Rock and Soil Mechanics

Volume 42 | Issue 2 Article 8

6-18-2021

# A multiparameter non-proportional shear strength reduction method for slope stability analysis based on energy evolution theory

Feng LU
*School of Civil Engineering, Southwest Jiaotong University, Chengdu, Sichuan 610031, China*

Wen-ge QIU
*School of Civil Engineering, Southwest Jiaotong University, Chengdu, Sichuan 610031, China*, qiuwen_qw@163.com

## Custom Citation

LU Feng, QIU Wen-ge, . A multiparameter non-proportional shear strength reduction method for slope stability analysis based on energy evolution theory[J]. Rock and Soil Mechanics, 2021, 42(2): 547-557.

Rock and Soil Mechanics 2021 42(2): 547−557
https: //doi.org/10.16285/j.rsm.2020.5435

ISSN 1000−7598
rocksoilmech.researchcommons.org/journal

# A multiparameter non-proportional shear strength reduction method for slope stability analysis based on energy evolution theory

LU Feng[1, 2],  QIU Wen-ge[1, 2]

1. Key Laboratory of Transportation Tunnel Engineering of the Ministry of Education, Southwest Jiaotong University, Chengdu, Sichuan 610031, China
2. School of Civil Engineering, Southwest Jiaotong University, Chengdu, Sichuan 610031, China

**Abstract:** In the failure process of slopes, the contribution of different mechanical or material parameters to the stability is different and dynamic. In the context of shear strength reduction method (SRM) for stability analysis, it is of great significance to determine the factor of safety (FOS) by considering non-proportional reduction for these different mechanical or material parameters. This study firstly examines the mechanism of energy evolution in the process of slope failure. Then, the contribution of different mechanical parameters to dissipated energy evolution is weighted in conducting SRM, based on which we proposed a new method to calculate the FOS with multiparameter non-proportional reduction of shear strength parameters. This method can reflect the characteristic of FOS following the whole reduction path of different shear strength parameters. Subsequently, the proposed method is verified by examples. Finally, the influence of the reduction step and reduction path of the non-proportional reduction factors on the calculated FOS results is discussed.

**Keywords:** energy evolution; shear strength reduction (SRM); factor of safety (FOS); degradation; reduction path

## 1 Introduction

It is well known that the methods commonly used for slope stability analysis mainly include limit analysis method (upper limit method and lower limit method), limit equilibrium method and shear strength reduction method (SRM). The strength reduction technique was first proposed and applied by Zienkiewicz et al.[1] in 1975, and then named the SRM by Matsui et al. [2] in 1992. Compared with the limit analysis method and the limit equilibrium method, SRM has the following advantages: on the one hand, it does not need to presuppose the shape or position of the fracture surface and the internal force between the bars, it can monitor the entire destruction process; on the other hand, complex conditions such as seismic forces, pore water pressure and supporting structures can be calculated. With the improvement of computer performance and the development of numerical calculation software, SRM has been widely used by many researchers in the stability analysis of geotechnical engineering[3–9].

Based on the Mohr-Coulomb strength criterion and the concept of strength reserve, the traditional SRM usually adopts a proportional reduction method by which the strength parameter cohesion $c$ and the internal friction angle $\varphi$ use the same reduction factor to obtain the new cohesive force and internal friction angle, then gradually increase the reduction factor to make the slope reach the limit state, and the reduction factor at this time is defined as the factor of safety (FOS). In fact, the strength parameters($c$ and $\varphi$) of the slope in the failure process have different and dynamic contributions to maintain the stability of the slope[10–11]. A large number of tests have shown that the degradation rate and degree of cohesive force $c$ and internal friction angle $\varphi$ of geotechnical materials are different when they deteriorate[12–19]. When using SRM for stability analysis, the difference of different mechanical parameters of materials should be considered in the reduction process, that is, the non-proportional reduction between the parameters. In addition, a variety of geotechnical materials or supporting structures are often involved in geotechnical engineering, and the attenuation rate and degree of mechanical parameters among different geotechnical materials, geotechnical materials and supporting materials are not the same. In the traditional SRM, a variety of geotechnical materials are often reduced in equal proportions and the reduction of supporting materials is often not considered; or the rock and soil mass and supporting structure are reduced in equal proportions without considering the difference between materials in the process of reduction. Therefore, it is of great significance to study the multiparameter non-proportional reduction for different mechanical parameters of the same material and different materials. The number of reduction parameters can be two or more, depending on the selected strength criterion[20] and the number of reduced materials.

