

9-16-2021

## Influence of rock mass spatial variability on probability of tunnel roof wedge failure

Wen-gang ZHANG

*National Joint Engineering Research Center of Geohazards Prevention in the Reservoir Areas, Chongqing University, Chongqing 400045, China*

Qi WANG

*School of Civil Engineering, Chongqing University, Chongqing 400045, China*

Han-long LIU

*National Joint Engineering Research Center of Geohazards Prevention in the Reservoir Areas, Chongqing University, Chongqing 400045, China*

Fu-yong CHEN

*School of Civil Engineering, Chongqing University, Chongqing 400045, China, chenfuyong@cqu.edu.cn*

Follow this and additional works at: <https://rocksoilmech.researchcommons.org/journal>



Part of the [Geotechnical Engineering Commons](#)

---

### Custom Citation

ZHANG Wen-gang, WANG Qi, LIU Han-long, CHEN Fu-yong, . Influence of rock mass spatial variability on probability of tunnel roof wedge failure[J]. Rock and Soil Mechanics, 2021, 42(5): 1462-1472.

This Article is brought to you for free and open access by Rock and Soil Mechanics. It has been accepted for inclusion in Rock and Soil Mechanics by an authorized editor of Rock and Soil Mechanics.

# Influence of rock mass spatial variability on probability of tunnel roof wedge failure

ZHANG Wen-gang<sup>1, 2, 3</sup>, WANG Qi<sup>2</sup>, LIU Han-long<sup>1, 2, 3</sup>, CHEN Fu-yong<sup>2</sup>

1. Key Laboratory of New Technology for Construction of Cities in Mountain Area, Chongqing University, Chongqing 400045, China

2. School of Civil Engineering, Chongqing University, Chongqing 400045, China

3. National Joint Engineering Research Center of Geohazards Prevention in the Reservoir Areas, Chongqing University, Chongqing 400045, China

**Abstract:** In tunnel design and construction, the stability evaluation of tunnel roof wedge usually adopts traditional deterministic analysis methods, which cannot appropriately reflect the spatial variability of rock mass. Based on limit equilibrium method, an efficient approach for evaluating the safety factor integral expression of tunnel roof wedge is presented in this study. This approach considers the influence of spatial variability of rock mass joint friction angle and is validated by universal distinct element code (UDEC). Based on random field method and Monte Carlo simulation (MCS), a generated random field is substituted into the established safety factor integral expression to calculate the safety factor of the tunnel wedge, in which the spatial variability of rock mass joint friction angle is well considered. The failure probability of roof wedge is calculated with consideration of geological and geometry parameters uncertainties. The results indicate that the spatial variability of rock mass joint friction angle has a significant influence on the failure probability of tunnel roof wedge. Ignoring the spatial variability of rock mass mechanical parameters will cause an overestimated failure probability of tunnel roof wedge.

**Keywords:** spatial variability; tunnel roof wedge; limit equilibrium method; factor of safety; failure probability

## 1 Introduction

With the rapid development of underground space, more and more tunnels are excavated in rock mass, and the instability and collapse of the tunnel roof occur frequently during construction, resulting in serious casualties and economic losses [1–4]. Therefore, it is of great significance to measure the stability of tunnel wedge to ensure the safe construction of the tunnel. At present, deterministic analysis methods are often used to evaluate stability by calculating the safety factor of tunnel roof. Common analysis methods include limit equilibrium method [1, 5–7] and numerical simulation [3, 8–11]. When using a deterministic method to evaluate the project stability, it is often necessary to select a series of representative geological parameters to calculate the factor of safety [12–13]. However, existing studies have shown that even if the calculated factor of safety is much higher than the critical value, ruin still occurs in the project. This is because the geological parameters uncertainty is ignored in the deterministic analysis [14–18].

In recent years, some scholars have systematically considered the geological parameters uncertainty to calculate the failure probability of the tunnel roof based on reliability analysis methods during tunnel excavation [19–24]. Low et al. [19] and Liu et al. [22] considered the joint friction angle, lateral pressure coefficient and joint stiffness ratio as random variables through the first-order reliability method (FORM) and second-order reliability method (SORM) to calculate the failure probability

of wedge in circular tunnel roof. Luo et al. [20] calculated the failure probability of three-dimensional wedge of a rectangular tunnel using first-order reliability method and Monte-Carlo simulation (MCS) based on Hoek-Brown criterion. Yang et al. [21] used response surface method (RSM) and MCS to calculate the failure probability of a rectangular tunnel roof wedge under the action of seepage pressure. Zhou et al. [23] used the RSM method to predict the roof stability of a shallow rectangular tunnel by taking the density and dynamic elastic modulus of rock and soil materials as random variable. However, the current research on the failure probability of tunnel roof only considers the rock mass parameters as random variables, and ignores the spatial variability of parameters, which leads to inability of accurately describing inherent variability of rock mass parameters [2]. At present, most of the literature on rock masses spatial variability research focus on slope engineering [25–27], and the spatial variability of rock and soil is less considered in tunnel engineering relatively. Now, the only literatures that involve studies of the spatial variability in tunnel engineering focused on stress analysis, displacement analysis, and stability evaluation of tunnel faces [28–30], and there is no in-depth study of the influence of spatial variability of rock mass mechanical parameters on failure probability of tunnel roof wedge. It is hence necessary to propose a calculation method for the failure probability of tunnel roof wedge that can describe the spatial variability of

Received: 10 August 2020

Revised: 6 January 2021

This work was supported by the Program of China Scholarships Council (201906050026) and the Construction of Science and Technology Program of Chongqing (2019-0045).

First author: ZHANG Wen-gang, born in 1983, PhD, Professor, Doctoral advisor, research interests: rock engineering reliability and risk management. E-mail: zhangwg@cqu.edu.cn

Corresponding author: CHEN Fu-yong, male, born in 1994, PhD candidate, research interests: rock engineering reliability and risk management. E-mail: chenfuyong@cqu.edu.cn

rock mass mechanical parameters, and to analyze the influence of the spatial variability on the wedge failure probability.

Based on limit equilibrium theory and random field theory, this paper establishes an effective method to calculate the failure probability of tunnel roof wedge when considering the spatial variability of rock mass mechanical parameters, and to analyze the effect of rock mass parameters spatial variability on the failure probability. Firstly, a formula for calculating the factor of safety of the tunnel roof wedge that considers the spatial variability of joint friction angle is deduced by the limit equilibrium method, and the correctness of the formula is verified by discrete element method. And then, the random field of joint friction angle is generated using the Cholesky decomposition method, it is substituted into the derived formula for factor of safety, and the failure probability of tunnel roof wedge is calculated based on Monte Carlo simulation. Finally, a systematic parameter analysis is carried out to analyse the influence of tunnel geometry and geological parameters uncertainties on the wedge failure probability.

## 2 Stability analysis

The classic failure mode of symmetrical roof wedge of a circular tunnel proposed in the literature<sup>[31]</sup> is presented in Fig.1. Because the wedge failure model has the advantages of clear failure mechanism and simple calculation, it is widely used in stability analysis of tunnel wedge<sup>[5, 22, 31]</sup>. Therefore, this classic failure mode is also used to analyze the stability of tunnel roof wedge in this paper.

