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General analytical solution for consolidation of sand-drained ground considering the vacuum loading process and the time-dependent surcharge loading

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Abstract: Based on Barron's equal strain consolidation theory, the consolidation governing equation of sand-drained ground under vacuum combined surcharge preloading is derived, and a general analytical solution is obtained, which considers the vacuum loading process, the time-dependent surcharge loading, the characteristics of the vacuum pressure decreasing along with the depth and radial direction, and the smear effect of vertical drains and the vertical flow. The correctness of the analytical solution proposed in this paper is verified by comparing the degenerate analytical solution with the existing analytical solution and the finite difference solution. Based on the analytical solution, the consolidation behaviors of sand-drained foundation are analyzed. The results show that the consolidation rate of sand-drained ground is accelerated with the increase of vacuum loading factor η. However, when η increases to a certain extent, the influence of the vacuum loading process on the consolidation rate of sand-drained ground can be ignored. The consolidation rate of sand-drained ground decreases with the increase of vacuum pressure attenuation coefficients k_1 and k_2. The consolidation rate of sand-drained ground decreases with the increase of vacuum pressure p_0 and increases with the increase of final surcharge loading q_u. With the increase of loading time T_{h1}, the consolidation rate of sand-drained ground decreases gradually, and the consolidation rate is the largest under instantaneous surcharge loading.

Keywords: vacuum loading process; surcharge loading; vacuum combined surcharge preloading; sand-drained ground; general analytical solution

1 Introduction

Vacuum preloading method is commonly used to strengthen soft soil ground in engineering[1]. Considering the transfer characteristics of vacuum negative pressure, this method is often used in combination with sand drain (mostly plastic drainage plate currently). In the existing theoretical research, the consolidation and drainage problems of soil under the vacuum preloading method are mainly studied by the sand-drained ground consolidation theory[2−5]. Indraratna et al.[2] derived a radial consolidation analytical solution of sand-drained ground by assuming that the negative vacuum pressure in the sand drain is linearly attenuated along with the depth. Based on the double logarithmic compression model, Han et al.[4] obtained a nonlinear consolidation analytical solution of sand-drained ground under vacuum preloading. Zhang et al.[5] considered varying well resistance with time to study the consolidation of sand-drained ground with varying radial permeability coefficient under vacuum preloading.

To enhance the consolidation and drainage effects of soft soil ground, the vacuum preloading method is often combined with surcharge loading in engineering[6−10]. Rujikiatkamjorn et al.[6] analyzed the consolidation and drainage of sand-drained ground under the action of vacuum combined surcharge preloading, considering both the radial and vertical flow and drainage in soil. Guo et al.[7] derived a general consolidation analytical solution of sand-drained ground under vacuum combined surcharge preloading, considering the attenuation of the vacuum pressure along with the depth and the change of the additional stress with time and depth caused by surcharge loading. Guo et al.[9] obtained a nonlinear consolidation analytical solution of sand-drained ground under vacuum combined surcharge preloading by considering the nonlinear compression and permeability relationship of soil.

However, in the above theoretical studies on the consolidation of sand-drained ground, the vacuum loading is assumed to be applied instantaneously, and the influence of the vacuum loading process on the consolidation properties of sand-drained ground has not been considered. Peng et al.[11] used an exponential function to research the vacuum loading process, took into account the factors such as smear effect and well resistance, and obtained the consolidation solution of sand-drained ground considering the vacuum loading process. Based on the elliptical cylinder equivalent model, Tian et al.[12] considered the time effect of the

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vacuum loading and derived the analytical consolidation solution of sand-drained ground under vacuum preloading. However, a specific exponential function is used to consider the one single loading process of vacuum pressure in the above research, which is not universal.

In the actual soft soil ground treatment, it is advisable to consider the stepped vacuum loading to reduce the clogging degree of the drainage channel to avoid too large a hydraulic gradient caused by one single vacuum loading[13−14]. The existing model test results show that the final settlement of sand-drained ground under stepped vacuum loading is greater, and the strength of soil after treatment is also greater[15−16]. At the same time, the test results show that the vacuum pressure will attenuate not only along with the depth of soil but also the radial direction of soil[17−18]. Therefore, the attenuation of vacuum pressure along the depth and radial direction should be considered[19−20]. In addition, the vacuum preloading method is usually used combining with surcharge loading to strengthen the soft soil ground, and the surcharge is often not applied at one time. The influence of time-dependent surcharge loading on the consolidation behavior of sand-drained ground should be considered[7].

Therefore, the consolidation and drainage issues of sand-drained ground should be studied in combination with considering the vacuum loading process and time-dependent surcharge loading. Based on Barron's equal strain consolidation theory of sand-drained ground[21], the consolidation and drainage governing equation under vacuum combined surcharge preloading is derived and established in this paper, considering the vacuum loading process, the time-dependent surcharge loading, the characteristics of the vacuum pressure decreasing along with the depth and radial direction, the smear effect, and the vertical flow. A more general consolidation analytical solution is obtained by series transformation. The correctness of the analytical solution in this paper is verified by comparing and analyzing the degenerate solution in the paper with the existing analytical solution and the analytical solution with the finite difference solution. Based on the present analytical solution, the consolidation behavior of sand-drained ground is analyzed.

2 Consolidation governing equation of sand-drained ground

2.1 Calculation diagram

For the convenience of analysis, the single sand-drained ground is taken as the research object. Figure 1 shows the calculation diagram of consolidation and drainage of sand-drained ground considering vacuum loading process and time-dependent surcharge loading. It is assumed that the top of sand-drained ground is a drainage boundary, and the bottom is an undrained boundary. In Fig. 1, r and z are the radial and vertical coordinates of sand-drained ground, respectively; the length of the sand drain is l; r_w, r_s, and r_e are the radius of the sand drain, the radius of smear zone of the sand drain, and the radius of influence zone of the sand drain, respectively; k_s, k_h, and k_v are the radial permeability coefficient of the soil in smear zone, the radial permeability coefficient of the soil in the undisturbed zone, and the vertical permeability coefficient of the soil in influence zone, respectively; $q(t)$ is the uniformly distributed load changing with time t acting on the surface of sand-drained ground. In order to consider the vacuum loading process, it is assumed that the vacuum pressure applied at a given time t is $p_t(t)$. According to the references[19−20], it is assumed that the attenuation of vacuum pressure along with depth and radial direction. At time t, it is assumed that the vacuum pressure decreases from $p_t(t)$ to $k_1 p_t(t)$ along with the depth direction and decreases from $p(z, r_w, t)$ to $k_2 p(z, r_w, t)$ in the radial direction.

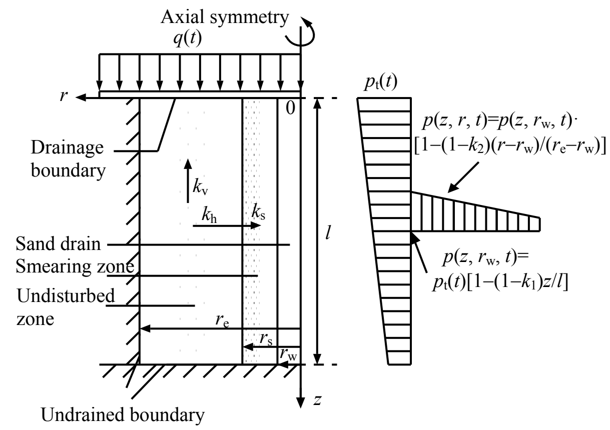


Fig. 1 Calculation diagram of sand-drained ground

2.2 Basic assumptions

In this paper, the following basic assumptions are made based on Barron's equal strain consolidation theory of sand-drained ground[21]:

(1) Soil is completely saturated. Soil particles and water are incompressible. The deformation and consolidation of soil are only caused by the discharge of pore water.