The research on the two-parameter non-proportional reduction method (also known as the double reduction method) currently focuses on homogeneous slopes. Scholars have achieved certain research results. As early as 1948, Taylor[21] believed that $c$ and $\varphi$ should

---

Received: 16 April 2020 Revised: 5 November 2020
This work was supported by the National Key R&D Program of China (2017YFC0806000), the National Natural Science Foundation of China (51991395, U1434206, 51678497).

Fist author: LU Feng, male, born in 1989, PhD candidate, mainly engaged in safety evaluation and disaster control technology research in the whole process of tunnel life. E-mail: fenglu0901@foxmail.com
Corresponding author: QIU Wen-ge, male, born in 1959, PhD, Professor, PhD supervisor, mainly engaged in the study of mechanical behavior and control technology in the whole process of tunnel life。E-mail: qiuwen_qw@163.com

adopt different factors of safety, and proposed that the working order and degree of $c$ and $\varphi$ are different when the slope is sliding. Tang et al.[22], Jiang et al.[23], also believed that the attenuation speed and degree of $c$ and $\varphi$ are different, and proposed that the strength parameters ($c$ and $\varphi$) should adopt different reduction factors to reflect the contribution to the slope stability, thus different reduction factors or factors of safety should be used. Bai et al.[20] proposed a new framework for defining factors of safety reference slopes, which theoretically discussed the relationship between the double reduction method and the traditional SRM, providing a theoretical basis for the study of multi-parameter non-proportional reduction. This section summarizes the representative research results of the non-proportional SRM for homogeneous slopes in recent years, as shown in Table 1. Among them, the mathematical average method obtains the reduction factors of $c$ and $\varphi$ firstly when the slope is reduced from the initial state to the instability failure, then the average value is processed to get the comprehensive FOS. Its physical significance needs to be further clarified. The strength reduction path method obtains the reduction path diagram of $c$ and $\varphi$ by calculation, and then derives the calculation formula of the comprehensive FOS from special cases to general conditions according to the length of the reduction path. In the limit equilibrium method, the FOS for a circular slip surface is actually the ratio of the anti-slide moment to the slide moment. Although the FOS possesses a clear physical meaning, it needs to presume the shape and position of the slip surface, which is artificial and cumbersome to calculate. For non-circular arc and non-linear sliding surface, the shear force on the sliding surface is neither the vector sum of the force in space nor the algebraic sum of the force projected in a certain direction. Therefore, its rationality has been questioned by some scholars[8]. In the reference slope method, the critical maximum common tangent line is calculated, and then the ratio of the sum of the initial shear strength at all common tangent points of the slope to be assessed and the sum of the reduced shear strength at all common tangent points of the critically damaged slope is defined as the comprehensive FOS. The method has clear physical meaning and provides new ideas for the study of non-proportional shear strength reduction.

**Table 1 Definitions of FOS using double non-proportional reduction method**

| Method | Comprehensive definition of FOS | Literature source |
|---|---|---|
| Mathematical average method | $\mathrm{FOS}=(\mathrm{SRF}^c+\mathrm{SRF}^\varphi)/2$ | Tang et al.(2007)[22], Jiang et al.(2013)[23] |
| | $\mathrm{FOS}=2/(1/\mathrm{SRF}^c+1/\mathrm{SRF}^\varphi)$ | Wu et al.(2018)[24] |
| | $\mathrm{FOS}=\sqrt{\mathrm{SRF}^c\cdot\mathrm{SRF}^\varphi}$ | Yuan et al.(2016)[25],Zhu et al.(2018)[26] |
| | $\mathrm{FOS}=\sqrt{(\mathrm{SRF}^{c2}+\mathrm{SRF}^{\varphi 2})/2}$ | Deng etc.(2019)[11] |
| Shear strength reduction path method | $\mathrm{FOS}=\sqrt{2}\mathrm{SRF}^c\mathrm{SRF}^\varphi\sqrt{\mathrm{SRF}^{c2}+\mathrm{SRF}^{\varphi 2}}$ | Yuan et al.(2013)[27] |
| | $\mathrm{FOS}=1/\left[1-\sqrt{(1-1/\mathrm{SRF}^c)^2+(1-1/\mathrm{SRF}^\varphi)^2}/\sqrt{2}\right]$ | Isakov et al.(2014)[28] |
| Limit equilibrium method | $\mathrm{FOS}=\min\limits_{s\in S}\int_S[\tau]\mathrm{d}l\Big/\int_S\tau\mathrm{d}l=\int(c+\sigma\tan\varphi)\mathrm{d}l\Big/\int(c/SRF^c+\sigma\tan\varphi/\mathrm{SRF}^\varphi)\mathrm{d}l$ | Zheng et al.(2005)[8],Wu etc.(2018)[24] |
| Reference slope method | $\mathrm{FOS}=\sum\tau_0\Big/\sum\tau_c=2c+\tan\varphi_0\left[\sum\limits_{i=1}^{N}\sigma_{o_i}-\sin\varphi_c\sum\limits_{i=1}^{N}r_i\right]\Big/\cos\varphi_c\sum\limits_{i=1}^{N}r_i$ | Bai et al.(2014，2015)[29, 20] |

