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Statistical damage constitutive model of high temperature rock based on Weibull distribution and its verification

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Abstract: The decrease of rock stability is caused by the possible deterioration of rock mechanical properties under high temperature environment. Therefore, the study of the constitutive behavior of rocks under high temperature is of great significance. Based on the recent researches of statistical rock damage constitutive model, the statistical damage constitutive model of rock after high temperature is established by adopting M-C criterion with the thermal damage variable and Weibull distribution function and the parameter expression is determined. The model is compared with the theoretical curve to verify its rationality. Finally, the model is verified by the uniaxial compression test results of sandstone under different temperature conditions (e.g., 25 ℃, 80 ℃, 100 ℃, 150 ℃). The results show that the theoretical curve of statistical damage constitutive model of the high-temperature rock established in this paper has the same trend as the theoretical curve in the literature, proving that the established constitutive model is reasonable. The theoretical curve of the model is in good agreement with the curve obtained in the experiments, implying that it can represent the stress-strain characteristics of sandstone failure under the condition of uniaxial test. This model does not contain unconventional mechanical parameters, and the physical meaning is clear. The research results can provide theoretical support for related calculations and numerical simulations of rock mechanics after high-temperature treatment.

Keywords: Weibull distribution; high temperature rock; Mohr-Coulomb criterion; thermal damage; constitutive model

1 Introduction

With the continuous development of high-temperature rock masses and underground engineering construction such as geothermal energy development and deep mineral resource mining, there are more and more researches related to the physical and mechanical properties of rocks under high-temperature[1−6]. The mechanical properties of rocks are usually degraded at high temperatures, which will affect the bearing capacity and stability of rock mass. Therefore, a reasonable constitutive model is the key to understand and judge the mechanical properties of rocks under high temperatures.

Many scholars have studied the influence of high temperatures on the physical and mechanical properties of rocks from different aspects through experiments and constitutive models. In terms of experiments, Hu et al. [7] carried out physical and mechanical tests of granite treated at different temperatures; Zhao et al. [8] used X-ray diffraction and other test methods to study the variation of mechanical properties of sandstone at different temperatures (250 ℃−900 ℃), showing that the number of pores will increase, causing the decrease of strength as the temperature increased. Fang et al.[9] defined the thermal damage variables and studied the thermal damage characteristics of granite under tensile failure from the perspective of damage mechanics, showing that as the temperature increases, the brittle-plastic transformation will occur in the granite samples. All these experiments confirmed that high temperatures caused thermal damage to rocks. In terms of the constitutive model, the statistical damage constitutive model can reasonably describe the defects evolution process in the rocks, and can better reflect the mechanical mechanism of rock damage under high-temperature. Among them, Cao et al.[10] considered that the key of the damage constitutive model was the Weibull parameter. The rock damage statistical constitutive model was established based on the Drucker-Prager (D-P) criterion. Chaki et al.[11] studied the failure mechanism of rock thermal damage through experiments and established a thermal damage constitutive model to study the failure characteristics. Wang et al. [12] established a rock damage statistical constitutive model considering the damage correction coefficient based on experiments. Li et al.[13] also used the Weibull distribution function to establish a constitutive model without considering the high temperature applicability. Li et al.[14] considered the

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influence of temperature on the Weibull function and introduced the D-P criterion to establish a thermal-mechanical damage constitutive model considering the temperature effect, but also did not verify the applicability of high temperature by comparing experimental results at three different temperatures. In the above studies, the D-P criterion was often considered in determining the strength of the micro-element. In fact, the M-C criterion is more suitable compared to the D-P criterion during the rock failure stage. In this paper, the strength of the micro-element is assumed to obey the Weibull distribution, a statistical damage constitutive model of high-temperature rock is established considering the impacts of thermal damage variable, damage correction coefficient, threshold stress, threshold strain, etc. In this model, the M-C criterion is used in studying the strength of micro-element with temperature effect. The rationality and applicability of the proposed model were verified by comparing the uniaxial test data in Xu et al. [6] with the triaxial test data in this paper.

2 Statistical damage constitutive model of high-temperature rock

2.1 Damage variable and statistical distribution function

Based on the definition of thermal damage[15], showing that a large number of mesoscopic cracks will inevitably occur in the rock at high temperature, and they will gradually propagate as the temperature increases, resulting in a significant decrease in the elastic modulus. It indicates that the temperature caused rock damage. Therefore the elastic modulus is selected to represent the thermal damage variable[15], focusing on the effect of temperature on the mechanical performance of the rock. The thermal damage D_{T} is defined as

$$D_{\mathrm{T}} = 1 - \frac{E_{\mathrm{T}}}{E_0} \tag{1}$$

where E_{T} is the elastic modulus of rocks at temperature T; E_0 is the elastic modulus of rocks at room temperature (25 ℃).