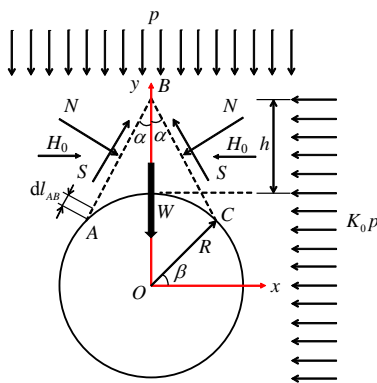


Fig. 1 Typical failure mode of tunnel roof wedge

Similar to most slope stability analysis, the factor of safety FS of tunnel roof wedge is defined as the ratio of the resistant force to the sliding force<sup>[5]</sup>:

$$FS = \frac{2S \cos \alpha}{2N \sin \alpha + W} \quad (1)$$

where  $S$  is the shear force;  $N$  is the normal force;  $W$  is the weight of the wedge; and  $\alpha$  is the semi-apical angle of tunnel wedge. The calculation formulas of the parameters are as follows<sup>[22, 31]</sup>:

$$N = \frac{H_0 (k_s \cos^2 \alpha + k_n \sin^2 \alpha) \cos \varphi}{k_n \sin \alpha \sin \varphi + k_s \cos \alpha \cos \varphi} \quad (2)$$

$$S = \frac{H_0 (k_s \cos^2 \alpha + k_n \sin^2 \alpha) \sin \varphi}{k_n \sin \alpha \sin \varphi + k_s \cos \alpha \cos \varphi} \quad (3)$$

$$W = \gamma R^2 [\cos^2 \beta (\tan \beta + \cot \alpha) - \frac{\pi}{2} + \beta] \quad (4)$$

where  $H_0$  is the horizontal force inside the wedge;  $\varphi$  is the friction angle of the joint;  $k_s$  is the shear stiffness of the joint;  $k_n$  is the normal stiffness of the joint;  $R$  is the radius of the circular tunnel;  $\gamma$  is the unit weight of rock; and  $\beta$  is the angle between  $OC$  and  $x$ -axis, and it is related to the point where the joint cuts into the tunnel, as shown in Fig.1. The cohesion of joints is ignored in the calculation of factor of safety for the tunnel roof wedge. And the calculation formulae for  $H_0$  and  $\beta$  are as follows<sup>[19, 22]</sup>:

$$H_0 = \frac{1}{2} p R [(1 + K_0) C_{H1} - (1 - K_0) C_{H2}] \quad (5)$$

$$\beta = \cos^{-1} \left[ \left( \frac{h}{R} + 1 \right) \sin \alpha \right] + \alpha \quad (6)$$

where  $h$  is the center height of the wedge in Fig. 1;  $p$  is the vertical in situ stress;  $K_0$  is the coefficient of horizontal in situ stress; and  $C_{H1}$  and  $C_{H2}$  are two intermediate variables, which can be calculated by the following two formulas:

$$C_{H1} = \left( \frac{h}{R} + 1 \right) - \frac{1}{(h/R + 1)} \quad (7)$$

$$C_{H2} = \left( \frac{h}{R} + 1 \right) - \frac{1}{(h/R + 1)^3} \quad (8)$$

It should be noted that the failure of wedge only occurs when the geometric parameters of the circular tunnel satisfy the following formula:

$$\alpha \leq \sin^{-1} \left( \frac{1}{1 + h/R} \right) \quad (9)$$

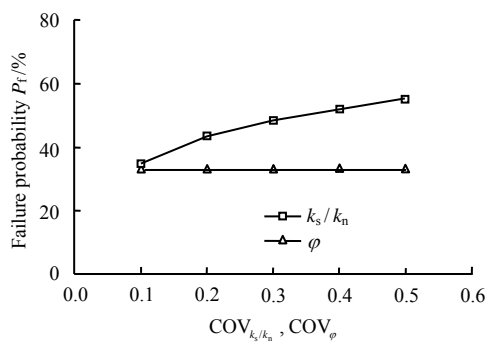
When the left and right of Eq. (9) are equal, the tunnel is just tangent to the joint. If Eq. (9) is not true, the joint will not intersect the circular tunnel, so the tunnel roof wedge will not be formed. At this moment, it is considered that the instability and falling failure of tunnel roof wedge will not occur.

In this section, only the variability of the parameters  $\varphi$ ,  $k_s/k_n$ ,  $p$  and  $K_0$ <sup>[19, 22]</sup> are considered, and the remaining parameters are constants (the variability of joint geometric parameters is considered later). Literatures [19, 22] are referred for parameter values, as shown in Table 1. From Eqs. (1) to (9), it can be seen that the only joint mechanical parameters that affect the factor of safety for the tunnel roof wedge are the joint friction angle  $\varphi$  and the ratio of shear to normal stiffness  $k_s/k_n$ . In order to reasonably simplify the computational process of the factor of safety, the degrees of influence of  $\varphi$  and  $k_s/k_n$  on the wedge failure probability are determined by using a simple sensitivity analysis. Fig. 2 shows that the variation of coefficient  $COV_{k_s/k_n}$  of  $k_s/k_n$  has almost no effect on the probability of failure  $P_f$ , while the coefficient of variation  $COV_\varphi$  of the friction angle  $\varphi$  has a significant effect on the

probability of failure  $P_f$ . Hence, the follow-up study of this paper will only consider the spatial variability of the joint friction angle  $\varphi$ .

**Table 1** Parameters of tunnel roof wedge shown in Fig.1

Parameter	Symbol	Distribution type	Mean value	COV
Joint friction angle	$\varphi / (^\circ)$	Lognormal	35	0.086
Ratio of shear to normal stiffness	$k_s/k_n$	Lognormal	0.1	0.250
Vertical stress	$p/\text{MPa}$	Lognormal	0.5	0.100
Coefficient of horizontal in situ stress	$K_0$	Lognormal	0.5	0.250
Semi-apical angle	$\alpha / (^\circ)$	Lognormal	25	0.080
Wedge height	$h/\text{m}$	Lognormal	5.1	0.080
Rock unit weight	$\gamma / (\text{kN} \cdot \text{m}^{-3})$	Lognormal	27	0.080
Tunnel radius	$R/\text{m}$	—	6	—



**Fig. 2** Influence of coefficients of variation of  $\varphi$  and  $k_s/k_n$  on failure probability  $P_f$

In order to describe the spatial variability of joint parameters, the infinitesimal method is used to calculate the factor of safety when considering the spatial variability of the friction angle. The micro-element  $dl_{AB}$  is selected on the joint  $AB$ , as shown in Fig.1. Suppose the friction angle on the micro-element is  $\varphi_e$ , the normal stress on the joint  $AB$  is  $\sigma_n$ , and the normal force  $N$  and shear force  $S$  can be obtained by integrating the normal stress  $\sigma_n$  along  $AB$  according to the Mohr-Coulomb criterion:

$$N = \int_{AB} \sigma_n dl_{AB} \quad (10)$$

$$S = \int_{AB} \sigma_n \tan \varphi_e dl_{AB} \quad (11)$$

In this paper, the relaxation method [31] is used to calculate the factor of safety of the wedge of tunnel roof.  $N$  and  $S$  can be represented by the vertical displacement of the wedge  $u_y$ :

$$N = H_0 \cos \alpha - k_n u_y \sin \alpha \quad (12)$$

$$S = H_0 \sin \alpha + k_s u_y \cos \alpha \quad (13)$$

Substituting Eqs. (10) and (11) into Eqs. (12) and (13) gives

$$u_y = \frac{H_0 \cos \alpha \int_{AB} \tan \varphi_e dl_{AB} - H_0 \sin \alpha}{k_n \sin \alpha \int_{AB} \tan \varphi_e dl_{AB} + k_s \cos \alpha} \quad (14)$$

Substituting Eq. (14) into Eqs. (12) and (13) leads to

$$N = \frac{H_0 (k_s \cos^2 \alpha + k_n \sin^2 \alpha)}{k_n \sin \alpha \int_{AB} \tan \varphi_e dl_{AB} + k_s \cos \alpha} \quad (15)$$

$$S = \frac{H_0 \int_{AB} \tan \varphi_e dl_{AB} (k_s \cos^2 \alpha + k_n \sin^2 \alpha)}{k_n \sin \alpha \int_{AB} \tan \varphi_e dl_{AB} + k_s \cos \alpha} \quad (16)$$

When calculating the factor of safety with consideration of the spatial variability of rock mass joints friction angle, the shear force  $S$  and normal force  $N$  are calculated first by Eqs. (15) and (16), and then  $S$  and  $N$  are substituted into Eq. (1) to obtain the factor of safety. From Eqs. (15) and (16), it can be found that when the friction angle  $\varphi_e$  of each micro-element  $dl_{AB}$  in the joint  $AB$  is equal, Eqs. (15) and (16) are equivalent to Eqs. (2) and (3).