(2) The equal strain condition is satisfied, that is, the vertical deformation of each point on the same horizontal surface is the same within the range of the sand drain influence zone.

(3) The seepage and drainage law of soil follows Darcy's law.

(4) Considering the smear effect of sand-drained ground, the radial permeability coefficient of the soil in the smear zone is smaller than that of the soil in the undisturbed zone. Except for the radial permeability coefficient, other properties of the soil in the smear zone are the same as those in the undisturbed zone.

(5) During the consolidation and drainage process, the volumetric compressibility and permeability coefficient of soil remain constants.

(6) Considering the vacuum loading process[11−16] and the attenuation of the vacuum pressure along with the depth and radial direction of soil[19−20, 22−23], it is assumed that the vacuum pressure $p(z,r,t)$ in any position within sand drain influence zone can be expressed as follow:

$$p(z,r,t) = -p_0 \cdot g(t) \cdot h(z) \cdot o(r) \tag{1}$$

where p_0 is the final design vacuum pressure value, $p_t(t) = -p_0 g(t)$; $g(t)$ is a time function of any form considering the vacuum loading process, $0 \leqslant g(t) \leqslant 1$; $o(r) = [1-(1-k_2)(r-r_w)/(r_e - r_w)]$; $h(z) = [1-(1-k_1)z/l]$, k_1, k_2 are the negative pressure attenuation coefficients considering the depth and radial attenuation of vacuum pressure, respectively. The values of k_1 and k_2 are greater than 0 but not greater than 1. Generally speaking, the coefficients k_1 and k_2 will change during the consolidation process, but it is usually assumed that they are constants during the consolidation process to simplify the calculation[3, 19−20]. The determination method of the coefficients k_1 and k_2 is shown in the literature[22−23].

(7) At any depth, the amount of water flowing from the soil into the sand drain equals the increment of water flowing upward in the sand drain at that position.

2.3 Consolidation governing equation

According to assumption (6), the vacuum pressure u_{vac} at any point of soil in sand drain influence zone is

$$u_{vac} = -p_t(t)\left[1-(1-k_1)\frac{z}{l}\right]\left[1-(1-k_2)\frac{(r-r_w)}{(r_e-r_w)}\right] \tag{2}$$

According to Eq. (2), the excess pore water pressure head h at any point in the soil can be expressed as

$$h = \frac{1}{\gamma_w}(u + u_{vac}) = \frac{1}{\gamma_w}\left\{u + p_0 \cdot g(t) \cdot h(z)\left[1-(1-k_2)\frac{(r-r_w)}{(r_e-r_w)}\right]\right\} \tag{3}$$

where γ_w is the unit weight of water; u is excess pore water pressure.

Deriving r using Eq. (3), the radial hydraulic gradient i_r is obtained

$$i_r = \frac{1}{\gamma_w}\left[\frac{\partial u}{\partial r} - p_0 \cdot g(t) \cdot h(z)\frac{(1-k_2)}{(r_e - r_w)}\right] \tag{4}$$

According to Eq. (4), the radial seepage velocity v_r in smear zone and undisturbed zone is

$$v_r = \begin{cases} \dfrac{k_s}{\gamma_w}\left[\dfrac{\partial u}{\partial r} - p_0 \cdot g(t) \cdot h(z)\dfrac{(1-k_2)}{(r_e - r_w)}\right], & r_w \leqslant r \leqslant r_s \\ \dfrac{k_h}{\gamma_w}\left[\dfrac{\partial u}{\partial r} - p_0 \cdot g(t) \cdot h(z)\dfrac{(1-k_2)}{(r_e - r_w)}\right], & r_s \leqslant r \leqslant r_e \end{cases} \tag{5}$$

Based on the assumptions of equal strain theory[7−8], we can obtain:

$$\frac{\partial \varepsilon_v}{\partial t} = m_v\left(\frac{dq}{dt} - \frac{\partial \bar{u}}{\partial t}\right) \tag{6}$$

where ε_v is the volumetric strain (equal to vertical strain) at any point in sand drain influence zone; m_v is the volume compressibility coefficient; $\bar{u}(z,t)$ is the average excess pore water pressure at any depth in sand drain influence zone, and the expression of $\bar{u}(z,t)$ is as follows:

$$\bar{u}(z,t) = \frac{1}{\pi(r_e^2 - r_w^2)}\int_{r_w}^{r_e} 2\pi r u(r,z,t)\,dr \tag{7}$$

Based on the above assumptions, the consolidation equation of sand-drained ground considering radial and vertical seepage is[7−8]

$$-\frac{1}{r}\frac{\partial (r \cdot v_r)}{\partial r} - \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\partial \varepsilon_v}{\partial t} \tag{8}$$

The radial boundary conditions are

$$i_r\big|_{r=r_e} = 0 \tag{9}$$

$$u\big|_{r=r_w} = p(z, r_w, t) \tag{10}$$

Sorting out Eq. (8) and integrating r on both sides of the equation, we can use the boundary condition Eq. (9) to obtain

$$v_r = \left(\frac{\partial \varepsilon_v}{\partial t} + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2}\right)\frac{(r_e^2 - r^2)}{2r} \tag{11}$$

Substituting Eq. (11) into Eq. (5), we can obtain

$$\frac{\partial u}{\partial r} = \begin{cases} \dfrac{\gamma_w}{2k_s}\left(\dfrac{\partial \varepsilon_v}{\partial t} + \dfrac{k_v}{\gamma_w}\dfrac{\partial^2 \bar{u}}{\partial z^2}\right)\dfrac{(r_e^2 - r^2)}{r} + p_0 \cdot g(t) \cdot h(z)\dfrac{(1-k_2)}{(r_e - r_w)}, \\ r_w \leqslant r \leqslant r_s \\ \dfrac{\gamma_w}{2k_h}\left(\dfrac{\partial \varepsilon_v}{\partial t} + \dfrac{k_v}{\gamma_w}\dfrac{\partial^2 \bar{u}}{\partial z^2}\right)\dfrac{(r_e^2 - r^2)}{r} + p_0 \cdot g(t) \cdot h(z)\dfrac{(1-k_2)}{(r_e - r_w)}, \\ r_s \leqslant r \leqslant r_e \end{cases} \tag{12}$$

Integrating r on both sides of Eq. (12), then we can use the boundary condition Eq. (10) to obtain

$$u=\begin{cases}\dfrac{\gamma_w}{2k_s}\left(\dfrac{\partial\varepsilon_v}{\partial t}+\dfrac{k_v}{\gamma_w}\dfrac{\partial^2\overline{u}}{\partial z^2}\right)\left[r_e^2\ln\dfrac{r}{r_w}-\dfrac{(r^2-r_w^2)}{2}\right]-\\ p_0h(z)g(t)\left[1-(1-k_2)\dfrac{r-r_w}{r_e-r_w}\right],\ r_w\leqslant r\leqslant r_s\\ \dfrac{\gamma_w}{2k_h}\left(\dfrac{\partial\varepsilon_v}{\partial t}+\dfrac{k_v}{\gamma_w}\dfrac{\partial^2\overline{u}}{\partial z^2}\right)\left[\left(r_e^2\ln\dfrac{r}{r_s}-\dfrac{r^2-r_s^2}{2}\right)+\dfrac{k_h}{k_s}(r_e^2\cdot\\ \ln\dfrac{r_s}{r_w}-\dfrac{r_s^2-r_w^2}{2})\right]-p_0h(z)g(t)\left[1-(1-k_2)\dfrac{r-r_w}{r_e-r_w}\right],\\ r_s\leqslant r\leqslant r_e\end{cases}\tag{13}$$