Due to the uneven and random distribution of rock material particles at high-temperature, and a large number of micro-cracks and fissures exist in the rock micro-element body, resulting in a random variation in the rock strength. According to the references[16–18], the Weibull probability distribution function has the characteristics of easy integration, and a value range greater than 0, which satisfies the statistical characteristics of rock failure under compression. Therefore, it is assumed that the strength of rock micro-element body at high temperature obeys the Weibull distribution function, so the Weibull distribution is introduced and the probability density function at room temperature is analogous to obtain:

The density function $f(x)$ is

$$f(x) = \begin{cases} \dfrac{m}{K}\left(\dfrac{x}{K}\right)^{m-1} \exp\left[-\left(\dfrac{x}{K}\right)^{m}\right] \\ 0 \end{cases} \tag{2}$$

The distribution function $F(x)$ is

$$F(x) = 1 - \exp\left[-\left(\frac{x}{K}\right)^{m}\right] \tag{3}$$

where x is the strength of the micro-element body; m and K are the parameters affecting the shape and size of the rock micro-element body depending on temperature, which obey the Weibull distribution function. Therefore, thermal damage at high-temperature will affect m and K[19–22]. In this paper, the variation rules of $K(T)$ and $m(T)$ are similar with that of previous studies. The Weibull parameters of rocks under different temperatures are

$$\left.\begin{aligned} m(T) &= m_0\left(1 - D_{\mathrm{T}}\right) \\ K(T) &= K_0\left(1 - D_{\mathrm{T}}\right) \end{aligned}\right\} \tag{4}$$

where m_0 and K_0 are the Weibull parameters of the rock at 25 ℃; $m(T)$ and $K(T)$ are the Weibull parameters of rocks at different temperature T.

According to the thermal damage concept and damage mechanics theory [21] put forward by Liu et al.[15], the damage of rocks at high temperature is mostly statistical thermal damage, and the rock micro-elements obey the statistical distribution. The ultimate failure of the rock micro-element body that continuously accumulates microcracks and pores will reduce the rock strength and cause the damage of rocks. According to the Liu et al.[15], the number of damaged micro-elements is a hypothetical random amount. Therefore, this paper defines the total rock damage variable D as the ratio of the number of damaged micro-element bodies N_F under certain stress and high-temperature state to the total number of micro-element N[17, 22]:

$$D = \frac{N_{\mathrm{F}}}{N} \tag{5}$$

The rock is composed of random heterogeneous granular micro-elements. When the strength of micro-elements $f(\sigma)$ exceeds a certain strength, the micro-elements begin to fracture successively, and the number of damaged micro-elements N_{F} is

$$N_{\mathrm{F}} = \int_0^{f(\sigma)} N \cdot f(\sigma)\mathrm{d}\sigma = N\int_0^{f(\sigma)} f(\sigma)\mathrm{d}\sigma \tag{6}$$

where σ is the stress value.

Therefore, according to Eqs. (4)–(6), the total damage value of rock is

$$D=\frac{N\int_0^{f(\sigma)}f(\sigma)\mathrm{d}\sigma}{N}=\int_0^{f(\sigma)}f(\sigma)\mathrm{d}\sigma \tag{7}$$

Substituting Eqs. (3) and (4) into Eq. (7) yields

$$D=1-\exp\left[-\left(\frac{f(\sigma)}{K_0(1-D_\mathrm{T})}\right)^{m_0(1-D_\mathrm{T})}\right] \tag{8}$$

According to Eq. (8), it can be seen that the total rock damage value D is related to the thermal damage value D_T, which reflects the thermal-mechanical coupling effect.

According to the Lemaitre strain equivalence principle[16] and the effective stress concept, the strain of the damaged rock under the nominal stress measured in the test is equal to the effective strain produced by the damaged rock under the effective stress condition. Meanwhile, due to the friction and confining pressure of the specimen, it is found that the internal micro-element body still has the ability to transmit compressive and shear stress after failure during the triaxial test, and there is a certain residual strength[17]. Therefore, the damage correction factor η that is in the range of 0 and 1 is introduced. Then the relationship between the three-dimensional isotropic nominal stress σ_i ($i=1,2,3$) and the effective stress σ_i^* ($i=1,2,3$) is

$$\sigma_i^*=\frac{\sigma_i}{1-\eta D}\ (i=1,2,3) \tag{9}$$

Tian et al.[18] studied that the rock stress−strain under high-temperature has an obvious elastic stage. Therefore, the strain and stress relationship based on the generalized Hooke's law can be written as

$$\varepsilon_i=\frac{1}{E}\left[\sigma_i^*-\mu\left(\sigma_j^*+\sigma_k^*\right)\right],\ (i,j,k=1,2,3) \tag{10}$$

where ε_i is the strain value; μ is the Poisson's ratio; E is the Elastic modulus.