Note: $\mathrm{SRF}^c$ is the cohesion reduction factor; $\mathrm{SRF}^\varphi$ is the internal friction angle reduction factor; $S$ is the set of all potential slip lines of the slope; $s$ is a specific slip line in the set $S$; $\tau$ is the shear stress on $s$; $[\tau]$ is the shear strength that meets the Mohr-Coulomb criterion on $s$; $\tau_0$ and $\tau_c$ are the sum of the shear strength of the Mohr-Coulomb strength line and the tangent point of the stress circle at initial state and failure, respectively; and other parameters are detailed in the corresponding literature.

From above results, it is not difficult to find that the existing non-proportional reduction method usually sets a reduction ratio $k$ ($k=\mathrm{SRF}^c\big/\mathrm{SRF}^\varphi$) in advance, and then reduces a stable slope to the limit state at a time according to the reduction ratio $k$. The key idea is to find a limit state which is the reference state[20] under a given reduction ratio, and then use the method in Table 1 to calculate the FOS of the current state relative to the limit state, which reflects the current state of the slope relative to the safety reserve of the limit state, while little attention is paid to the calculation of FOS evolving with the reduction path. In fact, as mentioned above, under the influence of external environment, the mechanical parameters of slope rock and soil and supporting materials usually undergo different degrees and dynamic changes of degradation processes, and the evolution process of the slope failure exists objectively. Corresponding to parameter reduction, on the one hand, the process of parameter degradation is actually the process of parameter reduction[20]; on the other hand, the path of different parameter reduction in the process of parameter reduction is variable, that is, it is not necessarily linear, the contribution of different parameters to slope stability in the whole reduction process is different and dynamic. In the process of the slope reduces from the initial state to the limit state, the FOS evolves dynamically along the reduction path. In addition, the engineers and technicians not only need to learn about the FOS of slopes at current stage, but also need to understand its evolution mechanism over time, so that they can choose the right time to carry out targeted reinforcement. Therefore, from the perspective that the slope serviceability capacity may change from time to time, the study of the evolution mechanism of FOS is of both engineering and academic significance.

Essentially, the slope failure is an energy-driven process[30] and a process of gradual destruction. When the mechanical parameters of rock-soil or supporting materials deteriorate, the stress and strain within the slope system will be redistributed and energy will evolve, which conforms to the law of energy conservation. Compared with the commonly used three failure criteria i.e., plastic zone penetration, non-convergence calculation and characteristic point displacement mutation, the use of energy evolution criterion to evaluate slope stability is more accurate and has clear physical meaning[30–31] when performing the strength reduction. In view of the above research background, this paper firstly analyzes the energy evolution law of slope system when the shear strength is reduced based on the energy evolution theory, then takes the contribution of different mechanical parameters to the evolution of dissipated energy in the reduction process as the weight of different reduction factors. Based on the definition framework of factors of safety of reference slope [20], a method for getting the FOS of multiparameter non-proportional reduction considering the correlation of the parameter reduction process is proposed. This method can obtain the characteristic of FOS along the reduction path, and the weight of contribution of mechanical parameters. Then, a numerical example is given to verify the rationality of the proposed method. Finally, the effect of reduction step size and reduction path on the calculated FOS is discussed.