Substituting Eq. (13) into Eq. (7), we can obtain

$$\overline{u}=\frac{\gamma_w r_e^2}{2k_h}R\left(\frac{\partial\varepsilon_v}{\partial t}+\frac{k_v}{\gamma_w}\frac{\partial^2\overline{u}}{\partial z^2}\right)-p_0G\cdot h(z)\cdot g(t)\tag{14}$$

where

$$R=\frac{n^2}{n^2-1}\left(\ln\frac{n}{s}+\frac{1}{\rho}\ln s-\frac{3}{4}\right)+\frac{s^2}{n^2-1}\left(1-\frac{1}{\rho}\right)\left(1-\frac{s^2}{4n^2}\right)+\frac{1}{\rho(n^2-1)}\left(1-\frac{1}{4n^2}\right)\tag{15}$$

$$G=\frac{n+2+k_2(2n+1)}{3(n+1)}\tag{16}$$

where $n=r_e/r_w$; $s=r_s/r_w$; $\rho=k_s/k_h$; G is the coefficient related to the radial attenuation of vacuum pressure.

Substituting Eq. (6) into Eq. (14), we can obtain

$$\overline{u}=B\frac{\mathrm{d}q}{\mathrm{d}t}-B\frac{\partial\overline{u}}{\partial t}+A\frac{\partial^2\overline{u}}{\partial z^2}-p_0G\cdot h(z)\cdot g(t)\tag{17}$$

where $A=\dfrac{k_v r_e^2R}{2k_h}$; $B=\dfrac{r_e^2R}{2C_h}$; $C_h=\dfrac{k_h}{m_v\gamma_w}$; C_h is the radial consolidation coefficient.

Equation (17) is the consolidation governing equation of sand-drained ground considering the vacuum loading process and time-dependent surcharge loading. The equation also considers the attenuation of the vacuum pressure along with the depth and radial direction, smear effect, and vertical seepage. Therefore, Eq. (17) is closer to the actual working conditions of soft soil ground reinforcement under vacuum-surcharge preloading, which has certain generality.

The boundary conditions of Eq. (17) are

$$\overline{u}\big|_{z=0}=-p_0G\cdot g(t)\tag{18}$$

$$\left.\frac{\partial\overline{u}}{\partial z}\right|_{z=l}=0\tag{19}$$

The initial condition is

$$\overline{u}\big|_{t=0}=q(0)\tag{20}$$

3 Solution of consolidation governing equation

According to the pattern of the consolidation governing equation and the boundary conditions, it is assumed that the solution can be expressed as

$$\overline{u}=-p_0G\cdot h(z)\cdot g(t)+\sum_{m=1}^{\infty}T_m(t)\sin\left(\frac{M}{l}z\right)\tag{21}$$

where $T_m(t)$ is time function in the solution; $M=(2m-1)\pi/2$, $m=1,\ 2,\ 3,\ \cdots$.

Equation (21) has satisfied the corresponding boundary conditions Eqs. (18)–(20). Substituting Eq. (21) into the initial condition Eq. (20), we can obtain

$$\sum_{m=1}^{\infty}T_m(0)\sin\left(\frac{M}{l}z\right)=p_0G\cdot h(z)g(0)+q_0\tag{22}$$

where q_0 is the initial surcharge loading, $q_0=q(0)$.

Then we can obtain

$$T_m(0)=\frac{2}{l}\int_0^l[p_0G\cdot h(z)g(0)+q_0]\sin\left(\frac{M}{l}z\right)\mathrm{d}z\tag{23}$$

Substituting Eq. (21) into the governing Eq. (17), we can obtain

$$\sum_{m=1}^{\infty}T_m(t)\sin\left(\frac{M}{l}z\right)+A\frac{M^2}{H^2}\sum_{m=1}^{\infty}T_m(t)\sin\left(\frac{M}{l}z\right)+B\sum_{m=1}^{\infty}T_m'(t)\sin\left(\frac{M}{l}z\right)=B\left[\frac{\mathrm{d}q}{\mathrm{d}t}+p_0G\cdot h(z)\cdot g'(t)\right]\tag{24}$$

To obtain the analytical solution of Eq. (24), the left term of Eq. (24) can be transformed by using the orthogonality of trigonometric series:

$$B\left[\frac{\mathrm{d}q}{\mathrm{d}t}+p_0G\cdot h(z)\cdot g'(t)\right]=B\sum_{m=1}^{\infty}Q_m(t)\sin\left(\frac{M}{l}z\right)\tag{25}$$

$$Q_m(t)=\frac{2}{l}\int_0^l\left[\frac{\mathrm{d}q}{\mathrm{d}t}+p_0G\cdot h(z)\cdot g'(t)\right]\sin\left(\frac{M}{l}z\right)\mathrm{d}z\tag{26}$$

Substituting Eq. (25) into Eq. (24), we can obtain

$$T_m'(t)+\beta_mT_m(t)=Q_m(t)\tag{27}$$

where $\beta_m=\dfrac{1}{B}+\dfrac{A}{B}\dfrac{M^2}{l^2}=\dfrac{2C_h}{r_e^2R}+C_v\dfrac{M^2}{l^2}$; $C_v=\dfrac{k_v}{m_v\gamma_w}$; C_v is the vertical consolidation coefficient.

Equation (27) is a first-order linear differential equation related to $T_m(t)$. According to the initial condition Eq. (22), the solution of Eq. (27) can be obtained

$$T_m(t)=\mathrm{e}^{-\beta_mt}\left[\int_0^tQ_m(\tau)\mathrm{e}^{\beta_m\tau}\,\mathrm{d}\tau+T_m(0)\right]\tag{28}$$

where τ is an intermediate variable related to t.

Therefore, a general consolidation analytical solution of sand-drained ground considering the vacuum loading process and time-dependent surcharge loading can be written as

$$\bar{u}(z,t) = -p_0 G \cdot h(z) g(t) + \sum_{m=1}^{\infty} \left[\int_0^t Q_m(\tau) e^{\beta_m \tau} d\tau + T_m(0) \right] e^{-\beta_m t} \sin\left(\frac{M}{l} z \right) \tag{29}$$

According to the expression of function $h(z)$ in Eq. (1), Eq. (29) can be further expressed as

$$\bar{u}(z,t) = -p_0 G \cdot \left[1-(1-k_1)\frac{z}{l} \right] g(t) + \sum_{m=1}^{\infty} 2 \left\{ \int_0^t \left[\frac{1}{M}\frac{dq}{d\tau} + D_m \cdot g'(\tau) \right] e^{\beta_m \tau} d\tau + \frac{q_0}{M} + D_m \cdot g(0) \right\} \cdot e^{-\beta_m t} \sin\left(\frac{M}{l} z \right) \tag{30}$$

where $D_m = \dfrac{p_0 G[M-(1-k_1)\sin M]}{M^2}$.