Substituting Eq. (9) into Eq. (10) leads to

$$\varepsilon_i=\frac{1}{E(1-\eta D)}\left[\sigma_i-\mu\left(\sigma_j+\sigma_k\right)\right],\ (i,j,k=1,2,3) \tag{11}$$

Substituting Eq. (8) into Eq. (11) we have

$$\varepsilon_i=\frac{1}{E\left(1-\eta+\eta\exp\left[-\left(\frac{f(\sigma)}{K_0(1-D_\mathrm{T})}\right)^{m_0(1-D_\mathrm{T})}\right]\right)}\cdot\left[\sigma_i-\mu\left(\sigma_j+\sigma_k\right)\right],\ (i,j,k=1,2,3) \tag{12}$$

The above equation can be rewritten as,

$$\sigma_i=\varepsilon_i E\left(1-\eta+\eta\exp\left[-\left(\frac{f(\sigma)}{K_0(1-D_\mathrm{T})}\right)^{m_0(1-D_\mathrm{T})}\right]\right)+\mu\left(\sigma_j+\sigma_k\right) \tag{13}$$

Substituting Eq. (1) into Eq. (13), the rock damage equation considering the temperature effect is written as

$$\sigma_i=E_0(1-D_\mathrm{T})\varepsilon_i\left(1-\eta+\eta\exp\left[-\left(\frac{f(\sigma)}{K_0(1-D_\mathrm{T})}\right)^{m_0(1-D_\mathrm{T})}\right]\right)+\mu\left(\sigma_j+\sigma_k\right) \tag{14}$$

2.2 Micro-element strength

The micro-element, which is the smallest unit of microscopic damage statistics, can be regarded as a mass point of continuous damage mechanics. It also has the stress−strain mechanical characteristics. The micro-element body follows a certain type of statistical distribution characteristic, which reflects the heterogeneity of particles and the distribution of micro-cracks. The statistical characteristics of all micro-element bodies determine the macroscopic characteristics of the material.

With the increase of temperature, the internal friction angle of rock gradually increases, and the cohesion decreases during the rock uniaxial and triaxial tests. The parameters in the M-C strength criterion are much easier to be obtained, making it suitable for rock analysis. The M-C strength criterion considering the temperature effect is written by Tian et al. [18−19] as follows:

$$\sigma_1^*-\sigma_3^*-\left(\sigma_1^*+\sigma_3^*\right)\sin\varphi_\mathrm{T}-2c_\mathrm{T}\cos\varphi_\mathrm{T}=0 \tag{15}$$

where c_T, φ_T are the cohesion and internal friction angle of rock under different temperatures, respectively.

According to Tian et al.[19] and Eq. (15), the strength of rock micro-element $f(\sigma)$ under high temperature is

$$f(\sigma)=\sigma_1^*-\sigma_3^*-\left(\sigma_1^*+\sigma_3^*\right)\sin\varphi_\mathrm{T} \tag{16}$$

Since the effective stress is used, the nominal stress is also required. The conversion relationship between nominal stress and effective stress (9) can be rewritten as

$$\sigma_1^*=\frac{\sigma_1}{1-\eta D} \tag{17}$$

$$\sigma_3^*=\frac{\sigma_3}{1-\eta D} \tag{18}$$

According to the conversion of Equation (11),

$$\sigma_1^*=\frac{\sigma_1}{1-\eta D}=\frac{\sigma_1\varepsilon_1 E}{\sigma_1-\mu\left(\sigma_2+\sigma_3\right)} \tag{19}$$

$$\sigma_3^*=\frac{\sigma_3}{1-\eta D}=\frac{\sigma_3\varepsilon_1 E}{\sigma_1-\mu\left(\sigma_2+\sigma_3\right)} \tag{20}$$

Substituting Eqs. (19) and (20) into Eq. (16), based on M-C failure criterion, the strength of rock micro-element under high-temperature is given as

$$f(\sigma)=\frac{\sigma_1\varepsilon_1 E-\sigma_3\varepsilon_1 E-\sin\varphi_{\mathrm{T}}\left(\sigma_1\varepsilon_1 E+\sigma_3\varepsilon_1 E\right)}{\sigma_1-\mu\left(\sigma_2+\sigma_3\right)} \tag{21}$$

2.3 High temperature statistical damage constitutive model

Under the action of external force, the deformation and failure of rock is the process of internal crack generation, compaction, expansion and convergence[23–24]. When the stress is lower than the critical cracking stress, the crack propagation is not observed and no damage occurs in the rock. Therefore, there is a threshold point in the rock deformation and failure process. In this paper, the stress corresponding to the threshold point is defined as the threshold stress σ_{C}, and the strain is defined as threshold strain ε_{C}. There is a direct relationship between threshold stress and threshold strain and temperature[17, 22]. With the increase of temperature, the threshold stress decreases and the threshold strain increases. The threshold stress and strain are related to the peak stress that is different under different temperatures. Therefore, the threshold stress and the threshold strain are different at different temperatures.

Under the continuous load, the initial cracks in the rock gradually close and undergo a linear elastic stage. It is difficult to obtain the threshold stress in the stress−strain curve. Thus the damage threshold point is also difficult to be obtained. Martin et al.[20] put forward a method to calculate the threshold stress of granite through triaxial compression test on Lac du Bonnet granite and determined that $\sigma_{\mathrm{C}}=40\%\sigma_{\mathrm{D}}$ (σ_{D} is the peak stress, as shown in Fig.1). Therefore, in this paper, 40% of the peak stress is regarded as the threshold stress, that is, $\sigma_{\mathrm{C}}=40\%\sigma_{\mathrm{D}}$, and the corresponding strain is the threshold strain ε_{C}.