### 3 Random field description of joint friction angle

At present, there are many methods to generate random fields, such as local average method, K-L series expansion method and Cholesky decomposition method. Since the Cholesky decomposition method has the advantages of simple calculation and easy program implementation [32], this paper uses the Cholesky decomposition method to generate random field  $\varphi_e$  of joint friction angle  $\varphi$ , and the random field variable at any spatial position  $y_i$  is assumed to be  $\varphi_e(y_i)$ :

$$\varphi_e(y_i) = \exp[u_{\ln} + \sigma_{\ln} G(y_i)] \quad (17)$$

where  $i$  is the element series number of the spatial random field;  $u_{\ln}$  and  $\sigma_{\ln}$  are the mean and standard deviation of the normal variable  $\ln \varphi$ , respectively;  $G(y_i)$  is the random field variable where the standard normal distribution random field  $\mathbf{G}$  is located at  $y_i$ ; and the calculation formula of the standard normal distribution random field  $\mathbf{G}$  is [33–34]:

$$\mathbf{G} = \mathbf{LZ} \quad (18)$$

where  $\mathbf{Z}$  is the independent standard normal random sample matrix; and  $\mathbf{L}$  is the triangular matrix obtained by Cholesky decomposition:

$$\mathbf{L} \times \mathbf{L}^T = \mathbf{C} \quad (19)$$

$$\mathbf{C} = \begin{bmatrix} 1 & \rho(\tau_{12}) & \cdots & \rho(\tau_{1n}) \\ \rho(\tau_{12}) & 1 & \cdots & \rho(\tau_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ \rho(\tau_{1n}) & \rho(\tau_{2n}) & \vdots & 1 \end{bmatrix} \quad (20)$$

where  $\mathbf{C}$  is the correlation matrix;  $n$  is the number of elements in the random field;  $\tau_{ij} = |y_i - y_j|$  is the vertical distance between the element  $i$  and the element  $j$ . In this paper, the exponential autocorrelation function  $\rho(\tau_{ij})$  [33–35] is used, and its expression is

$$\rho(\tau_{ij}) = \exp\left(-\frac{2\tau_{ij}}{\theta}\right) \quad (21)$$

where  $\theta$  is the scales of fluctuation in the vertical direction.

The change of joint friction angle with depth under different scales of fluctuation is compared as shown in Fig.3. It can be observed that with the increase of the scales of fluctuation  $\theta$ , the variation of the friction angle  $\varphi$  becomes smaller and smaller. When the scales of fluctuation  $\theta$  tends to infinity, the friction angle  $\varphi$  tends to a mean value, and there is almost no spatial variability. This is the situation in which the parameters are regarded as random variables in the traditional reliability analysis.

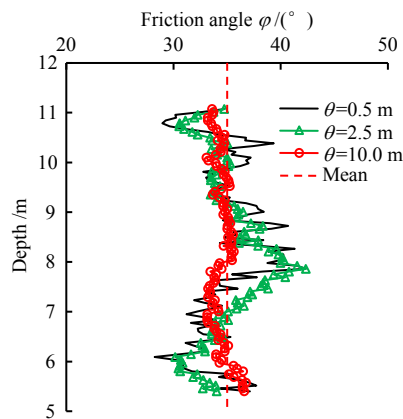


Fig.3 Spatial variability of friction angle  $\varphi$  for different scales of fluctuation

#### 4 Numerical test of the tunnel roof wedge

In order to verify the correctness of Eqs. (15) and (16) and assess the calculation error of the wedge model adopted, the discrete element software UDEC 6.0 [36] is employed to calculate the factor of safety under different parameter combinations, and the factor of safety obtained numerically is compared with the factor of safety calculated by the formula derived in

this paper.

##### 4.1 Building model

Figure 4 is a schematic diagram of the established UDEC model. In order to reduce the influence of boundary effect, the model length and height are both 40 m. Except for the upper boundary which is a free boundary, all other boundaries are subject to normal translational constraints, and the  $K_0p$  and  $p$  in Fig. 1 are applied to the model boundary. According to related literatures [19, 22], the various input parameters of tunnels are selected as shown in Table 2. The parameter combinations in Table 2 cover the range of values of tunnel wedge parameters. Since the orthogonal experiment design method can greatly reduce the amount of calculation while meeting the calculation requirements, the parameters in Table 2 are orthogonally designed here, and a total of 81 parameter combinations are obtained (due to space limitations, they are not listed here). The modelling process is mainly divided into three parts: ① The model reaches the initial stress equilibrium state. ② The circular tunnel is excavated. ③ The factors of safety for tunnel wedge is calculated by the strength reduction method.

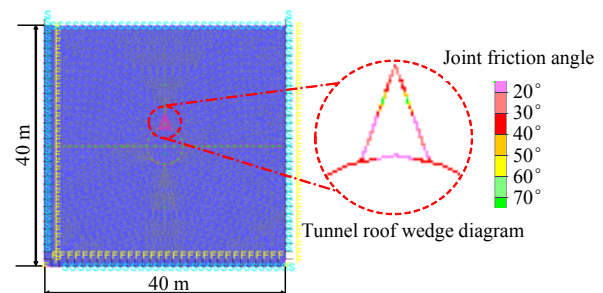


Fig.4 Numerical model of tunnel roof wedge

Table 2 Input parameters of numerical model

Wedge semi-apical angle $\alpha$ /( $^{\circ}$ )	Joint friction angle $\varphi$ /( $^{\circ}$ )	Ratio of shear to normal stiffness $k_s/k_n$	Vertical stress $p$ /MPa	Lateral pressure coefficient $K_0$	Wedge height $h$ /m	Tunnel radius $R$ /m	Rock unit weight $\gamma$ /(kN $\cdot$ m $^{-3}$ )	Friction angle COV $_{\varphi}$	Scales of fluctuation $\theta$ /m
20, 30, 40, 50, 60, 70	20, 30, 40, 50, 60, 70, 80	1, 0.1, 0.01, 0.001	0.1, 0.3, 0.5, 0.7, 0.9	0.5, 1.0, 1.5, 2.0	1, 2, 3, 4, 5	3, 4, 5, 6	25, 26, 27, 28, 29, 30	0.1, 0.3, 0.5, 0.7, 0.9	1, 10, 50, 100

##### 4.2 Simulation results

When the geometric parameters of the tunnel do not satisfy Eq. (9), the tunnel roof will not destroy. Therefore, this situation is not considered in the result statistics. Fig.5 intuitively presents the safety factors obtained from theoretical analysis and numerical simulation, i.e.  $FS_A$  and  $FS_N$ , respectively. It can be seen from Fig. 5 that there is a certain error between the safety factors obtained by theoretical calculation and numerical simulation. This is due to the assumption and simplification of the tunnel wedge failure model selected in this paper. But the error of the calculation results is basically within 30%, and the coefficient of determination  $R^2$  is 0.965, indicating that the calculation method of wedge safety factor deduced in this paper that considers the joint friction angle spatial variability

is in good agreement with the numerical simulation results. Consequently, the method for calculating factor of safety deduced in this paper can be used for the sub-

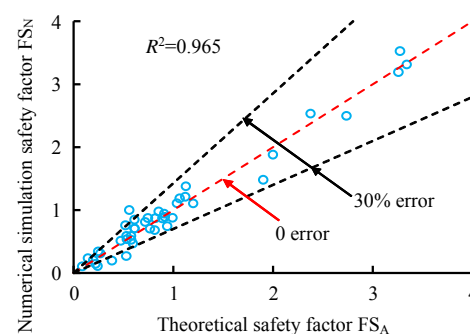


Fig.5 Comparison of  $FS_A$  versus  $FS_N$

sequent calculation of tunnel roof wedge failure probability.