The average consolidation degree U_p defined by pore water pressure can be expressed as

$$U_p = \frac{\int_0^l \Delta\bar{\sigma}'(z,t)dz}{\int_0^l \Delta\bar{\sigma}'(z,\infty)dz} \tag{31}$$

where $\Delta\bar{\sigma}'(z,t)$ is the average effective stress increment of ground at depth z and time t; $\Delta\bar{\sigma}'(z,\infty)$ is final average effective stress increment of ground at depth z.

According to the principle of effective stress, we can obtain

$$\Delta\bar{\sigma}'(z,t) = q(t) - \bar{u}(z,t) \tag{32}$$

$$\Delta\bar{\sigma}'(z,\infty) = q_u + p_0 G \cdot h(z) \tag{33}$$

where q_u is the final surcharge loading value.

Thereupon, the average pore water pressure consolidation degree U_p of sand-drain ground can be expressed as

$$U_p = \frac{2\int_0^l [q(t) - \bar{u}(z,t)] dz}{2q_u l + p_0 G(1+k_1) l} \tag{34}$$

The average consolidation degree U_s defined by the settlement can be expressed as

$$U_s = \frac{S_t}{S_\infty} \tag{35}$$

where S_t and S_∞ are the settlement at time t and final settlement of sand-drained ground, respectively.

The expression of final settlement S_∞ can be written as

$$S_\infty = m_v \int_0^l \Delta\bar{\sigma}'(z,\infty)dz = m_v l \left[q_u + p_0 G \frac{(1+k_1)}{2} \right] \tag{36}$$

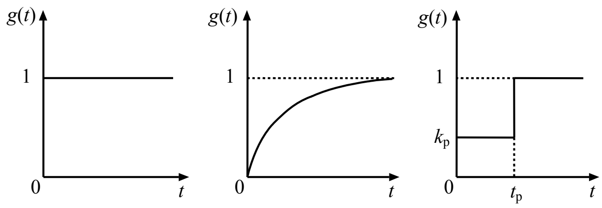
Based on the assumptions of small deformation and equal strain, the average pore water pressure consolidation degree U_p is equal to the average settlement consolidation degree U_s of sand-drained ground[6–7]. Then the settlement S_t of sand-drained ground at time t can be written as

$$S_t = U_s S_\infty = U_p S_\infty \tag{37}$$

Equations (30) and (34) are the consolidation analytical solutions of sand-drain ground considering the vacuum loading process and time-dependent surcharge loading with arbitrary time function. After time function $g(t)$ reflecting vacuum loading process and surcharge loading $q(t)$ are determined, the expressions of the average excess pore water pressure at any depth and the average consolidation degree of sand-drained ground are obtained by substituting the above equations into Eqs. (30) and (34).

4 Analytical solutions under several loading patterns

The variation patterns of several time functions $g(t)$ considering the vacuum loading process are shown in Fig. 2. Figure 2(a) is the case of assuming the vacuum pressure is applied instantaneously. Figure 2(b) is the case of simulating vacuum pressure exponential function loading. Figure 2(c) is the case of considering multi-step vacuum pressure loading.

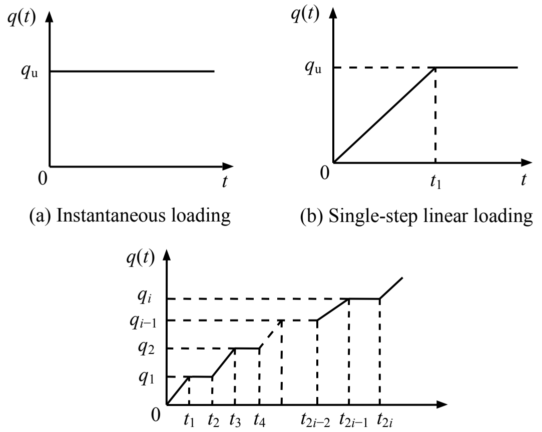


(a) Instantaneous loading (b) Exponential function loading (c) multi-step loading

Fig. 2 Several variations of the function $g(t)$

Figure 3 shows several patterns of time-dependent surcharge loading. Figure 3(a) is the case of instantaneous loading $q(t)$, which does not change with time. Figure 3(b) is the case of single-step linear loading $q(t)$, which increases linearly with time t first and then remains constant. Figure 3(c) is the case of multi-step linear loading $q(t)$, which increases linearly step by step (final surcharge loading is q_u).

In the following, the analytical solution of the consolidation governing equation is mainly solved according to the various patterns of time function $g(t)$ and surcharge loading $q(t)$ in Figs. 2 and 3. For the convenience of expressing the analytical solution, we assume $T_h = C_h t/(4r_e^2)$ that can be deduced as $t = 4T_h r_e^2 / C_h$. At the same time, we introduce a factor λ_m, which is a dimensionless parameter related to β_m and can be expressed as $\lambda_m = 4\beta_m r_e^2 / C_h$.



(c) Multi-step linear loading

Fig. 3 Several surcharge loading patterns

4.1 Case 1

When the strength of ground is high, the vacuum loading process can be ignored. If the rapid surcharge loading is carried out after the vacuum preloading, in other words, the vacuum loading and surcharge loading patterns are shown and adopted in Figs. 2(a) and 3(a), $g(T_{\rm h})=1$ and $q(T_{\rm h})=q_{\rm u}$ are satisfied. Therefore, we can obtain $g'(T_{\rm h})=0$.

Substituting $g(T_{\rm h})=1$, $q(T_{\rm h})=q_{\rm u}$, and $g'(T_{\rm h})=0$ into Eqs. (30) and (34), the average excess pore water pressure at any depth and the average consolidation degree of sand-drained ground can be expressed as

$$\bar{u}(z,T_{\rm h})=-p_0G\left[1-(1-k_1)\frac{z}{l}\right]+\sum_{m=1}^{\infty}2\left\{\frac{q_{\rm u}}{M}+D_m\right\}{\rm e}^{-\lambda_m T_{\rm h}}\sin\left(\frac{M}{l}z\right) \tag{38}$$

$$U_{\rm s}=U_{\rm p}=1-\sum_{m=1}^{\infty}\left\{D_m+\frac{q_{\rm u}}{M}\right\}\frac{4{\rm e}^{-\lambda_m T_{\rm h}}}{M[2q_{\rm u}+p_0G(1+k_1)]} \tag{39}$$

4.2 Case 2

When the area of soft soil ground is large, it is necessary to consider the loading process of vacuum pressure[22, 24], and when the linear surcharge preloading method is used to strengthen the consolidation and drainage of the ground after vacuum preloading, the exponential function pattern shown in Fig. 2(b) is used to simulate the vacuum loading and the pattern shown in Fig. 3(b) is used to simulate the single-step linear surcharge loading process. According to the research of Peng et al.[11], Tian et al.[12], and Liu et al.[24], time function $g(t)$ can be expressed as

$$g(t)=1-{\rm e}^{-\alpha t} \tag{40}$$

where α is the vacuum loading coefficient, which can be obtained by fitting the loading curve of actual vacuum negative pressure. The dimensionless parameter η related to α can be expressed as $\eta=4\alpha r_{\rm e}^2/C_{\rm h}$.