By substituting Eq. (21) into Eq. (14), the rock damage constitutive model after the damage threshold point can be obtained:

$$\sigma_i=E_0\left(1-D_{\mathrm{T}}\right)\varepsilon_i\left(1-\eta+\eta\exp\left[-\left(\frac{\sigma_1\varepsilon_1 E-\sigma_3\varepsilon_1 E-\sin\varphi_{\mathrm{T}}\left(\sigma_1\varepsilon_1 E+\sigma_3\varepsilon_1 E\right)}{\left[\sigma_1-\mu\left(\sigma_2+\sigma_3\right)\right]K_0\left(1-D_{\mathrm{T}}\right)}\right)^{m_0\left(1-D_{\mathrm{T}}\right)}\right]\right)+\mu\left(\sigma_j+\sigma_k\right) \tag{22}$$

Before the damage threshold point, the constitutive relationship of the rock can be obtained by fitting with the quadratic function. The stress−strain curve passes through the coordinate origin, so the quadratic function can be set as

$$\sigma_i=A\varepsilon_i^2+B\varepsilon_i \tag{23}$$

where A and B are the fitting coefficients of the quadratic function.

Combining Eqs. (22) and (23), the statistical damage constitutive model for rock at high temperature is obtained based on Weibull distribution and M-C failure criterion as follows:

$$\sigma_i=\begin{cases} A\varepsilon_i^2+B\varepsilon_i,\ 0\leqslant\varepsilon\leqslant\varepsilon_{\mathrm{C}} \\ E_0\left(1-D_{\mathrm{T}}\right)\varepsilon_i\left\{1-\eta+\eta\exp\left[-\left(\dfrac{f(\sigma)}{K_0\left(1-D_{\mathrm{T}}\right)}\right)^{m_0\left(1-D_{\mathrm{T}}\right)}\right]\right\}+ \\ \mu\left(\sigma_j+\sigma_k\right),\varepsilon>\varepsilon_{\mathrm{C}} \end{cases} \tag{24}$$

3 Determination of parameters in the constitutive model

Many parameters are required to build the constitutive model proposed in this paper, including E, μ, c_{T} and ϕ_{T}, which can be obtained by conventional uniaxial and triaxial rock tests at high-temperature. The determination of coefficients depends on obtaining the peak stress and peak strain at different temperature. In the high temperature rock triaxial mechanical tests, parameters such as nominal stresses σ_1, σ_2, σ_3 ($\sigma_2=\sigma_3$) and strain ε_1 can be obtained.

3.1 Parameters *m* and *K*

When $\varepsilon>\varepsilon_{\mathrm{C}}$, the Eq. (21) is substituted into Eq. (24) to solve the parameters. Eq.(25) is obtained after simplification

$$\sigma_1=E\varepsilon_1\left\{1-\eta+\eta\exp\left[-\left(\frac{f(\sigma)}{K}\right)^m\right]\right\}+2\mu\sigma_3 \tag{25}$$

where

$$f(\sigma)=\frac{\sigma_1\varepsilon_1 E-\sigma_3\varepsilon_1 E-\sin\varphi_{\mathrm{T}}\left(\sigma_1\varepsilon_1 E+\sigma_3\varepsilon_1 E\right)}{\sigma_1-\mu\left(\sigma_2+\sigma_3\right)} \tag{26}$$

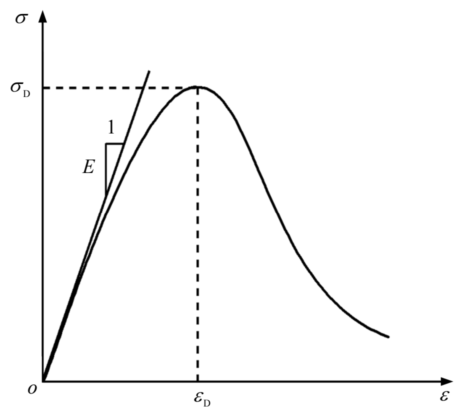


Fig. 1 The stress−strain curve under triaxial compression state

As shown in Fig.1, the strain at the peak point is ε_{D}, and the stress at the peak point is σ_{D}. The stress−strain curve satisfies the constitutive model at the peak stress, namely:

$$\sigma|_{\varepsilon_1=\varepsilon_D}=\sigma_D \tag{27}$$

The first derivative of the stress−strain curve at the peak is 0,

$$\left.\frac{\partial\sigma_1}{\partial\varepsilon_1}\right|_{\substack{\sigma=\sigma_D\\ \varepsilon=\varepsilon_D}}=0=E\left\{1-\eta+\eta\left\{\exp\left[-\left(\frac{f(\sigma_D)}{K}\right)^m\right]+\varepsilon_D e^{\left[-\left(\frac{f(\sigma_D)}{K}\right)^m\right]}\left\{-m\left(\frac{f(\sigma_D)}{K}\right)^{m-1}\right\}\frac{\sigma_D E-\sigma_3 E-\sin\varphi_T(\sigma_D E+\sigma_3 E)}{K(\sigma_D-2\mu\sigma_3)}\right\}\right\} \tag{28}$$

where

$$f(\sigma_D)=\frac{\sigma_D\varepsilon_D E-\sigma_3\varepsilon_D E-\sin\varphi_T(\sigma_D\varepsilon_D E+\sigma_3\varepsilon_D E)}{\sigma_D-2\mu\sigma_3} \tag{29}$$