## 5 Monte Carlo simulation to calculate the wedge failure probability

In this paper, the MCS method is used to assess the failure probability of engineering cases. It inputs a large amount of random data  $\hat{X}$  into the analysis model, and then analyzes the probability of event occurrence. In this paper, the failure probability  $P_f$  is defined as follows:

$$P_f = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} I[FS_i(\hat{X}_i) < 1] \quad (22)$$

$$I = \begin{cases} 0, & FS_i \geq 1 \\ 1, & FS_i < 1 \end{cases} \quad (23)$$

where  $N_{MC}$  is the number of MCS;  $FS_i$  represents the factor of safety calculated for the  $i$ -th time; and  $I$  is an indicative function.

As shown in Eq. (22), the failure probability  $P_f$  is related to the number of MCS. Thus, the convergence analysis of failure probability must be carried out when the MCS method is used to calculate the failure probability. As the number of MCS increases, the failure probability will gradually converge to a fixed value. When the coefficient of variation of the failure probability is less than a certain value, the failure probability is the failure probability of tunnel roof wedge. The specific implementation process will be elaborated in Section 6. In this paper, the joint friction angle  $\varphi$  is considered as a random field variable, and geological parameters ( $k_s/k_n$ ,  $p$ ,  $K_0$ ,  $\gamma$ ) and joint geometric parameters ( $\alpha$ ,  $h$ ) are taken as random variables, and the radius of the tunnel is regarded as a constant. In order to avoid negative values in the sampling process, a lognormal distribution is used to describe the parameter uncertainty.

## 6 Computational process of failure probability based on random field theory

In order to facilitate the understanding of the method for calculating tunnel wedge failure probability that considers the spatial variability of joints friction angle proposed in this article, the author uses Fig. 6 to explain the failure probability calculation process in detail, which is mainly divided into 6 steps:

(1) First determine the geological and geometric parameters of the tunnel roof wedge, and then describe the uncertainty of parameters through the mean, coefficient of variation, autocorrelation function and scales of fluctuation.

(2) Generate  $N_{MC}$  sets of random variable samples of geological and geometric parameters through Monte Carlo sampling.

(3) In order to meet the accuracy requirements, the joint  $AB$  is divided into 1 000 micro-element [37], the center coordinates of each micro-element are obtained, and the corresponding correlation matrix  $C$  is generated according to the autocorrelation function through the

coordinate relationship between the micro-elements.

(4) Carry out the Cholesky decomposition of the correlation matrix  $C$  to obtain the triangular matrix  $L$ . Then the lognormal random field of the friction angle  $\varphi$  is gained by Eq. (17).

(5) Based on the relaxation method, the normal force  $N$  and shear force  $S$  are obtained by integrating along the joint  $AB$ , and substituted into Eq.(1) to calculate the factor of safety of the wedge of tunnel roof.

(6) According to Eqs. (22) and (23), Monte Carlo simulation method is used to calculate the failure probability  $P_f$  until the coefficient of variation  $COV_{P_f}$  of  $P_f$  is less than the threshold  $T_{COV}$ .

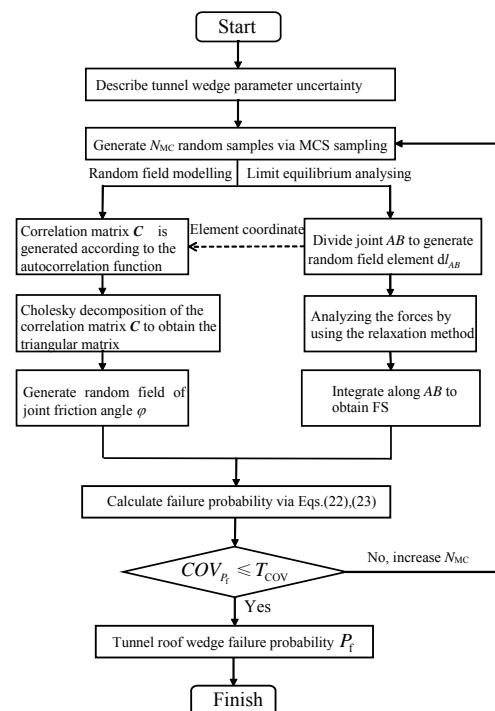


Fig. 6 Flowchart for calculating failure probability of tunnel roof wedge based on random field

## 7 Failure probability analysis of the tunnel roof wedge-shaped body

This section mainly analyzes the system parameters to study the influence of geological and geometric parameters uncertainties on the failure probability of the tunnel roof wedge when considering the joint friction angle spatial variability. The multi-layer perceptron neural network is applied to the parameters sensitivity analysis of tunnel wedge to identify the influence degree of the wedge parameters on the probability of failure. The specific values of the parameters have been given in Table 1.

### 7.1 Influence of geological parameters

Figures 7 to 11 show the influences of the mean value and variation coefficient of different geological parameters on the failure probability of the tunnel vault wedge in different scales of fluctuations ( $\theta$  is 1, 4, 10 m and  $+\infty$ ). From Figs. 7(a), 8(a), 9(a), 10(a), 11(a), it can be seen that with the increase of the mean value of friction angle  $\varphi$ , the ratio of shear to normal stiffness of the joint  $k_s/k_n$ , the vertical in situ stress  $p$  and the



coefficient of horizontal in situ stresses  $K_0$ , the failure probability of the tunnel roof wedge  $P_f$  gradually decreases, but the failure probability increases with the increase of  $\mu_\gamma$ . Figs. 7(b), 8(b), 9(b), 10(b), and 11(b) reflect the influence of parameter COV change on the failure probability, and the increase in the coefficient of variation of  $\varphi$  and  $K_0$  can lead to a significant

increase in the failure probability of tunnel roof wedge-shaped body. The increase in the variation coefficient of  $k_s/k_n$  and  $p$  results in insignificant change in the failure probability of tunnel roof wedge. However, the increase of the  $COV_\gamma$  of rock unit weight will bring a decrease in the failure probability. It can be seen from Eqs. (1) to (8) that the increase of  $\varphi$ ,  $k_s/k_n$ ,  $p$  and  $K_0$

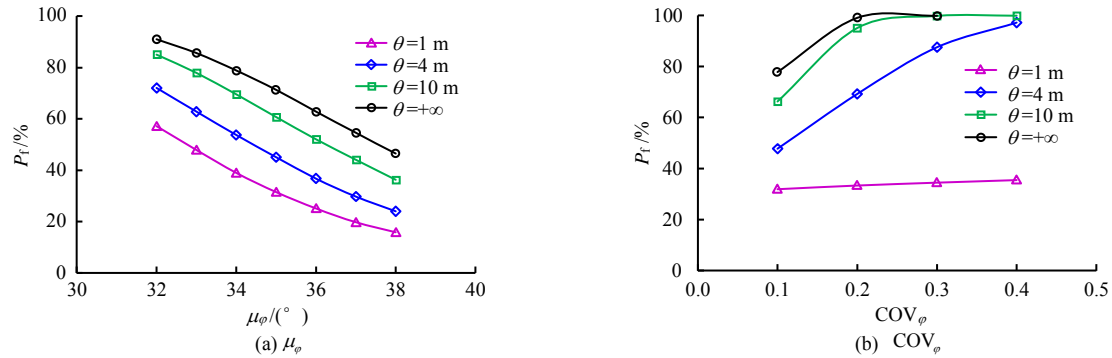


Fig. 7 Influences of  $\varphi$  on failure probability  $P_f$  with different  $\theta$