When the surcharge loading is single-step linear, we can obtain

$$q(t)=\begin{cases}q_{\rm u}\cdot t/t_1, 0\leqslant t\leqslant t_1\\ q_{\rm u},\ t>t_1\end{cases} \tag{41}$$

where t_1 is the linear loading time. The time factor $T_{\rm h1}$ of t_1 can be expressed as $T_{\rm h1}=C_{\rm h}t_1/4r_{\rm e}^2$.

Substituting Eqs. (40)-(41), and $g'(T_{\rm h})=\eta{\rm e}^{-\eta T_{\rm h}}$ into Eqs. (30) and (34), the average excess pore water pressure at any depth and average consolidation degree of sand-drained ground can be expressed as

When $0\leqslant T_{\rm h}\leqslant T_{\rm h1}$ is satisfied, we can obtain

$$\bar{u}(z,T_{\rm h})=-p_0G\left[1-(1-k_1)\frac{z}{l}\right](1-{\rm e}^{-\eta T_{\rm h}})+\sum_{m=1}^{\infty}2\left[D_m\eta\frac{({\rm e}^{-\eta T_{\rm h}}-{\rm e}^{-\lambda_m T_{\rm h}})}{(\lambda_m-\eta)}+\frac{q_{\rm u}(1-{\rm e}^{-\lambda_m T_{\rm h}})}{M\lambda_m T_{\rm h1}}\right]\sin\left(\frac{M}{l}z\right) \tag{42}$$

$$U_{\rm s}=U_{\rm p}=\frac{2(q_{\rm u}\cdot T_{\rm h}/T_{\rm h1})+p_0G(1+k_1)(1-{\rm e}^{-\eta T_{\rm h}})}{2q_{\rm u}+p_0G(1+k_1)}-\sum_{m=1}^{\infty}\left[D_m\eta\frac{({\rm e}^{-\eta T_{\rm h}}-{\rm e}^{-\lambda_m T_{\rm h}})}{(\lambda_m-\eta)}+\frac{q_{\rm u}(1-{\rm e}^{-\lambda_m T_{\rm h}})}{M\lambda_m T_{\rm h1}}\right]\cdot\frac{4}{M[2q_{\rm u}+p_0G(1+k_1)]} \tag{43}$$

When $T_{\rm h}\geqslant T_{\rm h1}$ is satisfied, we can obtain

$$\bar{u}(z,T_{\rm h})=-p_0G\left[1-(1-k_1)\frac{z}{l}\right](1-{\rm e}^{-\eta T_{\rm h}})+\sum_{m=1}^{\infty}2\left\{D_m\eta\frac{({\rm e}^{-\eta T_{\rm h}}-{\rm e}^{-\lambda_m T_{\rm h}})}{(\lambda_m-\eta)}+\frac{q_{\rm u}[{\rm e}^{\lambda_m(T_{\rm h1}-T_{\rm h})}-{\rm e}^{-\lambda_m T_{\rm h}}]}{M\lambda_m T_{\rm h1}}\right\}\sin\left(\frac{M}{l}z\right) \tag{44}$$

$$U_{\rm s}=U_{\rm p}=\frac{2q_{\rm u}+p_0G(1+k_1)(1-{\rm e}^{-\eta T_{\rm h}})}{2q_{\rm u}+p_0G(1+k_1)}-\sum_{m=1}^{\infty}\left\{D_m\eta\frac{({\rm e}^{-\eta T_{\rm h}}-{\rm e}^{-\lambda_m T_{\rm h}})}{(\lambda_m-\eta)}+\frac{q_{\rm u}[{\rm e}^{\lambda_m(T_{\rm h1}-T_{\rm h})}-{\rm e}^{-\lambda_m T_{\rm h}}]}{M\lambda_m T_{\rm h1}}\right\}\cdot\frac{4}{M[2q_{\rm u}+p_0G(1+k_1)]} \tag{45}$$

It should be noted that the above solution assumes that η and λ_m are not equal. When η and λ_m are equal, the average excess pore water pressure at any depth and the average consolidation degree of sand-drained ground can also be obtained by integrating Eqs. (30) and (34). Due to the length limitation of the paper, not all of them are listed here.

4.3 Case 3

When the strength of ground is relatively small, multi-step vacuum preloading is adopted[13−16], and when multi-step vacuum loading and multi-step linear surcharge loading are used to further accelerate the consolidation and drainage of the ground. As shown in Fig. 2(c), the two-step loading pattern can be used to simulate the multi-step vacuum loading, and the pattern, as shown in Fig. 3(c), can be used to simulate the multi-step linear surcharge loading process. As for the two-step vacuum loading process, the time function $g(t)$ related to vacuum loading can be expressed as

$$g(t)=\begin{cases}k_{\mathrm{p}},\ 0\leqslant t\leqslant t_{\mathrm{p}}\\ 1,\quad t>t_{\mathrm{p}}\end{cases} \tag{46}$$

where k_{p} is the ratio of the first stage vacuum pressure value to the final vacuum pressure value, $0<k_{\mathrm{p}}<1$; t_{p} is the loading time of the first-step vacuum pressure. The time factor T_{hp} of t_{p} can be expressed as $T_{\mathrm{hp}}=C_{\mathrm{h}}t_{\mathrm{p}}/4r_{\mathrm{e}}^2$.

Then we can obtain

$$g'(T_{\mathrm{h}})=\begin{cases}0,\ t\neq t_{\mathrm{p}}\\ \lim\limits_{\Delta t\to 0}\dfrac{1-k_{\mathrm{p}}}{\Delta t},t=t_{\mathrm{p}}\end{cases} \tag{47}$$

When the surcharge loading $q(t)$ is multi-step linear, we can obtain

$$q(t)=\begin{cases}q_{i-1}+a_i(t-t_{2i-2}),\ t_{2i-2}\leqslant t\leqslant t_{2i-1}\\ q_i,\ t_{2i-1}\leqslant t\leqslant t_{2i}\end{cases} \tag{48}$$

where i is the number of loading steps, $i=1,2,\cdots,i$; q_i is a load of level i, q_0=0; $a_i=(q_i-q_{i-1})/(t_{2i-1}-t_{2i-2})$, a_i is the i^{th} step loading rate. The dimensionless parameter δ_i related to the loading rate a_i can be expressed as $\delta_i=4a_ir_{\mathrm{e}}^2/C_{\mathrm{h}}$. The time factor $T_{\mathrm{h}2i-1}$ of t_{2i-1} can be expressed as $T_{\mathrm{h}2i-1}=C_{\mathrm{h}}t_{2i-1}/4r_{\mathrm{e}}^2$. It is noted that the following analysis assumes that $T_{\mathrm{hp}}<T_{\mathrm{h1}}$ is satisfied.