Combining Eqs. (25), (27) and (28), the values of m and K can be determined.

3.2 Parameters *A* and *B*

The deformation and damage in the rock test is a continuous process, and the stress−strain curve changes regularly and continuously in the compression stage and the damage evolution stage,

$$E\varepsilon_C\left(1-\eta+\eta\exp\left[-\left(\frac{f(\sigma_C)}{K}\right)^m\right]\right)+2\mu\sigma_3=A\varepsilon_C^2+B\varepsilon_C \tag{30}$$

where,

$$f(\sigma_C)=\frac{\sigma_C\varepsilon_C E-\sigma_3\varepsilon_C E-\sin\varphi_T(\sigma_C\varepsilon_C E+\sigma_3\varepsilon_C E)}{\sigma_C-2\mu\sigma_3} \tag{31}$$

$$\left.\frac{\partial\sigma_1}{\partial\varepsilon_1}\right|_{\substack{\sigma=\sigma_C\\ \varepsilon=\varepsilon_C}}=E\left\{1-\eta+\eta\left\{\exp\left[-\left(\frac{f(\sigma_C)}{K}\right)^m\right]+\varepsilon_C e^{\left[-\left(\frac{f(\sigma_C)}{K}\right)^m\right]}\cdot\left\{-m\left(\frac{f(\sigma_C)}{K}\right)^{m-1}\right\}\frac{\sigma_C E-\sigma_3 E-\sin\varphi_T(\sigma_C E+\sigma_3 E)}{K(\sigma_C-2\mu\sigma_3)}\right\}\right\}= 2A\varepsilon_C+B \tag{32}$$

Combining Eqs. (30) and (32), both A and B can be calculated.

4 Model verification and analysis

4.1 Literature comparison

The uniaxial test ($\sigma_2=\sigma_3=0$) results of granite under different temperature treatments in the literature[6] with the correction coefficient η of 0.98 based on multiple fittings with the test results are used to verify the rationality of the statistical damage constitutive model of high temperature rock. The relevant data obtained from this experiment is shown in Table 1.

Table 1 The measured mechanical parameters at different temperatures

Temperature T/℃	Cohesion c_T / MPa	internal friction angle φ_T /(°)	Elastic modulus E /GPa	Peak stress σ_D /MPa	Peak strain ε_D /10^{-3}	Threshold stress σ_C /MPa	Threshold strain ε_C /10^{-3}	Poisson's ratio μ
25	19.81	47.36	35.438	120.05	4.14	48.02	2.25	0.14
200	14.65	52.86	31.188	121.15	4.78	48.46	2.41	0.15
400	14.41	52.20	32.650	97.30	4.21	38.92	2.58	0.18
600	17.58	53.29	14.927	54.49	5.87	21.80	3.18	0.14
800	15.15	50.87	12.053	41.84	5.44	16.74	3.32	0.17
1 000	16.48	50.00	3.870	19.27	6.49	7.71	3.52	0.19

Note: Threshold stress $\sigma_C=40\%\sigma_D$.

The variation of peak stress, peak strain and elastic modulus with temperature can be obtained in Table 1 and Fig.2. Due to the discreteness of experiments, the elastic modulus of granite increases at 400 ℃, but the compressive strength gradually decreases with the increase of temperature, indicating that the high-temperature effect will degrade the mechanical properties of granite, thereby reducing its strength and causing damage effects. As the temperature increases, the elastic modulus of the granite samples decrease and the overall trend of the peak strain gradually increase. It can be seen that the stress−strain curve will shift to the right direction with the increase in temperature, indicating that the plastic properties of granite increase, but the elastic properties decrease.

For the parameters of the constitutive model established in this paper, the parameters m, K, A, and B at different temperatures can be obtained according to the data in Table 1 and the above-mentioned parameter calculation methods, as shown in Table 2.

