of the field.

2.3 Pressure–suction element model of film water

According to the hydraulic driving force analysis of film water and the surface adsorption force analysis of water molecules, the pressure-suction element model of film water can be constructed. The specific distribution is shown in Fig.3.

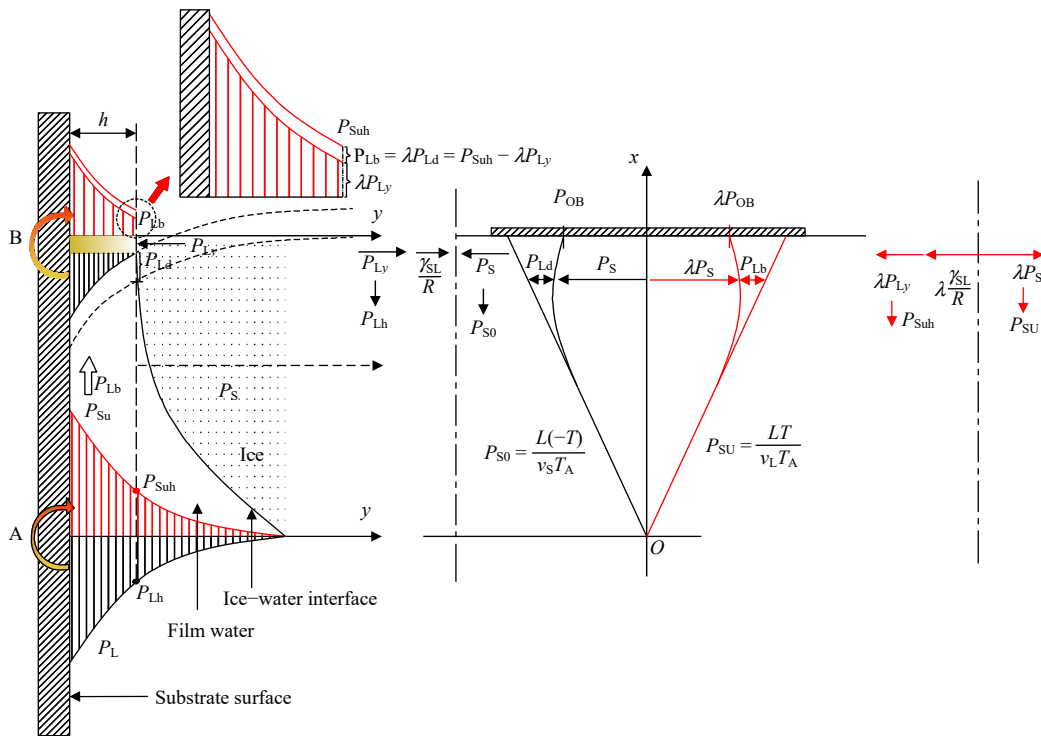


Fig. 3 Element model of film water in pressure and suction states

Through the two-dimensional stress analysis of the film water in Fig.3, it is found that the unfrozen film water at A is in a pressure-suction equilibrium state under the combined action of net suction and hydraulic pressure. However, freezing leads to the reduction of film water thickness and actual hydraulic pressure at B. In view of the constant net suction of the film water, the surface adsorption force will be generated under the dual action of net suction and actual hydraulic pressure (or theoretical suction and actual ice pressure), driving the water molecules to migrate tangentially along the substrate surface. In addition, since the surface adsorption force is independent of boundary conditions and is suitable for any form of unfrozen water, the surface adsorption force is the unified

source power of water migration.

3 Application of pressure–suction element model to the frozen fringe theory

3.1 Formation mechanism of segregated ice

By substituting the pressure–suction variable distribution of film water into the model of frozen fringe theory^[22], it was found that the actual ice pressure increased gradually with the decrease of temperature. According to Gilpin^[7], a new ice lens was generated when the actual ice pressure exceeded the sum of overburden pressure and separation pressure. Nixon^[9] defined the separation pressure as the additional pressure component that started the separation of soil skeleton. Therefore, the formation

conditions of segregated ice are

$$P_s \geq P_{OB} + P_{sep} \tag{16}$$

where P_{sep} is the separation pressure.