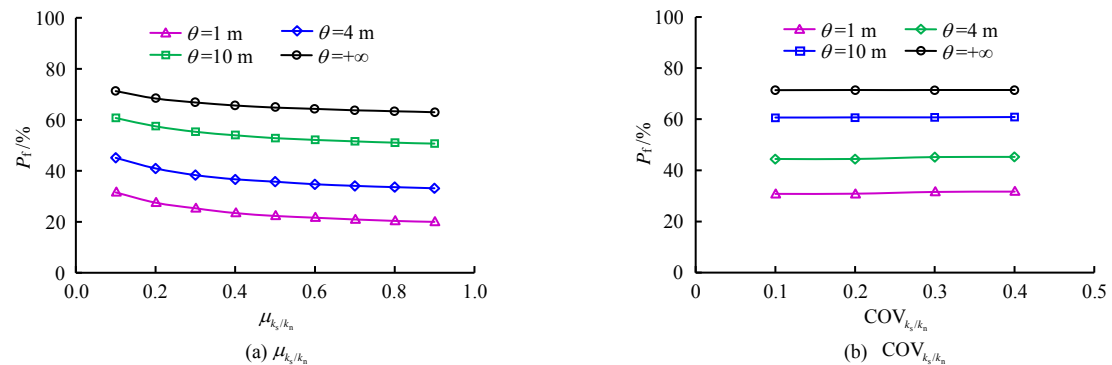


Fig. 8 Influences of  $k_s/k_n$  on failure probability  $P_f$  with different  $\theta$

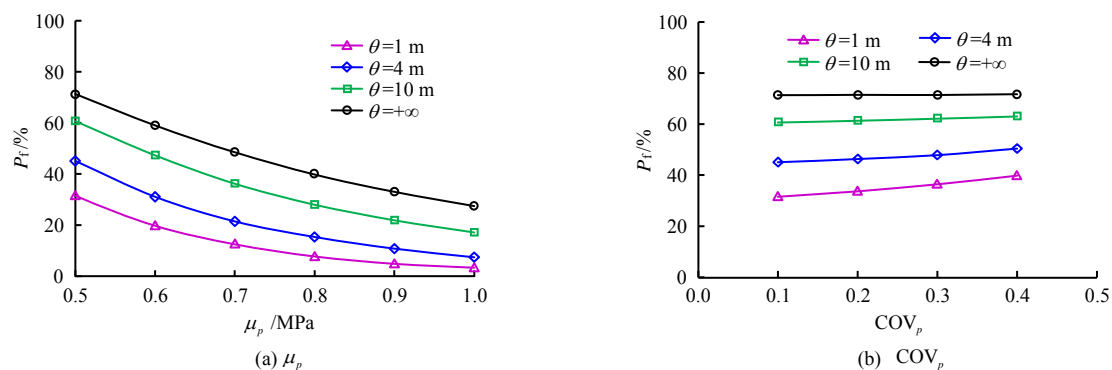


Fig. 9 Influences of  $p$  on failure probability  $P_f$  with different  $\theta$

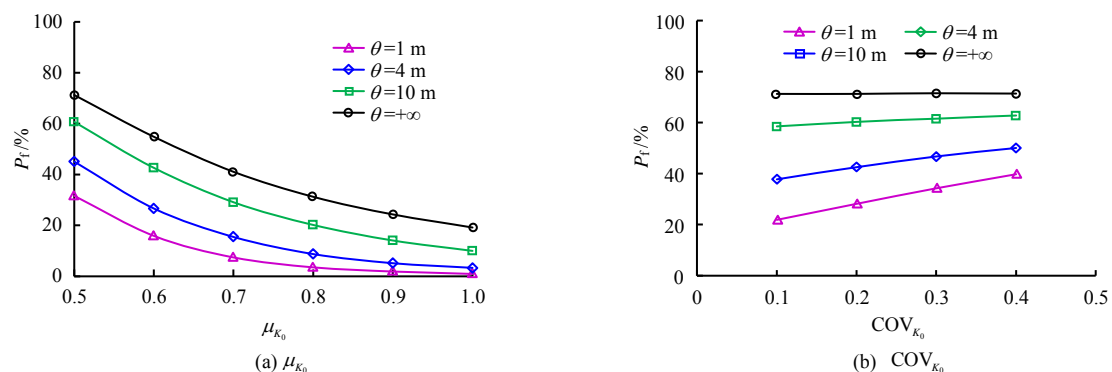


Fig. 10 Influences of  $K_0$  on failure probability  $P_f$  with different  $\theta$

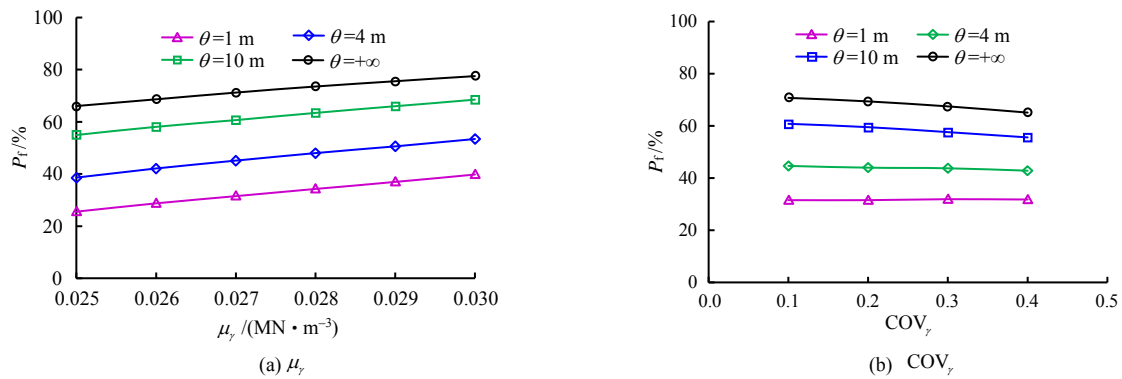


Fig. 11 Influences of  $\gamma$  on failure probability  $P_f$  with different  $\theta$

will increase the sliding resistance of the wedge, improve the stability and reduce the failure probability. An increase in the unit weight  $\gamma$  will increase the weight of the wedge, which leads to an increase in sliding force and a decrease in factor of safety for the wedge, which is not conducive to the stability of the wedge, and therefore the failure probability increases. It can be seen directly from the comparison of Figs. 7–11 that the joint friction angle  $\varphi$  has a greater impact on the failure probability, while the ratio of shear to normal stiffness of the joint  $k_s/k_n$  has the least impact on the failure probability, which is consistent with the law shown in Fig. 2.

Figures 7–11 also show the influence of the scales of fluctuation  $\theta$  on the failure probability of tunnel wedge. The scales of fluctuation  $\theta$  is a key parameter to measure the spatial variability of parameters. The larger the  $\theta$  is, the stronger the spatial correlation of the parameters and the weaker the spatial variability are. When  $\theta$  tends to infinity, the parameters tend to be homogeneous in space. It can be seen from the figure that the probability of failure of the wedge increases as the scales of fluctuation  $\theta$  of the joint friction angle  $\varphi$  increases. When the scales of fluctuation  $\theta$  tends to infinity, the failure probability calculated considering the spatial variability is consistent with the result of traditional spatially homogeneous rock mass. Compared with the failure probability calculated when ignoring spatial variability, considering the spatial variability of joint friction angle will reduce the failure probability to a certain extent. Hence, the traditional reliability analysis method ignores the inherent spatial variability of rock mass parameters when calculating the failure probability, which causes the calculation result to be larger and cannot truly reflect the failure risk to tunnel wedge, and leads to conservative support design in tunnel engineering, and the economic effectiveness of the support plan is reduced.