Substituting Eqs. (46)−(48) into Eqs. (30) and (34), the average excess pore water pressure at any depth and the average consolidation degree of sand-drained ground can be expressed as

When $0\leqslant T_{\mathrm{h}}\leqslant T_{\mathrm{hp}}$ is satisfied, we can obtain

$$\bar{u}(z,T_{\mathrm{h}})=-p_0G\left[1-(1-k_1)\frac{z}{l}\right]k_{\mathrm{p}}+\sum_{m=1}^{\infty}2\left[D_mk_{\mathrm{p}}\mathrm{e}^{-\lambda_mT_{\mathrm{h}}}+\frac{\delta_1(1-\mathrm{e}^{-\lambda_mT_{\mathrm{h}}})}{M\lambda_m}\right]\sin\left(\frac{M}{l}z\right) \tag{49}$$

$$U_{\mathrm{s}}=U_{\mathrm{p}}=\frac{2\delta_1T_{\mathrm{h}}+p_0G(1+k_1)k_{\mathrm{p}}}{2q_{\mathrm{u}}+p_0G(1+k_1)}-\sum_{m=1}^{\infty}\left[D_mk_{\mathrm{p}}\mathrm{e}^{-\lambda_mT_{\mathrm{h}}}+\frac{\delta_1(1-\mathrm{e}^{-\lambda_mT_{\mathrm{h}}})}{M\lambda_m}\right]\frac{4}{M[2q_{\mathrm{u}}+p_0G(1+k_1)]} \tag{50}$$

When $T_{\mathrm{hp}}\leqslant T_{\mathrm{h}}\leqslant T_{\mathrm{h1}}$ is satisfied, we can obtain

$$\begin{aligned}\bar{u}(z,T_{\mathrm{h}})&=-p_0G\left[1-(1-k_1)\frac{z}{l}\right]+\\&\sum_{m=1}^{\infty}2\left[\frac{D_m(1-k_{\mathrm{p}})\mathrm{e}^{\lambda_mT_{\mathrm{hp}}}}{\lambda_m}\lim_{\Delta T_{\mathrm{h}}\to 0}\frac{(\mathrm{e}^{\lambda_m\Delta T_{\mathrm{h}}}-1)}{\Delta T_{\mathrm{h}}}+D_mk_{\mathrm{p}}+\frac{\delta_1(\mathrm{e}^{\lambda_mT_{\mathrm{h}}}-1)}{M\lambda_m}\right]\mathrm{e}^{-\lambda_mT_{\mathrm{h}}}\sin\left(\frac{M}{l}z\right)=\\&-p_0G\left[1-(1-k_1)\frac{z}{l}\right]+\\&\sum_{m=1}^{\infty}2\left[(1-k_{\mathrm{p}})D_m\mathrm{e}^{\lambda_m(T_{\mathrm{hp}}-T_{\mathrm{h}})}+D_mk_{\mathrm{p}}\mathrm{e}^{-\lambda_mT_{\mathrm{h}}}+\frac{\delta_1(1-\mathrm{e}^{-\lambda_mT_{\mathrm{h}}})}{M\lambda_m}\right]\sin\left(\frac{M}{l}z\right)\end{aligned} \tag{51}$$

$$U_{\mathrm{s}}=U_{\mathrm{p}}=\frac{2\delta_1T_{\mathrm{h}}+p_0G(1+k_1)k_{\mathrm{p}}}{2q_{\mathrm{u}}+p_0G(1+k_1)}-\sum_{m=1}^{\infty}\left[(1-k_{\mathrm{p}})D_m\mathrm{e}^{\lambda_m(T_{\mathrm{hp}}-T_{\mathrm{h}})}+D_mk_{\mathrm{p}}\mathrm{e}^{-\lambda_mT_{\mathrm{h}}}+\frac{\delta_1(1-\mathrm{e}^{-\lambda_mT_{\mathrm{h}}})}{M\lambda_m}\right]\cdot\frac{4}{M[2q_{\mathrm{u}}+p_0G(1+k_1)]} \tag{52}$$

When $T_{\mathrm{h}2i-2}\leqslant T_{\mathrm{h}}\leqslant T_{\mathrm{h}2i-1}$ ($i=2,3,\cdots i$) is satisfied, we can obtain

$$\begin{aligned}\bar{u}(z,T_{\mathrm{h}})&=-p_0G\left[1-(1-k_1)\frac{z}{l}\right]+\\&\sum_{m=1}^{\infty}2\left\{(1-k_{\mathrm{p}})D_m\mathrm{e}^{\lambda_mT_{\mathrm{hp}}}+D_mk_{\mathrm{p}}+\left[\sum_{k=1}^{i-1}\left(\frac{\delta_k(\mathrm{e}^{\lambda_mT_{\mathrm{h}2k-1}}-\mathrm{e}^{\lambda_mT_{\mathrm{h}2k-2}})}{M\lambda_m}\right)\right]+\frac{\delta_i[\mathrm{e}^{\lambda_m(T_{\mathrm{h}}-T_{\mathrm{h}2i-2})}-1]}{M\lambda_m}\right\}\mathrm{e}^{-\lambda_mT_{\mathrm{h}}}\sin\left(\frac{M}{l}z\right)\end{aligned} \tag{53}$$

$$\begin{aligned}U_{\mathrm{s}}=U_{\mathrm{p}}&=\frac{2[q_{i-1}+\delta_i(T_{\mathrm{h}}-T_{\mathrm{h}2i-2})]+p_0G(1+k_1)(1-\mathrm{e}^{-\eta T_{\mathrm{h}}})}{2q_{\mathrm{u}}+p_0G(1+k_1)}-\\&\sum_{m=1}^{\infty}\left\{(1-k_{\mathrm{p}})D_m\mathrm{e}^{\lambda_mT_{\mathrm{hp}}}+D_mk_{\mathrm{p}}+\left[\sum_{k=1}^{i-1}\left(\frac{\delta_k(\mathrm{e}^{\lambda_mT_{\mathrm{h}2k-1}}-\mathrm{e}^{\lambda_mT_{\mathrm{h}2k-2}})}{M\lambda_m}\right)\right]+\frac{\delta_i[\mathrm{e}^{\lambda_m(T_{\mathrm{h}}-T_{\mathrm{h}2i-2})}-1]}{M\lambda_m}\right\}\frac{4\mathrm{e}^{-\lambda_mT_{\mathrm{h}}}}{M[2q_{\mathrm{u}}+p_0G(1+k_1)]}\end{aligned} \tag{54}$$

When $T_{\mathrm{h}2i-1}\leqslant T_{\mathrm{h}}\leqslant T_{\mathrm{h}2i}$ ($i=1,2,\cdots i$) is satisfied, we can obtain

$$\begin{aligned}\bar{u}(z,T_{\mathrm{h}})&=-p_0G\left[1-(1-k_1)\frac{z}{l}\right](1-\mathrm{e}^{-\eta T_{\mathrm{h}}})+\\&\sum_{m=1}^{\infty}2\left\{(1-k_{\mathrm{p}})D_m\mathrm{e}^{\lambda_mT_{\mathrm{hp}}}+D_mk_{\mathrm{p}}+\left[\sum_{k=1}^{i}\left(\frac{\delta_k(\mathrm{e}^{\lambda_mT_{\mathrm{h}2k-1}}-\mathrm{e}^{\lambda_mT_{\mathrm{h}2k-2}})}{M\lambda_m}\right)\right]\right\}\cdot\\&\mathrm{e}^{-\lambda_mT_{\mathrm{h}}}\sin\left(\frac{M}{l}z\right)\end{aligned} \tag{55}$$

$$U_{\mathrm{s}}=U_{\mathrm{p}}=\frac{2q_i+p_0G(1+k_1)(1-\mathrm{e}^{-\eta T_{\mathrm{h}}})}{2q_{\mathrm{u}}+p_0G(1+k_1)}-\sum_{m=1}^{\infty}\left\{(1-k_{\mathrm{p}})D_m\,\mathrm{e}^{\lambda_m T_{\mathrm{hp}}}+D_mk_{\mathrm{p}}+\left[\sum_{k=1}^{i}\left(\frac{\delta_k(\mathrm{e}^{\lambda_m T_{\mathrm{h}2k-1}}-\mathrm{e}^{\lambda_m T_{\mathrm{h}2k-2}})}{M\lambda_m}\right)\right]\right\}\cdot\frac{4\mathrm{e}^{-\lambda_m T_{\mathrm{h}}}}{M[2q_{\mathrm{u}}+p_0G(1+k_1)]} \quad (56)$$

As for other combinations of time function $g(t)$ and surcharge loading $q(t)$ in Figs. 2 and 3, the expressions of time function $g(t)$ and surcharge loading $q(t)$ can also be substituted into Eqs. (30) and (34) to obtain the expressions of the average excess pore water pressure at any depth and average consolidation degree of sand-drained ground, which are not listed here.