In Fig.4, T_f is the freezing temperature at the freezing front, which could be taken as $0\text{ }^\circ\text{C}$ ^[16]; T_{s0} is the segregation freezing temperature at the warm end of the old ice lens; and T_{s1} is the segregation freezing temperature at the warm end of the new ice lens. Curve I is the variation curve of ice pressure with temperature before the formation of new ice lens. Curve II is the variation curve of ice pressure with temperature during the formation of new ice lens.

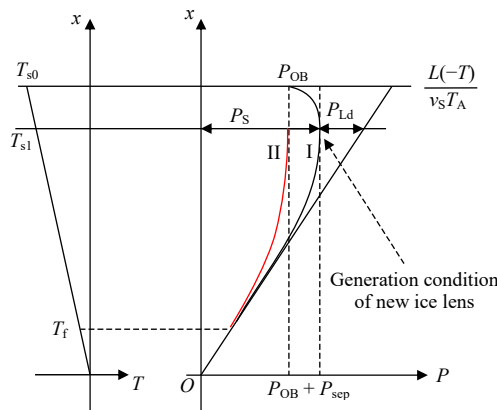


Fig. 4 Formation mechanism of segregated ice^[23]

3.2 Surface adsorption force at the warm end of ice lens

O’Neill and Miller^[8] believed that after the formation of the new ice lens, the actual ice pressure in the frozen fringe zone would decrease rapidly, i.e., it would quickly change from curve I to curve II, and reduce to the overburden pressure^[23] at the warm end of the new ice lens. So there is

$$P_s = P_{OB} \tag{17}$$

Combining Eqs. (15) and (17), the surface adsorption force equation of the warm end of the new ice lens can be obtained,

$$P_{Lb} = \lambda \left[\frac{L(-T_s)}{v_s T_A} - P_{OB} \right] = \frac{L T_s}{v_L T_A} - \lambda P_{OB} \tag{18}$$

It can be seen from Eq. (18) that the primary function of the overburden pressure is to counteract the theoretical suction at the warm end of the ice lens, and thus λP_{OB} can be defined as the overburden pressure offset factor.

In conclusion, the surface adsorption force at the warm end of the ice lens only depends on the difference between the theoretical suction corresponding to the segregation freezing temperature T_s and the overburden pressure offset factor, which is independent of the boundary conditions.

3.3 Segregation freezing temperature

Williams and Wood^[15] measured the actual ice pressure change rate with temperature in the frost heave test was $1.1\text{ MPa}/^\circ\text{C}$, while the theoretical ice pressure change rate with temperature calculated by the theory was $1.124\text{ MPa}/^\circ\text{C}$. Hypothesis 1: when a new ice lens is formed, the actual ice pressure P_s at the warm end of the ice lens is equal to the theoretical ice pressure P_{s0} . By combining with Eqs. (5) and (16), we can obtain

$$P_s = \frac{L(-T_s)}{v_s T_A} = P_{OB} + P_{sep} \tag{19}$$

The governing equation of segregation freezing temperature can be obtained by transforming Eq. (19),

$$T_s = -(P_{OB} + P_{sep}) \frac{v_s T_A}{L} \tag{20}$$

It can be seen from Eq. (20) that when the separation pressure P_{sep} remains unchanged, the higher the overburden pressure P_{OB} , the lower the corresponding segregation freezing temperature T_s . Hence, the overburden pressure is the main controlling factor of segregation freezing temperature.

3.4 Permeability coefficient of frozen fringe

Based on a large number of frost heave test data, Thomas et al.^[13] gave the fitting equation between the permeability coefficient of frozen fringe and segregation freezing temperature as follows:

$$k_{ff} = \begin{cases} k_{uf} & T > T_f \\ k_{uf} [1 - (T_s - T_f)]^{-\beta} & T_f \geq T \geq T_s \\ 0 & T < T_s \end{cases} \tag{21}$$

where k_{uf} is the permeability coefficient of saturated soil under ambient temperature; k_{ff} is the total permeability coefficient of frozen fringe; and β is power index. Zhou and Li^[16] considered that β of saturated silt could be taken as 8. It can be seen from Eq. (21) that the segregation freezing temperature and the permeability coefficient at the frozen fringe have a high-order power function relationship. Therefore, the lower the segregation freezing temperature, the smaller the corresponding permeability coefficient at the frozen fringe. When the internal temperature of the soil is lower than the segregation freezing temperature, the corresponding permeability coefficient decreases to 0, and the water migration stops.