## 2 Method for solving the FOS of multiparameter non-proportional shear strength reduction

### 2.1 Energy analysis principle of failure process of slopes

Under external load and self-weight stress, the energy of slope system is in a dynamic equilibrium process of transformation and dissipation. The work done by the external load to the material, that is, the input energy, part of which is stored in the form of elastic strain energy, and the other part is transformed into dissipated energy through damage and deformation. Since the capacity of slope to store elastic strain energy and dissipate energy through deformation is limited, when the total input energy is greater than the energy that the slope can withstand, the excess energy will be converted into kinetic energy. At this time, the slope reaches the limit equilibrium state and sliding failure occurs [32]. The process of slope energy input, transmission, conversion and dissipation always maintains dynamic balance and obeys the law of conservation of energy, namely

$$E^{\text{g}} = E^{\text{e}} + E^{\text{d}} + E^{\text{k}} \tag{1}$$

where $E^{\text{g}}$ is the work done by dead-weight stress; $E^{\text{e}}$ is the elastic strain energy; $E^{\text{d}}$ is the dissipative energy, and $E^{\text{k}}$ is the kinetic energy.

The failure of slope is essentially a process of energy dissipation. In the process of slope system transition from a stable state to a critical state, the material deformation continues to increase, and the material continuously damage in the system until slope failure occurs. Considering the reduction of strength parameter $c$ and $\varphi$, for strength reduction once, the energy evolution of the slope system can be expressed as [30–31]

$$\Delta E^{\text{g}} = \Delta E^{\text{e}} + \Delta E^{\text{d}} + \Delta E^{\text{k}} \tag{2}$$

where $\Delta E^{\text{g}}$, $\Delta E^{\text{e}}$, $\Delta E^{\text{d}}$ and $\Delta E^{\text{k}}$ are the increments of gravitational potential energy, elastic strain energy, dissipative energy and kinetic energy after strength reduction for once, respectively.

For the slope system under consideration, all kinds of energy can be calculated as follows [30–31]:

$$\Delta E^{\text{g}} = \int_V \rho g \Delta h \mathrm{d}V \tag{3}$$

$$\Delta E^{\text{e}} = \int_V \frac{1}{2} \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^{\text{e}} \mathrm{d}V \tag{4}$$

$$\Delta E^{\text{d}} = \int_V (U^{\text{total}} - \frac{1}{2} \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^{\text{e}}) \mathrm{d}V \tag{5}$$

$$\Delta E^{\text{k}} = \int_V \frac{1}{2} \rho \upsilon^2 \mathrm{d}V \tag{6}$$

where $\rho$ is the density; $g$ is the acceleration of gravity; $\Delta h$ is the height change of the unit centroid in the direction of gravity; $\boldsymbol{\sigma}$ is the stress tensor; $\boldsymbol{\varepsilon}^{\text{e}}$ is the elastic strain tensor, $\boldsymbol{\sigma} : \boldsymbol{\varepsilon}^{\text{e}} = \sigma_{ij} \varepsilon_{ij}^{\text{e}}$; $U^{\text{total}} = \int \boldsymbol{\sigma} : \mathrm{d}\boldsymbol{\varepsilon}^{\text{e}}$, is the total strain energy; $\upsilon$ is the velocity at the centroid of the element; and $V$ is the volume of the slope.

The dissipated energy increases with the rise of reduction coefficient. When the critical state is reached, the dissipated energy will have a sudden change, and the slope will lose its stability[30]. When a slope is reduced from the initial state to the critical state, the energy of the slope system will continuously transform and the energy dissipation will continue to increase. We take a homogeneous slope as an example. After the strength is reduced once, the total energy dissipation increment of the slope system is assumed to be $\Delta E^{\text{d}}$. The increment in dissipation energy generated by the reduction of cohesive force $c$ after strength reduction once is $\Delta E^{\text{d},c}$, and the increment in dissipation energy generated by the reduction of the internal friction angle $\varphi$ is $\Delta E^{\text{d},\varphi}$. The nonlinearity of the slope in conducting strength reduction[33] suggests that the increment in the dissipated energy of the slope system due to the joint reduction of $c$ and $\varphi$ is $\Delta E^{\text{d},c-\varphi}$. Therefore, the dissipated energy change of the slope system can be expressed as

$$\Delta E^{\text{d},c} + \Delta E^{\text{d},\varphi} + \Delta E^{\text{d},c\_\varphi} = \Delta E^{\text{d}} \tag{7}$$