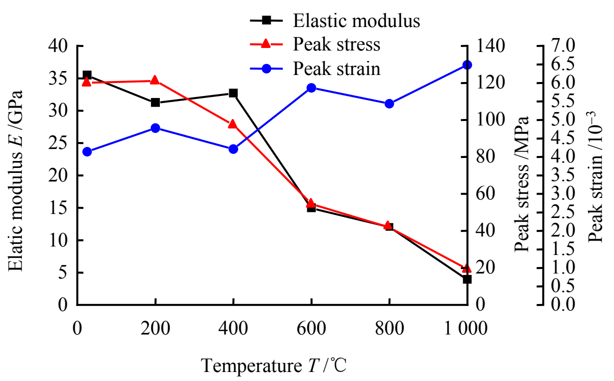


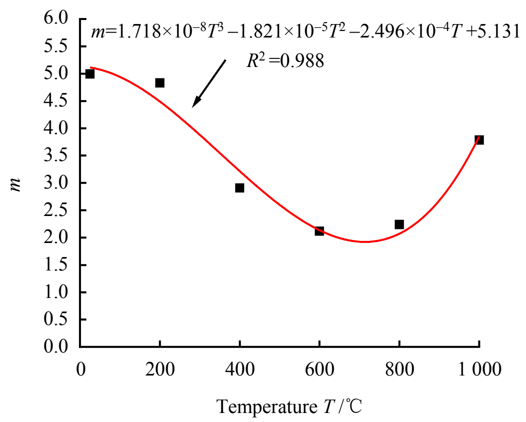
Fig. 2 Variation of peak stress, strain and elastic modulus with temperature

Table 2 Parameter values in constitutive model at different temperatures

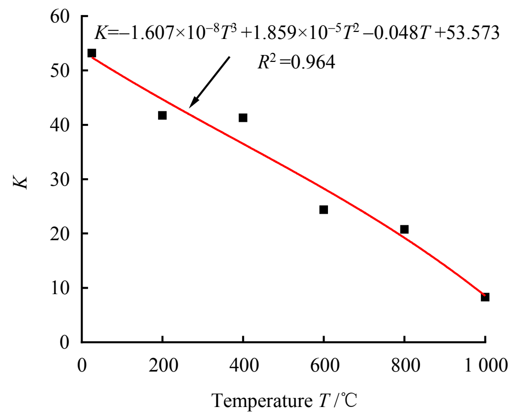
T /℃	m	K	A /10^5	B /10^5
25	4.997	53.179	−7.489	0.368
200	4.832	41.711	−1.725	0.314
400	2.907	41.260	−28.229	0.375
600	2.119	24.350	−11.352	0.167
800	2.240	20.755	−10.433	0.138
1 000	3.786	8.296	−1.061	0.041

At the same time, the variation curve of the Weibull distribution function parameters m and K with temperature in the constitutive equation can be obtained from Table 2, as shown in Fig.3. It can be seen from Figure 3 that m increases with the decrease of temperature T, and m is the smallest when the temperature reaches 600 ℃. The parameter K gradually decreases with the increase of temperature T between 25 and 1 000 ℃. This is because that the parameter m not only affects the geometry of micro-element body, but also reflects the plastic properties of rocks. As the temperature increases, the plasticity of rocks increases, while the elasticity of rocks is relatively decreased. Both the peak stress and K value attain the largest at 200 ℃, meaning that the K value determines the peak intensity. Therefore, the variation law of parameter conforms to the change trend of the parameters in the references[6], proving that the constitutive model established in this paper has a certain rationality.

The theoretical stress−strain curve of rocks at different temperatures are calculated based on Eq. (24) and the comparison between theoretical results and that of Xu et al.[6] can be seen in Fig. 4, showing similar variation trend. Both of them can reflect the variation law of strength and deformation in granite with temperature. It implies that the high-temperature rock statistical damage constitutive model based on Weibull distribution proposed in this paper can reflect the deformation characteristics of granite under different temperature and uniaxial compression tests. It can also efficiently characterize the rock stress−strain curve under different temperature effects. However, a certain deviation exists and it is in a reasonable range. The reason is that the parameters are different due to the selection of different failure criterion during the establishment of equations. There is a large deviation by using the constitutive model established by Xu et al.[6] with that of the experimental results. The constitutive model established in this paper is more in line with the experimental results at the post-peak stage of the stress−strain curve, illustrating that applying the model established in this paper in the uniaxial test is accurate and rational.