3.5 Water migration velocity

Hypothesis 2: water only moves to the warm end of the ice lens and conforms to Darcy’s law. The governing equation of water migration velocity can be given as

$$V_{ff} = ki = k \frac{\Delta h}{L_t} = \frac{k}{\rho_w g} \frac{\Delta P}{L_t} = -\frac{k_{ff}}{\rho_w g} \left(\frac{P_{Lb}}{L_t} + \rho_w g \right) \tag{22}$$

where k is the permeability coefficient variable; i is the

hydraulic gradient; Δh is the head difference; ΔP is the pressure difference; g is the gravitational acceleration; ρ_w is the water density; and L_t is the length of seepage path.

3.6 Distribution of pressure-suction variables in frozen fringe

In Fig.5, H is the height of the sample; C is the height of frozen area; A is the thickness of frozen fringe; Q is the height of unfrozen area; V_{uf} is the water migration velocity in the unfrozen area; V_{ff} is the water migration velocity in the frozen fringe area; V_f is the water migration velocity in the frozen area; x_f is the frozen front position; x_s is the warm end position of the ice lens; T_w is the hot end temperature of the sample; T_c is the cold end temperature of the sample; and P_w is the external water pressure. The above analysis can obtain the specific distribution of parameters such as temperature in the frozen fringe area, pressure-suction variables, permeability coefficient, and water migration velocity.

It can be seen from Fig.5 that the actual ice pressure and theoretical suction increase with the temperature decrease. The actual ice pressure determines the temperature and position of the formation of segregated ice, while the theoretical suction determines the direction of water

migration. Under the dual action of theoretical suction and actual ice pressure, the surface adsorption force is generated to drive the water to migrate to the ice lens's warm end and determine the velocity of water migration.

4 Test verification

For verifying the correctness of the above surface adsorption force equation, the test results of Konrad and Morgenstern^[19] are analyzed. The main physical and mechanical parameters are shown in Table 1.

Before the start of the test, the mass proportion of Devon silt passing 75 μm sieve was measured to be 95%. The saturated soil sample was consolidated under the pressure of 210 kPa, and the permeability coefficient of saturated silt was $(1.0 \pm 0.1) \times 10^{-7}$ cm/s. Then, the sample was put into the freezing bin. The top and bottom of the sample were adjusted to negative and positive temperatures, respectively, and the environmental bin temperature was constant at $+0.5$ °C, to ensure the axial freezing of the sample from top to bottom. Since it was an open system condition test, the water could be supplied through the base, and the head of the make-up water source was kept constant simultaneously.

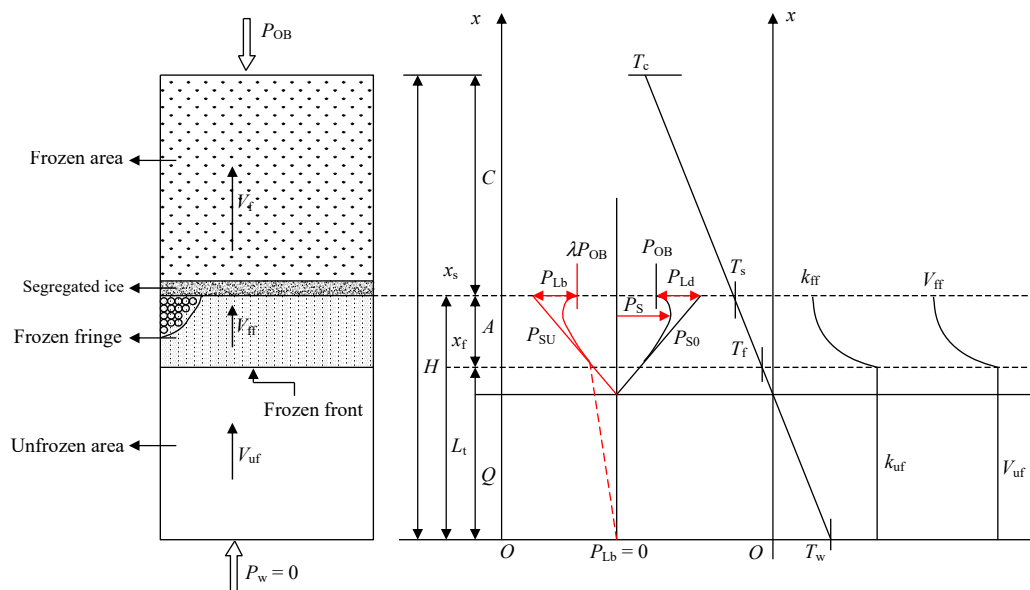


Fig. 5 Theoretical model of frozen fringe and its controlling parameters distribution