where $\Delta E^{\text{d}}$ is the total energy dissipation increment of the slope system after conducting strength reduction once; $\Delta E^{\text{d},c}$ is the increment in dissipation energy of the slope system caused by the reduction of $c$ alone after conducting reduction once; $\Delta E^{\mathrm{d},\varphi}$ is the increment in dissipation energy of the slope system induced by the reduction of $\varphi$ alone once; $\Delta E^{\mathrm{d},c-\varphi}$ is the increment in dissipated energy of the slope system caused by joint reduction of $c$ and $\varphi$ after reduction once, reflecting the increment in dissipated energy caused by nonlinearity and correlation when the two are reduced. In particular, when the energy change in conducting strength reduction is linear, $\Delta E^{\mathrm{d},c-\varphi}=0$ , nameley $\Delta E^{\mathrm{d},c}+\Delta E^{\mathrm{d},\varphi}=\Delta E^{\mathrm{d}}$ .

## 2.2 Multiparameter non-proportional reduction based on energy evolution theory to solve FOS over time

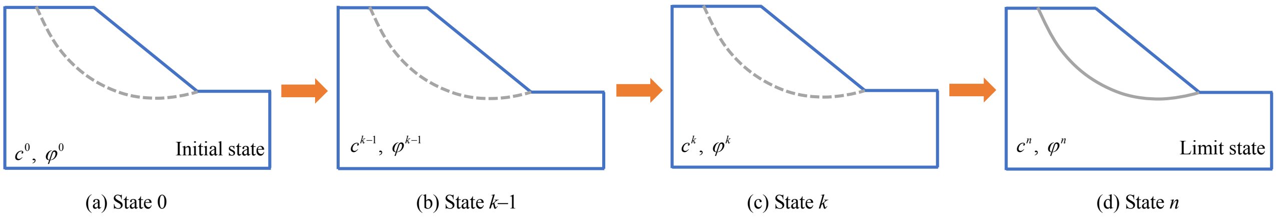
In reality, slope mechanical parameters will deteriorate, and the deterioration (reduction) of its mechanical parameters is a gradual process and follows a certain reduction path. Based on the Mohr-Coulomb strength criterion, considering the reduction of the strength parameters $c$ and $\varphi$, it is assumed that the strength reduction can be divided into $n$+1 states, such as state 0, ..., state $k$-1, state $k$, ..., state $n$, as shown in Fig. 1. According to the definition of safety factor of slope in literature[20], shear strength is selected as the key attribute of the slope studied, so the ratio of shear strength in one state of slope to that in reduced reference state is the safety factor of a state of slope relative to the reference state. The strength reduction process can be deemed as a process in which the reference state is further reduced and updated, approaching the limit state, until the reference state is just in the limit state( FOS = 1.0). Based on the above concept, the FOS of any state $k$-1 in Fig. 1 can be expressed as

$$\mathrm{FOS}(k-1)=\mathrm{SRF}_{k-1}\cdot\mathrm{SRF}_{k}\cdots\mathrm{SRF}_{n}\cdot\mathrm{FOS}(n),\quad 0<k\leqslant n,\ n\geqslant 1 \tag{8}$$

$$\mathrm{SRF}_{k}=f(\mathrm{SRF}^{c},\mathrm{SRF}^{\varphi}) \tag{9}$$

$$\mathrm{SRF}^{c}=c^{k-1}/c^{k},\mathrm{SRF}^{\varphi}=\tan\varphi^{k-1}/\tan\varphi^{k} \tag{10}$$

where $\mathrm{FOS}(k-1)$ is the comprehensive FOS when the slope is in state $k-1$; $\mathrm{FOS}(n)$ is the residual FOS after the slope is reduced $n$ times, if the state $n$ is just in limit state, then $\mathrm{FOS}(n)=1.0$ [20]; $c^{k-1}$ and $\varphi^{k-1}$ are respectively the cohesion and internal friction angle before the $k$th reduction; $c^{k}$ and $\varphi^{k}$ are respectively the cohesion and internal friction angle after the $k$th reduction; $\mathrm{SRF}^{c}$ and $\mathrm{SRF}^{\varphi}$ are the $k$th strength reduction coefficients of $c$ and $\varphi$, respectively, which can be determined by the reduction path; $\mathrm{SRF}_{k}$ is the comprehensive reduction coefficient when the slope is reduced for the $k$th time, which can be expressed as a function of $\mathrm{SRF}^{c}$ and $\mathrm{SRF}^{\varphi}$ [28]. When $\mathrm{SRF}^{c}=\mathrm{SRF}^{\varphi}$ and $n=1$, it is the traditional equal-proportional SRM.