5 Verification of solution

5.1 Comparison with existing analytical solutions

According to the assumptions made in this paper, the main difference between the present consolidation solution and the previous consolidation solution of the sand-drained ground is that the solution proposed in this paper considers the vacuum loading process and the attenuation of vacuum pressure along with the depth and radial direction. To verify the correctness of the present solution, the solution obtained when the vacuum pressure is applied instantaneously can be compared with the existing analytical solution.

When the vacuum pressure attenuation is not considered ($k_1=1$, $G=1$, $D_m=p_0/M$), the average excess pore water pressure at any depth of sand-drained ground $\bar{u}$ can be obtained combined with Eq. (30)

$$\bar{u}(z,t)=-p_0+\sum_{m=1}^{\infty}2\left\{\int_0^t\frac{1}{M}\frac{\mathrm{d}q}{\mathrm{d}\tau}\mathrm{e}^{\beta_m\tau}\,\mathrm{d}\tau+\frac{q_0}{M}+\frac{p_0}{M}\right\}\mathrm{e}^{-\beta_m t}\sin\left(\frac{M}{l}z\right) \quad (57)$$

Comparing Eq. (57) with the expression obtained by Guo et al.[7] in solving the average excess pore water pressure of soil at any depth of sand-drained ground under vacuum-surcharge preloading, it can be seen that the result has a good agreement in the form of that derived by Guo et al.[7] and physical meaning of the parameters are completely the same.

Furthermore, when the surcharge loading $q(t)$ is applied instantaneously, the expression of the average excess pore water pressure at any depth and the consolidation degree of sand-drained ground are expressed as

$$\bar{u}(z,t)=-p_0+\sum_{m=1}^{\infty}\left[\frac{2p_0}{M}+\frac{2q_{\mathrm{u}}}{M}\right]\mathrm{e}^{-\beta_m t}\sin\left(\frac{M}{l}z\right) \quad (58)$$

$$U_{\mathrm{s}}=U_{\mathrm{p}}=1-\sum_{m=1}^{\infty}\frac{2}{M^2}\mathrm{e}^{-\beta_m t} \quad (59)$$

It can be seen that Eqs. (58)−(59) are exactly the same as the consolidation analytical solution of sand-drained ground considering radial and vertical seepage and vacuum-surcharge preloading derived by Rujikiat-kamjorn et al.[6].

By comparing the solution proposed in this paper with the existing analytical solution without considering the vacuum loading process and the attenuation of vacuum pressure, it can be seen that the degenerated analytical solution in this paper is completely consistent with the existing analytical solution, which verifies the correctness of the present solution to a certain extent.

5.2 Comparison with finite difference solution

To further verify the correctness of the present solution, the finite difference form of Eq. (17) is developed. Then the finite difference solution is obtained using the corresponding boundary conditions and initial conditions and compared with the present consolidation analytical solution. The finite difference method can be referred to as Geng et al.[25] and Cao et al.[26]. Taking the analytical solution of case 2 as an example, the calculation parameters of sand-drained ground are shown in Table 1.

Table 1 Calculation parameters of sand-drained ground

H/m	r_{w}/m	s	n	ρ	k_{h}/(m • s^{-1})	k_{v}/(m • s^{-1})	p_0/kPa	q_{u}/kPa	m_{v}/MPa^{-1}	η	k_1	k_2	T_{h1}
10	0.07	4	10	0.2	5×10^{-9}	2×10^{-9}	50	50	0.5	2	0.5	0.5	0.1

Through the comparison between the present consolidation analytical solution and the finite difference solution in Fig. 4, the change law of the average consolidation degree with time under the two solutions is completely consistent, which further verifies the correctness of the analytical solution proposed in this paper.

6 Consolidation behavior analysis

To study the consolidation behaviors of sand-drained ground considering the change of external load with time, the solution in case 2 is taken as an example (the vacuum loading process is considered in the form of an exponential function, and the surcharge is a single-step linear loading) to analyze the consolidation behavior of sand-drained ground. According to Eqs. (42)−(45), it can be seen that the consolidation rate of sand-drained ground is affected by factors such as vacuum loading factor η, negative pressure attenuation coefficients k_1

and k_2, vacuum pressure p_0, final surcharge loading q_u, and loading time T_{h1}. Therefore, this paper mainly analyses the above factors. The calculation parameters of sand-drained ground are shown in Table 1.

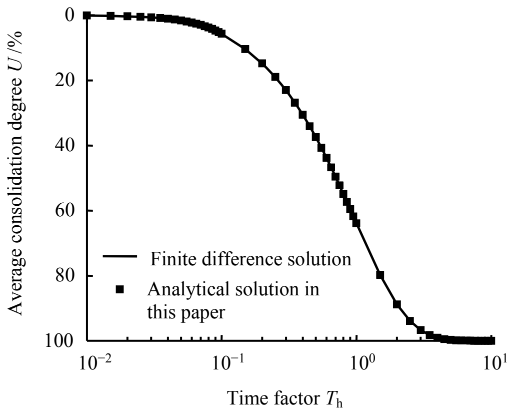


Fig. 4 Comparison of the analytical solution and finite difference solution

Figure 5 shows the average degree of consolidation curve of sand-drained ground under different vacuum loading factors η. It can be seen from Fig. 5 that η has a great influence on the consolidation rate of sand-drained ground, and the consolidation rate increases with the increase of η. At the same time, it can be seen that when $\eta \geqslant 10$ is satisfied, the increase of η has almost no effect on the consolidation rate. When $\eta = 100$ is satisfied, the average consolidation degree curve almost completely coincides with that without considering the vacuum loading process. In other words, when the vacuum loading factor η increases to a certain extent, the effect of the vacuum loading process on the consolidation rate of sand-drained ground can be ignored.

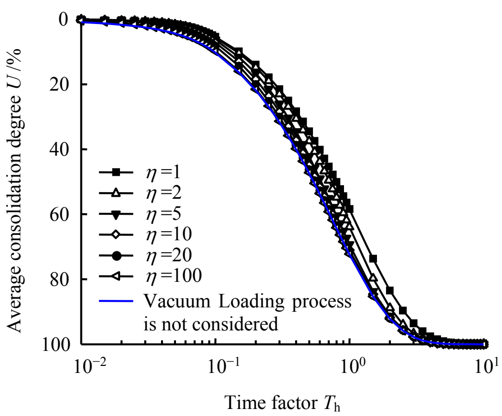


Fig. 5 Influence of vacuum loading factor η on average consolidation degree

Figure 6 reflects the effect of the vacuum loading factor η on the average excess pore water pressure. It can be seen from the figure that the larger the value of η is, the smaller the average excess pore water pressure under the same time factor is, and the average excess pore water pressure without considering the vacuum loading process is the smallest, consistent with the change law of the average consolidation degree curve in Fig. 5. In addition, it can be seen from Fig. 6 that when η is large enough (for example $\eta = 20$), the average excess pore water pressure considering the vacuum loading process is getting closer to that without considering the vacuum loading process with the consolidation process.