Table 1 Controlling parameters of the sample

Soil type	Liquid limit w_L /%	Plastic limit w_p /%	Specific gravity /Gs	Clay content /%
Silt	46.1	19.8	2.7	28

During the test, the temperature distribution, water migration in and out, and frost heave were monitored in the whole process. In addition, non-standard samples were used in the test. Different temperature gradients were used in the test process that the hot end temperature

T_w of the sample was constant at $+1 \pm 0.1$ °C, mainly adjusting the cold end temperature T_c .

After the test, the sample was cut into two halves along the axial direction. Half was measured for moisture content, and the other half needed to be measured for position and thickness of the segregated ice. It was found from the test that the segregation freezing temperature T_s was always maintained at about -0.1 °C. Other test parameters and values are able to be found in Table 2.

Table 2 Conditions of different tests at the beginning of growth of the last ice lens

Sample number	Hot end temperature $T_w/^\circ\text{C}$	Cold end temperature $T_c/^\circ\text{C}$	Sample height H/cm	Permeability coefficient $k_{uf}/(10^{-7} \text{ cm} \cdot \text{s}^{-1})$	Seepage path length L_t/cm	Temperature gradient $\text{grad } T/(^\circ\text{C} \cdot \text{cm}^{-1})$	Water migration velocity $V_{ff}/(10^{-6} \text{ mm} \cdot \text{s}^{-1})$
NS-1	+1.1	-3.4	10.4	1.00	3.20	0.37	31.5
NS-2	+1.1	-4.8	10.4	0.90	2.35	0.51	41.5
NS-4	+1.1	-2.5	7.6	1.10	3.00	0.40	40.0
NS-5	+1.1	-6.2	10.0	1.00	1.80	0.67	60.0
NS-6	+1.1	-3.4	6.4	0.95	1.80	0.67	59.0
NS-7	+1.1	-3.5	12.0	1.00	3.25	0.37	36.0
E-8	+1.1	-4.2	8.3	1.00	2.80	0.43	40.7
NS-9	+1.0	-6.0	28.0	1.00	10.60	0.10	9.0
NS-10	+1.0	-6.0	18.0	1.00	4.50	0.24	18.5

Note: k_{uf} values of NS-1, NS-2, NS-4, and NS-6 were given the test values, while k_{uf} values of other samples were taken the average values, i.e., $k_{uf} = 10^{-7} \text{ cm/s}$.

Table 2 shows the monitored values of cold end and hot end temperatures, sample height, permeability coefficient of saturated unfrozen soil, seepage path length of flow, temperature gradient, water migration velocity, and other variables.

In addition, Nixon^[9], Sheng et al.^[10] considered that the water migration velocity in the test could be measured first, and then the surface adsorption force at the warm end of the ice lens could be deduced. Based on this, the empirical equation of surface adsorption force can be obtained by transforming Eq. (22),

$$P_{Lb} = -\left(\frac{V_{ff}}{k_{uf}} + 1\right)\rho_w g L_t = -\left[\frac{V_{ff}}{k_{uf}}(1 - T_s)^\beta + 1\right]\rho_w g L_t \quad (23)$$

Taking NS-1 as an example, the corresponding surface adsorption force can be calculated by substituting the main parameters in Table 2 into the surface adsorption force Eq. (23),

$$P_{Lb} = -\left[\frac{V_{ff}}{k_{uf}}(1 - T_s)^\beta + 1\right]\rho_w g L_t = -\left[\frac{3.15 \times 10^{-6} \text{ cm/s}}{1.0 \times 10^{-7} \text{ cm/s}}(1 + 0.1)^8 + 1\right] \times 1.0 \times 10^3 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 3.2 \text{ cm} = -68.523 \times 10^3 \times 9.81 \times 3.2 \times 10^{-2} \text{ kg/(m} \cdot \text{s}^2) = -21.511 \text{ kPa}$$

Similarly, the theoretically calculated values of surface adsorption force of other samples in Table 2 can be obtained, detailed in Table 3.