(a) State 0 (b) State $k$–1 (c) State $k$ (d) State $n$

**Fig. 1 Illustration of SRM for slope stability analysis**

When the slope is reduced non-proportionally, $\mathrm{SRF}^{c}\neq\mathrm{SRF}^{\varphi}$ , which reduction coefficient is used to characterize the comprehensive FOS of the slope is a difficult problem. Slope failure is essentially a process of energy dissipation. In the process of an actual slope transition from a state to instability failure, the contributions of different mechanical parameters to maintaining slope stability differ and change dynamically[10–11]. In accordance with the energy principle in section 2.1, we can see that in a strength reduction, when $\Delta E^{\mathrm{d},c}>\Delta E^{\mathrm{d},\varphi}$ , this indicates that the cohesion in this reduction interval contributes more to the slope stability than the internal friction angle, that is, in this reduction, the reduction of cohesion is more unfavorable to the stability of the slope, and the cohesion reduction coefficient accounts for a larger proportion in the comprehensive reduction coefficient in this reduction process. When $\Delta E^{\mathrm{d},c}<\Delta E^{\mathrm{d},\varphi}$, it shows that the contribution of the internal friction angle in this reduction interval to the slope stability is greater than the cohesive force, that is, in this reduction, the reduction of the internal friction angle is more detrimental to the stability of the slope, and the reduction coefficient of the internal friction angle dominates the comprehensive reduction coefficient. When $\Delta E^{\mathrm{d},c}=\Delta E^{\mathrm{d},\varphi}$ , it shows that the cohesive force and internal friction angle in this reduction have the same contribution to the slope stability. Therefore, in this paper, the contribution to the energy dissipation of the slope in the reduction process of different mechanical parameters is used as the weight of different reduction coefficients, and Eq. (9) can be written as

$$\mathrm{SRF}_{k}=f(\mathrm{SRF}^{c},\mathrm{SRF}^{\varphi})=\omega^{c}\mathrm{SRF}^{c}+\omega^{\varphi}\mathrm{SRF}^{\varphi}+\omega^{c-\varphi}g(\mathrm{SRF}^{c},\mathrm{SRF}^{\varphi}) \tag{11}$$

$$\left.\begin{aligned}&\omega^{c}=\Delta E^{\mathrm{d},c}/\Delta E^{\mathrm{d}},\omega^{\varphi}=\Delta E^{\mathrm{d},\varphi}/\Delta E^{\mathrm{d}}\\&\omega^{c}+\omega^{\varphi}+\omega^{c-\varphi}=1.0\end{aligned}\right\} \tag{12}$$

$$g(\mathrm{SRF}^{c},\mathrm{SRF}^{\varphi})=\frac{2}{(\mathrm{SRF}^{c}+\mathrm{SRF}^{\varphi})/(\mathrm{SRF}^{c}\cdot\mathrm{SRF}^{\varphi})} \tag{13}$$

where $\omega^c$, $\omega^\varphi$ and $\omega^{c-\varphi}$ are the weights of contributions of $c$ alone reduction, $\varphi$ alone reduction, and $c$ and $\varphi$ reduction together to variation of dissipated energy in slope system; $g(\mathrm{SRF}^c,\mathrm{SRF}^\varphi)$ is the harmonic mean of the reduction coefficients $c$ and $\varphi$, which represents the joint effect of the reduction coefficients during non-proportional reduction, and reflects the influence of the nonlinearity and correlation of the reduction parameters. It should be noted that the harmonic average value selected here is to consider that the harmonic average value has been used in the double reduction method to characterize the comprehensive reduction coefficient among different reduction coefficients[24]. Moreover, the reduction coefficient calculated by the harmonic mean is the smallest among the common mean methods and is relatively safe.