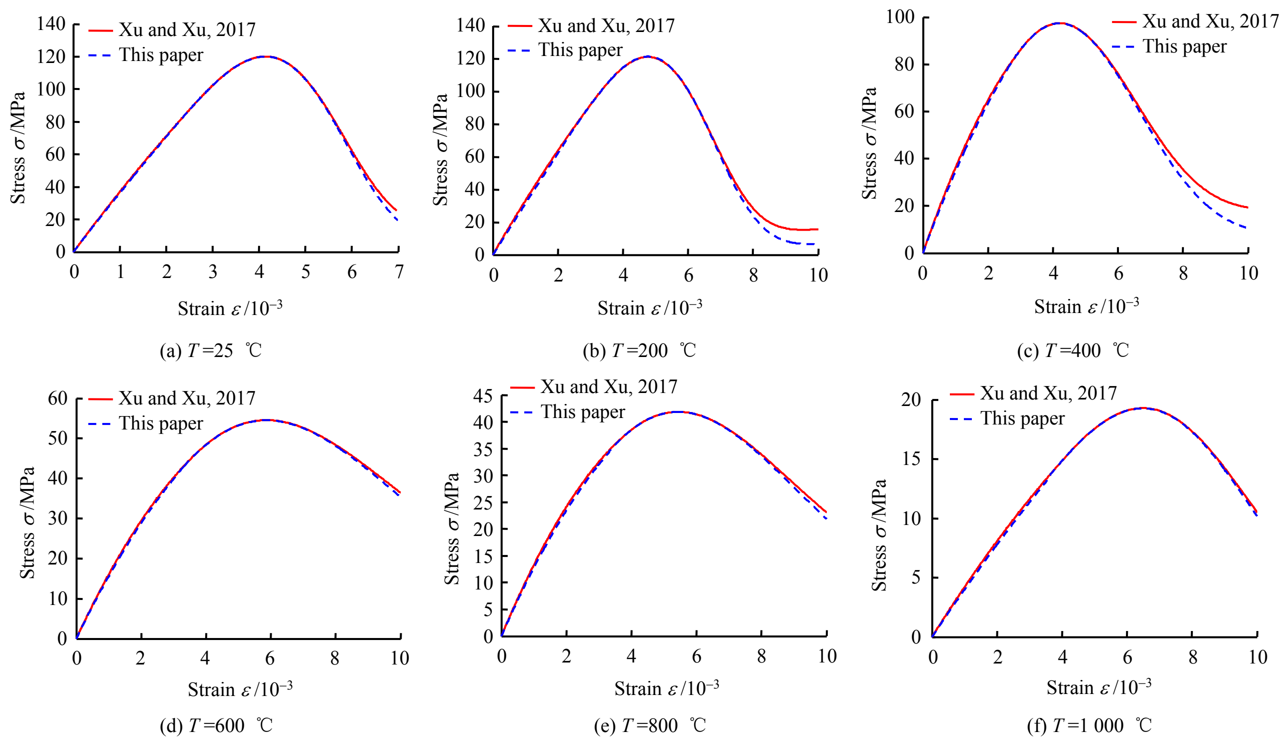


(a) m–T curve



(b) K–T curve

Fig. 3 Variation of m and K with temperature



(a) T=25 ℃ (b) T=200 ℃ (c) T=400 ℃

(d) T=600 ℃ (e) T=800 ℃ (f) T=1 000 ℃

Fig. 4 Comparison of stress–strain curves calculated in this paper with that of Xu's model[6] at different temperatures

4.2 Test verification

In order to verify the applicability of the model established in this paper, the triaxial test results of sandstone under different temperature conditions (25, 60, 100, 150 ℃) are used.

The rock samples were taken from the sandstone stratum of a tunnel project in Hunan province. The rock block from the same tunnel face is taken to the laboratory and the ZS-200 automatic coring machine and the SHM-200 double-end face grinder are used to make the cylindrical standard samples with a diameter of 50 mm and a height of 100 mm. The multi-functional RLW-2000 rock triaxial instrument jointly developed by Dalian Maritime University and Changchun Chaoyang Testing Machine Factory are used in the tests. Fig.5 shows the rock sample and instrument. The allowable height deviation of rock is not more than 5 mm and the diameter deviation is not more than 0.3 mm.

The real-time heating rate is controlled at 4 ℃/min. When the temperature reaches the preset temperature, the heating is stopped and a constant temperature is kept for 4 h. The displacement loading mode, with the displacement loading speed of 0.05 mm/min, is applied. Combining the experimental data and the solution process of the above-mentioned constitutive model parameters (e.g., η =0.98), the parameter values of the constitutive model at each temperature can be obtained, see in Table 3. According to Eq. (24), the triaxial stress–strain curve of the rock at various temperatures can be compared with the experimental data, as shown in Fig.6.