Table 3 Theoretical values of surface adsorption force for different samples

Sample number	Surface adsorption force P_{Lb}/kPa	Sample number	Surface adsorption force P_{Lb}/kPa
NS-1	-21.511	NS-7	-24.922
NS-2	-23.017	E-8	-24.239
NS-4	-23.235	NS-9	-21.101
NS-5	-22.888	NS-10	-17.948
NS-6	-23.684	—	—

Note: the relative error of NS-10 was large, indicating that the actual value of k_{uf} of NS-10 deviated from the average value of k_{uf} , which belonged to calculation error and could be ignored.

The theoretically calculated value of the above surface adsorption force is tested via t -test with the number of samples n taken as 8. Firstly, the average value $\overline{P_{Lb}}$ is calculated,

$$\overline{P_{Lb}} = -\frac{1}{8}(21.511 + 23.017 + \dots + 21.101) = -23.074 \text{ kPa}$$

The expected value μ of surface adsorption force is assumed as -23.0 kPa . Accordingly, the corresponding sample standard deviation S_X is

$$S_X = \sqrt{\frac{\sum_{i=1}^8 (P_{Lb} - \mu)^2}{8 - 1}} = 1.287 \quad (24)$$

At this time, the t -test statistic of single population is

$$t = \frac{\overline{P_{Lb}} - \mu}{S_X / \sqrt{n}} = \frac{|23.074 - 23.0|}{1.287 / 2.828} = 0.163 \quad (25)$$

By querying the test critical value table^[24] with a significance level of 0.1 and a degree of freedom of 7, it can be seen that the average value of the surface adsorption force is not statistically different from the expected value. So the two results are basically the same. According to this, substituting the segregation freezing temperature of $-0.1 \text{ }^\circ\text{C}$ and the surface adsorption force of -23.0 kPa into Eq. (18), we can obtain:

$$P_{Lb} = -122.5 + 1.09P_{OB} = -23.0 \text{ kPa}$$

Hence the overburden pressure is $P_{OB} = 91.284 \text{ kPa}$.

Since the frost heave test led by Konrad and Morgenstern^[19] was carried out in the back pressure system, there was no additional load, but the contact pressure between the sample and the cold source was equal to 92% of the back pressure^[19]. Therefore, the contact pressure was the overburden pressure P_{OB} . In addition, from Eqs. (18) and (19), we could get,

$$P_{Lb} = \lambda \left[\frac{L(-T_s)}{v_s T_A} - P_{OB} \right] = \lambda P_{sep} \quad (26)$$

Substituting the surface adsorption force of -23.0 kPa into Eq. (26), we can obtain $P_{sep} = 21.101 \text{ kPa}$.

The recommended value of separation pressure of saturated silt in literature [25] was 20 kPa, while for Nixon^[9], it was 25 kPa. Obviously, the above separation pressure is reasonable.

According to $P_{OB} = 91.284$ kPa, $P_{sep} = 21.101$ kPa and combined with Eq.(20), the theoretical calculation value of segregation freezing temperature can be obtained:

$$T_s = -\frac{91.284 \text{ kPa} + 21 \text{ kPa}}{1.124 \times 10^{-3} \text{ kPa} / ^\circ\text{C}} = -0.100 \text{ }^\circ\text{C}$$

Evidently, the theoretical calculation value of the segregation freezing temperature is entirely consistent with the above test results. As a result, the overburden and separation pressures are taken as 91.284 kPa and 21.101 kPa, respectively, which is completely reasonable and correct.

In conclusion, even if the temperature gradient $\text{grad } T$ increases from 0.1 $^\circ\text{C} / \text{cm}$ to 0.67 $^\circ\text{C} / \text{cm}$ and the sample height H increases from 6.48 cm to 28 cm, as long as the segregation freezing temperature is constant at -0.1 $^\circ\text{C}$ and the overburden pressure is maintained at 91.284 kPa, the surface adsorption force at the warm end of the ice lens is always constant at -23 kPa. The results indicate that the surface adsorption force only depends on the difference between the theoretical suction corresponding to the segregation freezing temperature and the offset factor of the overburden pressure, while its value is independent of the temperature gradient and the sample height, which verifies the correctness of the surface adsorption force equation.