## 7.2 Influence of joint geometric parameters

The structural failure mode of tunnel rock mass is mainly controlled by the occurrence of rock mass structural plane. In the classic failure model of the wedge of the tunnel roof selected in this paper (Fig. 1), the semi-apical angle  $\alpha$  and the center height  $h$  are two key parameters that control the occurrence of rock mass structural plane and the distance between the wedge and the tunnel roof. Due to the uncertainty of the length

and direction of the joint and the measurement error itself, this paper treats the geometric parameters of the joint (semi-apical angle  $\alpha$  and center height  $h$ ) as random variables to study the influence of geometric parameter uncertainties of the joint on the failure probability of the wedge. Figs. 12 and 13 show the influence of the semi-apical angle  $\alpha$  and the center height  $h$  on the failure probability of tunnel roof wedge under different mean values and coefficients of variation. The influences of semi-apical angle  $\alpha$  and center height  $h$  on the trend of failure probability are similar. As the mean values of  $\alpha$  and  $h$  increase, the failure probability increases. When the coefficients of variation of  $\alpha$  and  $h$  increase, the failure probability decreases. From Eqs.(1)–(8), it can be noted that the increases of semi-apical angle  $\alpha$  and center height  $h$  result in an increase in the volume of the wedge, thereby reducing the stability of tunnel roof. Combining Figs. 12 and 13, it can be seen that the semi-apical angle  $\alpha$  has a slightly greater influence on the failure probability than the center height  $h$ . Therefore, in engineering surveys, the occurrence of rock mass structural planes should be accurately measured to minimize measurement errors. Figs. 12 and 13 also present the influence of scales of fluctuation on the failure probability, which is similar to the conclusion of Section 7.1, thus it will not repeat it here.

## 7.3 Influence of scales of fluctuation

Figure 14 further shows the influence of the scales of fluctuation on the failure probability under the coefficients of variation of the friction angle of different joints. As the scales of fluctuation of the joint friction angle increases, the failure probability of the wedge increases sharply first, and then slowly increases. After the scales of fluctuation  $\theta > 30$  m, the failure probability gradually approaches the spatially homogeneous failure probability. Just like the random field of joint friction angle  $\varphi$  shown in Fig. 3, when the scales of fluctuation becomes larger, the spatial variability of joint friction angle  $\varphi$  will become weaker and weaker, and its spatial correlation will be stronger, and its spatial distribution gradually tends to a mean value. When the coefficient of variation of joint friction angle is larger, the influence range of spatial variability on failure probability is also larger. More attention should therefore be paid to the spatial variability of rock mass parameters for surround-



ing rocks with larger variability. In summary, the spatial variability of the joint friction angle  $\varphi$  cannot be ignored in the failure probability calculation of the tunnel roof wedge, otherwise the failure probability of the wedge will be seriously overestimated, and the failure risk of

the tunnel wedge cannot be truly reflected. Inaccurate risk analysis of the tunnel wedge has led to the conservative design of the tunnel engineering, resulting in unnecessary waste of supporting materials and reducing the economics of the supporting scheme.

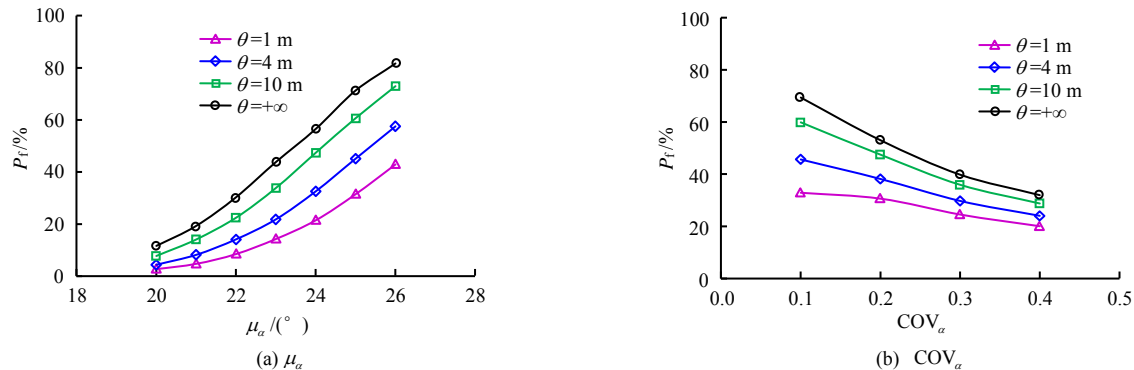


Fig. 12 Influences of  $\alpha$  on failure probability  $P_t$  with different  $\theta$

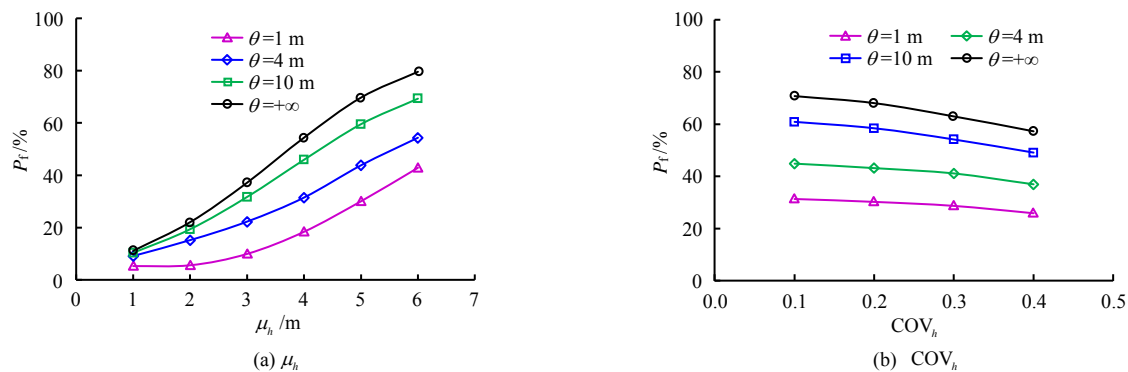


Fig. 13 Influences of  $h$  on failure probability  $P_t$  with different  $\theta$

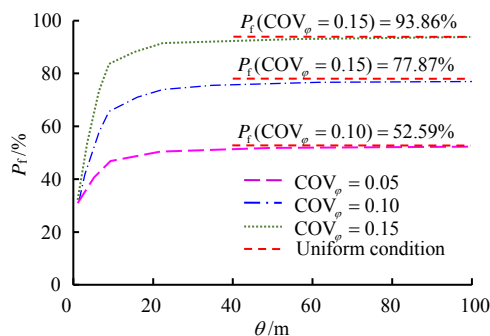


Fig. 14 Influences of  $\theta$  on failure probability  $P_t$  with different  $COV_\varphi$

#### 7.4 Influence of supporting force

To improve the stability of tunnel roof and reduce the failure probability of tunnel, reinforcement of shotcrete is often used to provide an upward supporting force for the tunnel roof to ensure the tunnel stability. However, most of the current theories on tunnel support are premised on the isotropic elastoplastic theory, which obviously does not apply to spatially variable rock masses [22]. When considering both support and spatial variability of tunnel risk analysis, the numerical simulation is an effective way to simulate the spatial variability of rock masses. However, because this paper considers the uncertainty of rock mass structural planes

occurrence (semi-apical angle  $\alpha$  and center height  $h$ ), the establishment of the corresponding numerical model is very complicated, and the use of numerical simulation for huge amount of Monte Carlo sampling calculation is also very time-consuming.