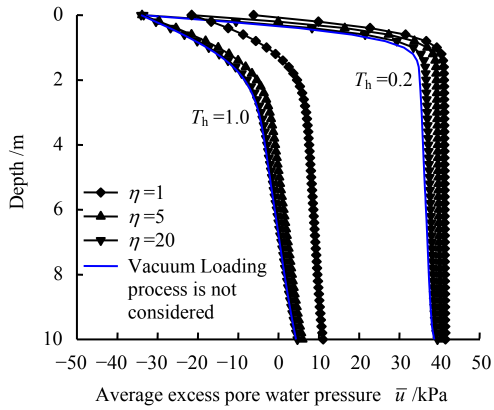


Fig. 6 Influence of vacuum loading factor η on average excess pore water pressure

Figure 7 shows the influence curve of negative pressure attenuation coefficients k_1 and k_2 on the average consolidation degree of sand-drained ground. It can be seen from Fig. 7 that the negative pressure attenuation coefficients k_1 and k_2 have the same influence on the consolidation rate of sand-drained ground. With the increase of k_1 and k_2, the consolidation rate of sand-drained ground decreases. However, k_1 and k_2 have little influence on the consolidation rate of sand-drained ground. The average consolidation degree curves under different k_1 and k_2 almost coincide.

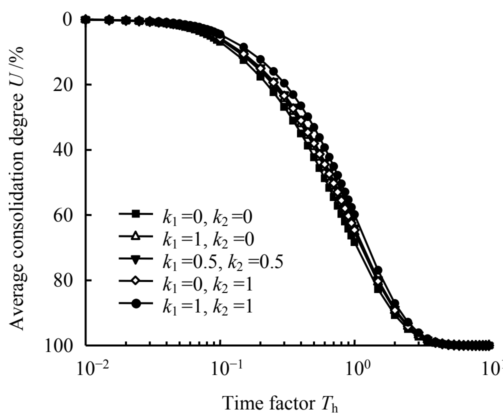


Fig. 7 Influence of vacuum pressure attenuation coefficient k_1 and k_2 on average consolidation degree

Figure 8 shows the average consolidation degree curve of sand-drained ground under different vacuum pressure p_0. It can be seen from Fig. 8 that the consolidation rate of sand-drained ground decreases with the increase of vacuum pressure p_0. When p_0 is small, the increase of p_0 greatly influences the consolidation rate of sand-drained ground. When p_0 is large, the increase of p_0 has little effect on the consolidation rate of sand-drained ground.

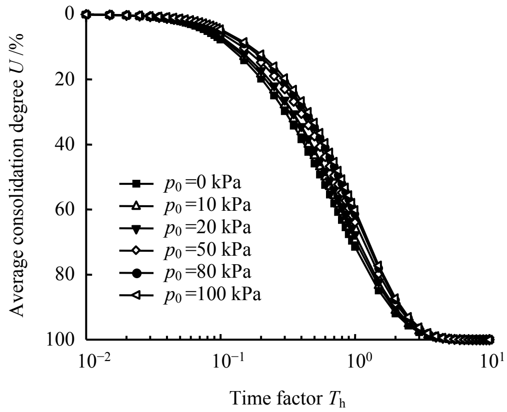


Fig. 8 Influence of vacuum pressure p_0 on average consolidation degree

Figure 9 shows the average consolidation degree curve of sand-drained ground under different final surcharge loading q_u. It can be seen from Fig. 9 that the consolidation rate of sand-drained ground increases with the increase of the final surcharge loading q_u, which is contrary to the change law of the consolidation rate with vacuum pressure p_0. At the same time, it can be seen that with the increase of q_u, the influence of increasing q_u on the consolidation rate of sand-drained ground gradually decreases.

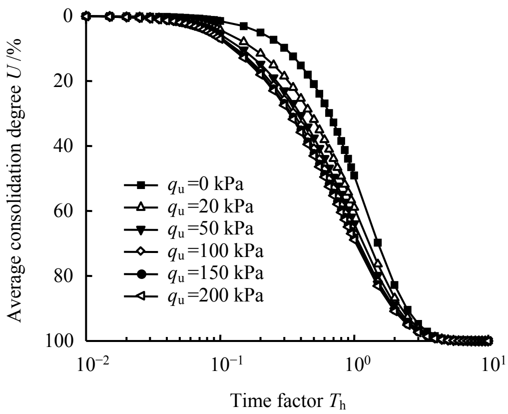


Fig. 9 Influence of final surcharge loading q_u on average consolidation degree

Figure 10 shows the average consolidation degree curve of sand-drained ground under different loading time T_{h1}. It can be seen from Fig. 10 that under the same final surcharge, the larger the loading time T_{h1} is, the smaller the consolidation rate of sand-drained ground is. In other words, the consolidation rate of sand-drained ground decreases with the increase of the loading time T_{h1}. When T_{h1}=0 is satisfied (the surcharge loading is applied instantaneously), the consolidation rate of sand-drained ground is the largest.

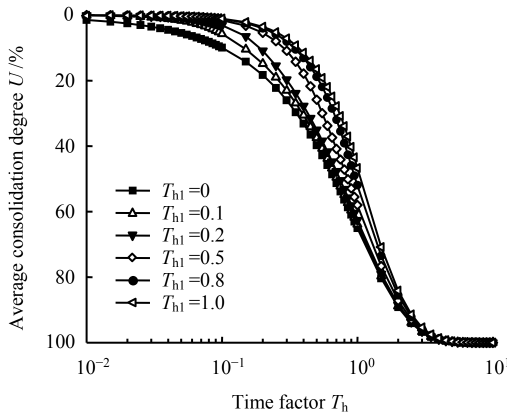


Fig. 10 Influence of loading time T_{h1} on average consolidation degree

7 Conclusions

(1) The consolidation rate of sand-drained ground is greatly affected by the vacuum loading factor η. With the increase of vacuum loading factor η, the consolidation rate of sand-drained ground increases. However, when η increases to a certain extent, the effect of the vacuum loading process on the consolidation rate of sand-drained ground can be ignored.

(2) The negative pressure attenuation coefficients k_1 and k_2 have the same influence on the consolidation rate of sand-drained ground. With the increase of k_1 and k_2, the consolidation rate of sand-drained ground decreases. However, the influence of k_1 and k_2 on the consolidation rate of sand-drained ground is small.

(3) The consolidation rate of sand-drained ground decreases with the increase of vacuum pressure p_0 and increases with the increase of final surcharge loading q_u. However, when p_0 or q_u increases to a certain value, the influence of further increasing p_0 or q_u on the consolidation rate of sand-drained ground is very small.

(4) The consolidation rate of sand-drained ground decreases with the increase of the loading time T_{h1}. When the surcharge loading is applied instantaneously, the consolidation rate of sand-drained ground reaches the largest value.

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