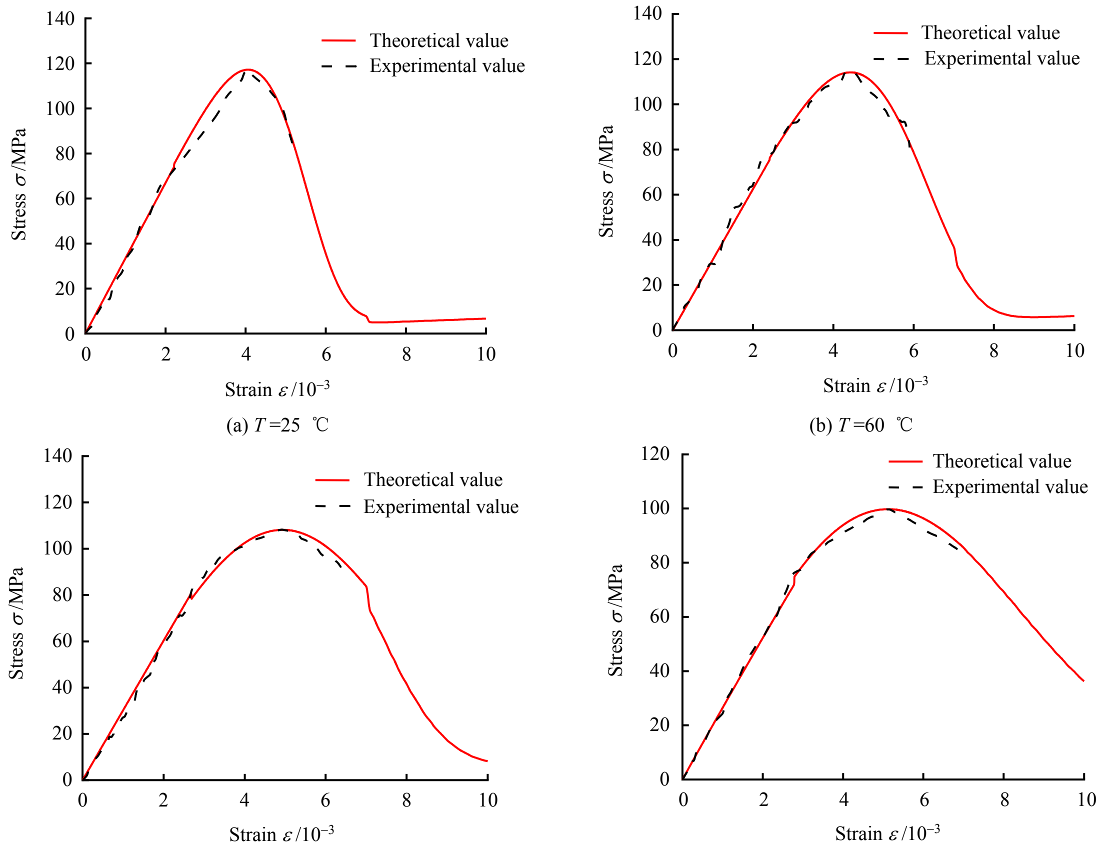
(a) Sandstone specimen (b) Triaxial apparatus

Fig. 5 Sandstone samples and triaxial instrument

Table 3 Mechanical parameters in the constitutive model at different temperatures

T/℃	$\sigma_2=\sigma_3$/MPa	c_T/MPa	φ_T/(°)	E/GPa	σ_D/MPa	ε_D/10^{-3}	σ_C/MPa	ε_C/10^{-3}	μ	m	K/10^6	A/10^6	B/10^6
25	5	18.24	46.58	33.452	117.21	4.06	46.88	2.21	0.15	6.25	36.05	−2 133.94	33 455.96
60	5	17.48	47.83	31.243	114.10	4.43	45.64	2.41	0.16	4.84	35.55	−6 804.22	31 256.01
100	5	17.22	47.92	30.174	108.21	4.94	43.28	2.68	0.15	3.00	38.95	−67 434.88	30 294.42
150	5	16.87	48.47	28.774	99.78	5.12	39.91	2.78	0.15	2.48	34.59	−332 456.83	26 772.24

Note: the threshold stress $\sigma_C=40\%\sigma_D$.



(a) T=25 ℃ (b) T=60 ℃ (c) T=100 ℃ (d) T=150 ℃

Fig. 6 Comparison of theoretical and experimental stress−strain curves under different temperatures

It can be seen from Fig.6 that the theoretical value of stress−strain curve calculated by the Weibull distribution-based high-temperature rock statistical damage model established in this paper shows a minor difference from the experimental value, which fully reflects the trend of stress−strain at the post-peak stage. It can also efficiently reflect the stress−strain relationship of sandstone at high-temperature under triaxial test conditions, revealing the deformation and failure characteristics of sandstone under different temperatures. Compared with t

he D-P criterion in Xu et al.[6], the M-C strength criterion that considers the temperature effect is more consistent with the failure law of rock at the post-peak stage. Therefore, the constitutive model established in this paper is proved to be reasonable and applicable by comparing results between the experiments and the model.

5 Conclusions

(1) First of all, the strength of the rock micro-element body at high temperature is assumed to obey the Weibull distribution. The rock damage equation considering the temperature effect is established based on Lemaitre strain equivalence principle, the effective stress theory and the damage correction coefficient η .

(2) On the basis of considering the existence of a damage threshold point in the process of rock deformation, the M-C strength criterion with temperature effect is used as the basis of the failure of rock micro-elements. A high-temperature rock statistical damage constitutive model based on Weibull distribution is established and the determination methods of parameters in the model are clarified.

(3) The parameters of the constitutive model established in this paper can be calculated from the results of conventional uniaxial and triaxial tests, making it very convenient to be applied. By comparison with the results of the literature, it is found that under uniaxial test conditions, there is a minor difference between the theoretical results and the calculated curve trends in the literature, especially at the post-peak stage. According to the comparison results between the triaxial test of sandstone at high-temperature and the theoretical curves calculated by the constitutive model, it is found that the model can better reflect the stress−strain relationship of sandstone at high-temperature under the triaxial test conditions, proving that the constitutive model established in this paper is applicable.

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