To analyze the failure probability of the tunnel roof wedge under the support condition, this paper adopts an approach similar to the literature [22]: simplify the interaction between the bolt, concrete, and rock mass, and introduce the upward support force  $T$ . The support force  $T$  is regarded as a constant, and the factor of safety can be calculated by the following formula:

$$FS_T = \frac{2S \cos \alpha + T}{2N \sin \alpha + W} \quad (24)$$

The calculation result is shown in Fig. 15. With the increase of the support force  $T$ , the failure probability decreases significantly. When the scales of fluctuation increases, the probability of failure of the wedge increases. If the impact of spatial variability on the project can be fully considered in the engineering design, then fewer supporting materials such as bolts and concrete can be used to achieve an ideal desired supporting effect when designing the tunnel roof support plan. For example, when the scales of fluctuation is close to infinity, providing a support force of 0.6 MN will make the failure probability of the wedge less than 0.2%, while when

the scales of fluctuation is 1 m, only 0.4 MN of the support force needs to be provided. When the scales of fluctuation tends to infinity, the friction angle of rock mass joints tends to be homogeneous in space, and the random field of rock mass parameters degenerates into a traditional random variable model. Consequently, the traditional reliability analysis methods commonly used in tunnel engineering will overestimate the actual failure probability of the tunnel roof wedge, resulting in a conservative design plan for the tunnel support, and unnecessary support material waste. As a result, the economic efficiency of the tunnel support plan is reduced. Therefore, in actual engineering, the spatial variability of rock mass materials should be fully considered, and the spatial variability of rock mass parameters should be accurately described, so as to truly reflect the failure probability of tunnel, and propose a robust tunnel support design scheme balancing between safety and economy.

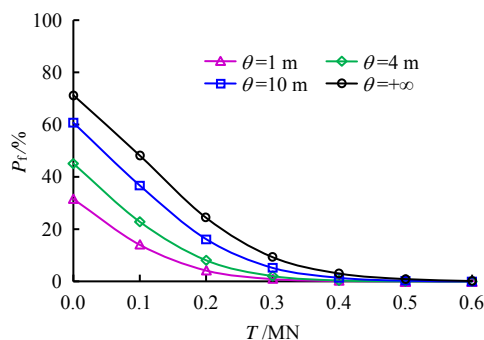


Fig. 15 Influences of  $T$  on failure probability  $P_f$  with different  $\theta$

### 7.5 Sensitivity analysis

In order to understand the degree of influence of the mean value and coefficient of variation of each parameter of the wedge on the failure probability, this paper uses a multilayer perceptron neural network to analyze the sensitivity of the parameters. The specific principle of the multilayer perceptron neural network can be referred to the literature [38], and its principle is not repeated here. This method uses relative importance and feature importance to evaluate the sensitivity of parameters.

The degrees of influence of 15 geological and geometric parameters on the tunnel roof failure probability ranked in Fig. 16 from largest to smallest are  $\mu_\alpha$ ,  $\mu_h$ ,  $\mu_\phi$ ,  $\mu_{K_0}$ ,  $\text{COV}_\phi$ ,  $\theta$ ,  $\text{COV}_\alpha$ ,  $\mu_p$ ,  $\text{COV}_h$ ,  $\mu_\gamma$ ,  $\text{COV}_{K_0}$ ,  $\mu_{k_s/k_n}$ ,  $\text{COV}_p$ ,  $\text{COV}_\gamma$  and  $\text{COV}_{k_s/k_n}$ . As mentioned in Section 7.2, the semi-apical angle  $\alpha$  and center height  $h$  will directly affect the quality of the wedge, leading to a significant change in the sliding force, and directly affecting the stability of the tunnel roof wedge, so  $\mu_\alpha$  and  $\mu_h$  have the greatest impact on the probability of failure. The coefficients of variation of different parameters have different effects on the failure probability, and the change in  $\text{COV}_\phi$  has the greatest impact. By visually comparing the importance of each parameter to the failure probability when

considering the spatial variability in Fig.16, it can be seen that the importance index of the scales of fluctuation  $\theta$  of the joint friction angle is greater than the coefficients of variation of  $\alpha$ ,  $h$ ,  $K_0$ ,  $p$  and  $\gamma$ . The influence of the scales of fluctuation  $\theta$  of joint friction angle on the failure probability is second only to the variability of joint friction angle. The sensitivity analysis in Fig. 16 further emphasizes that the spatial variability of rock mass parameters should be paid attention to in the reliability analysis and design of tunnel engineering.

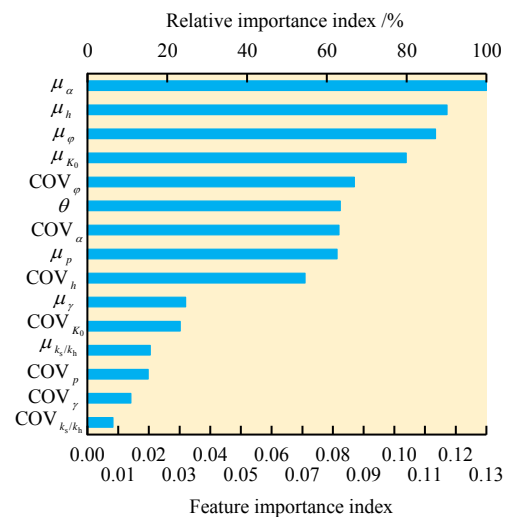


Fig. 16 Sensitivity analysis

## 8 Conclusion

Based on random field theory and limit equilibrium analysis, a method is developed in this paper to calculate the factor of safety for tunnel roof wedge considering the joint friction angle spatial variability. The influence of tunnel roof wedge parameters on the failure probability of the wedge is systematically studied, and the sensitivity analysis of tunnel roof wedge parameters is carried out with neural network. The main conclusions are as follows:

(1) The calculation formula for the factor of safety of the wedge considering the spatial variability of joint friction angle is deduced. When the friction angle of each joint micro-element is equal, the calculation result by the traditional method for the factor of safety is the same as that by the formula proposed in this study. The accuracy of the formula is verified by the discrete element numerical modeling. This calculation method considering the spatial variability of rock mass joints friction angle is effective.

(2) A method for calculation of failure probability considering the spatial variability of rock mass joints friction angle is developed. The concept of the method is simple and easy to implement. Based on this calculation method, the parametric analysis and sensitivity analysis of the wedge geological and geometric parameters are systematically carried out. It is found that  $\mu_\alpha$ ,  $\mu_h$ ,  $\mu_\phi$ ,  $\mu_{K_0}$ ,  $\text{COV}_\phi$ ,  $\theta$ ,  $\text{COV}_\alpha$  and  $\mu_p$  have a greater impact on the failure probability of tunnel roof wedge.

(3) The spatial variability of the rock mass joint friction angle has a significant impact on the failure probability of the wedge. When considering the spatial variability of the rock mass joints friction angle, the calculated failure probability of the tunnel roof wedge is less than the failure probability calculated by the traditional reliability analysis considering rock mass as homogeneous. Therefore, when evaluating the stability of the wedge in the engineering design, the natural spatial variability of the rock mass should be accurately described, and the failure risk of the wedge should be truly reflected, so as to design a reasonable support plan and take into account the safety of the tunnel design and construction and push the tunnel engineering design efficiency and economy in the direction of robustness and refinement.