5 Discussion

The pressure–suction element model of film water considers the surface adsorption force as the source power of water migration, which only depends on the difference between the theoretical suction corresponding to the segregation–freezing temperature T_s and the offset factor of the overburden pressure, and has nothing to do with the temperature gradient and the height of the sample. The segregation potential theory proposed by Konrad and Morgenstern^[6] held that the water migration velocity was proportional to the temperature gradient with the ratio of the segregation potential constant SP.

The main difference between the segregation potential theory and the model developed in paper is that it does not consider the effect of actual ice pressure. Hence, the source power of water migration is the theoretical suction P_{SU} . It can be seen from Eq. (10),

$$\frac{dP_{SU}}{dx} = \frac{L}{v_L T_A} \text{grad} T = \frac{L}{v_L T_A} \frac{T_s - T_w}{L_t} \quad (27)$$

Based on Eq. (27), the greater the temperature gradient,

the greater the corresponding theoretical suction gradient. Since the segregation freezing temperature T_s measured in Konrad and Morgenstern^[19] test was constant at -0.1 $^\circ\text{C}$, it is concluded that the greater the temperature gradient, the greater the theoretical suction gradient, the greater the water migration velocity.

In fact, in Eq. (27), the influences of temperature gradient and seepage path on water migration velocity are the same. Moreover, the seepage path is an important part of Darcy's law, and therefore the seepage path is the main factor affecting water migration velocity. In addition, Nixon^[9] found that the water migration velocity and driving force were not sensitive to the temperature gradient in the frost heave test. Meanwhile, the segregation potential SP was a semi-empirical equation, which is obviously lacking sufficient theoretical support to forcibly combine the water migration velocity and temperature gradient through the test.

Since then, Xu et al.^[26] and Zhang et al.^[27] also realized the shortcomings of the segregation potential theory, especially the segregation potential SP was challenging to determine under unstable conditions, which made it difficult to apply to engineering practice. Therefore, the study on the film water pressure–suction model analyzes the action mechanism and physical significance of various parameters and then finds that the surface adsorption force is the unified source power of water migration, eliminating the diversified discussion on the driving force of water migration.

6 Conclusion

(1) Based on the film water hydraulic driving force model and surface adsorption force model, the film water pressure–suction element model is developed from a two-dimensional perspective for the first time. The model analysis shows that the unfrozen film water is in pressure–suction equilibrium under the combined action of net suction and hydraulic pressure. However, freezing destroys the pressure–suction balance, decreasing film water thickness and hydraulic pressure. Since the net suction remains unchanged at this time, the surface adsorption force is generated under the dual action of net suction and actual hydraulic pressure, driving the water to migrate tangentially along the substrate surface. The surface adsorption force is irrelevant to the boundary conditions and is valid for any form of unfrozen water, which is the unified source power of water migration.

(2) Substituting the film water pressure–suction element model into the model of frozen fringe theory, it is found that the actual ice pressure and theoretical suction increase with the decrease of temperature. The

actual ice pressure determines the temperature and location of the formation of segregated ice, while the theoretical suction determines the direction of water migration. Under the dual action of the theoretical suction and the actual ice pressure, the surface adsorption force is generated, which drives the water to migrate to the warm end of the ice lens and determines the water migration velocity.

(3) According to the test analysis, when the segregation-freezing temperature is $-0.1\text{ }^{\circ}\text{C}$ and the overburden pressure is 91.284 kPa , even if the temperature gradient increases from $0.10\text{ }^{\circ}\text{C}/\text{cm}$ to $0.67\text{ }^{\circ}\text{C}/\text{cm}$ and the sample height increases from 6.4 cm to 28.0 cm , the surface adsorption force is always constant at -23.0 kPa . Hence, the surface adsorption force is independent of the temperature gradient and the sample height but only depends on the difference between the theoretical suction corresponding to the segregation freezing temperature and the offset factor of the overburden pressure, which verifies the correctness of the surface adsorption force equation.

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