## References

- [1] FRALDI M, GUARRACINO F. Limit analysis of collapse mechanisms in cavities and tunnels according to the Hoek-Brown failure criterion[J]. *International Journal of Rock Mechanics and Mining Sciences*, 2009, 46(4): 665–673.
- [2] CHEN F, WANG L, ZHANG W. Reliability assessment on stability of tunnelling perpendicularly beneath an existing tunnel considering spatial variabilities of rock mass properties[J]. *Tunnelling and Underground Space Technology*, 2019, 88: 276–289.
- [3] ZHANG R, CHEN G, ZOU J, et al. Study on roof collapse of deep circular cavities in jointed rock masses using adaptive finite element limit analysis[J]. *Computers and Geotechnics*, 2019, 111: 42–55.
- [4] HOU Gong-yu, XIE Bing-bing, JIANG Yu-sheng, et al. Theoretical and experimental study of the relationship between optical fiber strain and settlement of roof based on BOTDR technology[J]. *Rock and Soil Mechanics*, 2017, 38(5): 1298–1304.
- [5] ASADOLLAHI P, TONON F. Definition of factor of safety for rock blocks[J]. *International Journal of Rock Mechanics and Mining Sciences*, 2010, 8(47): 1384–1390.
- [6] LI T, YANG X. Limit analysis of failure mechanism of tunnel roof collapse considering variable detaching velocity along yield surface[J]. *International Journal of Rock Mechanics and Mining Sciences*, 2017, 100: 229–237.
- [7] WANG H T, WANG L G, LI S C, et al. Roof collapse mechanisms for a shallow tunnel in two-layer rock strata incorporating the influence of groundwater[J]. *Engineering Failure Analysis*, 2019, 98: 215–227.
- [8] HUANG Z, BROCH E, LU M. Cavern roof stability-mechanism of arching and stabilization by rockbolting[J]. *Tunnelling and Underground Space Technology*, 2002, 17(3): 249–261.
- [9] INDRARATNA B, OLIVEIRA D A F, BROWN E T, et al. Effect of soil-infilled joints on the stability of rock wedges formed in a tunnel roof[J]. *International Journal of Rock Mechanics and Mining Sciences*, 2010, 47(5): 739–751.
- [10] WU R, XU J H, LI C, et al. Stress distribution of mine roof with the boundary element method[J]. *Engineering Analysis with Boundary Elements*, 2015, 50: 39–46.
- [11] NAPA-GARCÍA G F, SANTOS R A, BECK A T, et al. Improvement of analytical factor of safety estimation of falling failure mode in roof wedge stability[J]. *International Journal of Rock Mechanics and Mining Sciences*, 2018, 103: 116–122.
- [12] QIN C, CHIAN S C. 2D and 3D stability analysis of tunnel roof collapse in stratified rock: a kinematic approach[J]. *International Journal of Rock Mechanics and Mining Sciences*, 2017, 100: 269–277.
- [13] CHEN F, ZHANG R, WANG Y, et al. Probabilistic stability analyses of slope reinforced with piles in spatially variable soils[J]. *International Journal of Approximate Reasoning*, 2020, 122: 66–79.
- [14] ZHANG J, WANG H, HUANG H W, et al. System reliability analysis of soil slopes stabilized with piles[J]. *Engineering Geology*, 2017, 229: 45–52.
- [15] LIU L L, CHENG Y M. System reliability analysis of soil slopes using an advanced kriging metamodel and quasi-Monte Carlo simulation[J]. *International Journal of Geomechanics*, 2018, 18(8): 06018019.
- [16] ZHENG F, JIAO Y Y, Sitar N. Generalized contact model for polyhedra in three-dimensional discontinuous deformation analysis[J]. *International Journal for Numerical and Analytical Methods in Geomechanics*, 2018, 42(13): 1471–1492.
- [17] ZHENG F, LEUNG Y F, ZHU J B, et al. Modified predictor-corrector solution approach for efficient discontinuous deformation analysis of jointed rock masses[J]. *International Journal for Numerical and Analytical Methods in Geomechanics*, 2019, 43(2): 599–624.
- [18] LI D Q, WANG L, CAO Z J, et al. Reliability analysis of unsaturated slope stability considering SWCC model selection and parameter uncertainties[J]. *Engineering Geology*, 2019, 260: 105207.
- [19] LOW B K, EINSTEIN H H. Reliability analysis of roof wedges and rockbolt forces in tunnels[J]. *Tunnelling and*

- Underground Space Technology, 2013, 38: 1–10.
- [20] LUO W, LI W. Reliability analysis of supporting pressure in tunnels based on three-dimensional failure mechanism[J]. *Journal of Central South University*, 2016, 23(5): 1243–1252.
- [21] YANG X L, ZHOU T, LI W T. Reliability analysis of tunnel roof in layered Hoek-Brown rock masses[J]. *Computers and Geotechnics*, 2018, 104: 302–309.
- [22] LIU H, LOW B K. Reliability-based design of tunnelling problems and insights for Eurocode 7[J]. *Computers and Geotechnics*, 2018, 97: 42–51.
- [23] ZHOU X P, HUANG X C, LIU P F, et al. A probabilistic method to analyze collapse failure of shallow rectangular tunnels[J]. *Tunnelling and Underground Space Technology*, 2018, 82: 9–19.
- [24] LI Shu-cai, HE Peng, LI Li-ping, et al. Reliability analysis method of sub-classification of tunnel rock mass and its engineering application[J]. *Rock and Soil Mechanics*, 2018, 39(3): 967–976.
- [25] XIAHOU Yun-shan, ZHANG Shu, TANG Hui-ming, et al. Study of structural cross-constraint random field simulation method considering spatial variation structure of parameters[J]. *Rock and Soil Mechanics*, 2019, 40(12): 4935–4945.
- [26] JIANG Shui-hua, LIU Yuan, ZHANG Hao-long, et al. Quantitatively evaluating the effects of prior probability distribution and likelihood function models on slope reliability assessment[J]. *Rock and Soil Mechanics*, 2020, 41(9): 3087–3097.
- [27] XUE Yang, WU Yi-ping, MIAO Fa-sheng, et al. Seepage and deformation analysis of Baishuihe landslide considering spatial variability of saturated hydraulic conductivity under reservoir water level fluctuation[J]. *Rock and Soil Mechanics*, 2020, 41(5): 1709–1720.
- [28] LÜ Q, XIAO Z, ZHENG J, et al. Probabilistic assessment of tunnel convergence considering spatial variability in rock mass properties using interpolated autocorrelation and response surface method[J]. *Geoscience Frontiers*, 2018, 9(6): 1619–1629.
- [29] BJURELAND W, JOHANSSON F, SPROSS J, et al. Influence of spatially varying thickness on load-bearing capacity of shotcrete[J]. *Tunnelling and Underground Space Technology*, 2020, 98: 103336.
- [30] WANG Chuan, LENG Xian-lun, LI Hai-lun, et al. Probabilistic stability analysis of underground caverns considering spatial variation of joint distribution[J]. *Rock and Soil Mechanics*, 2021, 42(1): 224–233.
- [31] BRADY B H G, BROWN E T. *Rock mechanics for underground mining*[M]. Berlin: Springer Science & Business Media, 1993.
- [32] LIU L L, DENG Z P, ZHANG S, et al. Simplified framework for system reliability analysis of slopes in spatially variable soils[J]. *Engineering Geology*, 2018, 239: 330–343.
- [33] LUO Z, ATAMTURKTUR S, CAI Y, et al. Simplified approach for reliability-based design against basal-heave failure in braced excavations considering spatial effect[J]. *Journal of Geotechnical and Geoenvironmental Engineering*, 2012, 138(4): 441–450.
- [34] HALDAR S, BABU G L S. Effect of soil spatial variability on the response of laterally loaded pile in undrained clay[J]. *Computers and Geotechnics*, 2008, 35(4): 537–547.
- [35] LI D Q, JIANG S H, CAO Z J, et al. A multiple response-surface method for slope reliability analysis considering spatial variability of soil properties[J]. *Engineering Geology*, 2015, 187: 60–72.
- [36] Itasca Consulting Group Inc. UDEC version 6.0 universal distinct element code[R]. [S. l.]: Itasca Consulting Group Inc., 2014.
- [37] CHING J, PHOON K K. Effect of element sizes in random field finite element simulations of soil shear strength[J]. *Computers & Structures*, 2013, 126: 120–134.
- [38] ZENG X, YEUNG D S. Sensitivity analysis of multilayer perceptron to input and weight perturbations[J]. *IEEE Transactions on Neural Networks*, 2001, 12(6): 1358–1366.