According to Eqs.(8), (11)–(13), for different strength materials (rock and soil, concrete, etc.) and different mechanical parameters (cohesion, internal friction angle, tensile strength, etc.) with unequal proportion SRM, the calculation can be carried out by the following formula:

$$\mathrm{FOS}(k-1)=\prod_{k=1}^{n}\left[\sum_{i=1}^{l}\sum_{j=1}^{m}\omega_i^j\mathrm{SRF}_i^j+\Big(1-\sum_{i=1}^{l}\sum_{j=1}^{m}\omega_i^j\Big)\cdot f(\mathrm{SRF}_i^j)\right]\cdot\mathrm{FOS}(n) \tag{14}$$

$$\omega_i^j=\Delta E_i^{\mathrm{d},j}\big/\Delta E^{\mathrm{d}} \tag{15}$$

$$f(\mathrm{SRF}_i^j)=\frac{lm}{\sum\mathrm{SRF}_i^j\big/\prod\mathrm{SRF}_i^j} \tag{16}$$

where $\mathrm{FOS}(k-1)$ is the comprehensive FOS of the slope in state $k-1$; $\mathrm{SRF}_i^j$ is the strength reduction factor of the $j$-th mechanical parameter of the $i$-th material; $\omega_i^j$ is the weight of the contribution of the reduction of the $j$-th mechanical parameter of the $i$-th material to the variation of dissipated energy in the slope system; $n$ is the number of reduction; $\mathrm{FOS}(n)$ is the residual FOS of the slope after reduction for $n$ times, $\mathrm{FOS}(n)\geqslant 1.0$; $\Delta E_i^{\mathrm{d},j}$ is the dissipation energy increment of the slope system after the reduction of the $i$-th material and the $j$-th mechanical parameter in conducting a strength reduction; $\Delta E^{\mathrm{d}}$ is the increment of dissipated energy of slope system after all mechanical parameters of all materials are reduced in conducting a strength reduction; $f(\mathrm{SRF}_i^j)$ is the harmonic mean of the reduction coefficient, which represents the joint effect of the reduction coefficients during non-proportional reduction, and reflects the influence of the nonlinearity and correlation of the reduction parameters on the comprehensive safety factor.

When the mechanical parameters of all materials are reduced in equal proportion, that is, when $\mathrm{SRF}_i^j$ = SRF is constant, $f(\mathrm{SRF}_i^j)=\mathrm{SRF}$ .This is the tradetional equal proportion SRM:

$$\mathrm{FOS}=\prod_{k=1}^{n}(\mathrm{SRF})\cdot\mathrm{FOS}(n)=\mathrm{SRF}^n\cdot\mathrm{FOS}(n) \tag{17}$$

The realization process of the whole process FOS solution method based on the energy evolution theory with multiparameter non-proportional reduction is shown in Fig. 2.

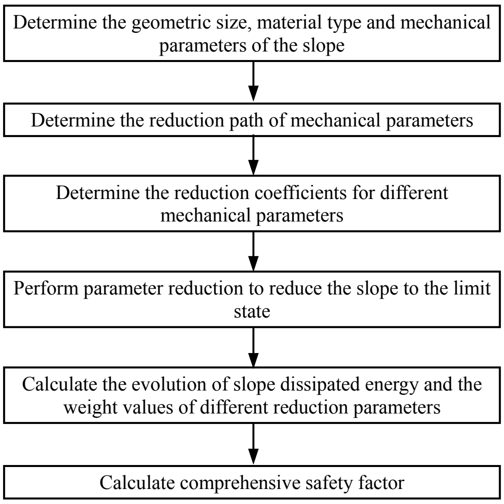


**Fig. 2 Flow chart for FOS calculations**

## 3 Example verification

In order to verify the applicability of the method proposed in this paper in the non-proportional reduction of homogeneous materials with different mechanical or material parameters, two classic examples in the literature are selected for verification and analysis. Example 1 is a homogeneous slope of one material, and example 2 is a slope supported by anti-slide piles of two materials.

### 3.1 Example 1—Homogeneous slope

This example is a homogeneous slope of one material, which is a classical example in the study of non-proportional strength reduction[29]. The size and grid division of the slope are shown in Fig.3. The slope material is an ideal elastic-plastic material obeying Mohr-Coulomb strength criterion. The unit weight of soil is $\gamma$ = 25 kN/m$^3$, the shear modulus is $K$ = 4.846 MPa, the bulk modulus is $G$ = 8.333 MPa, the cohesion is $c$ =42 kPa, and the internal friction angle is $\varphi$ = 27°. The reduction coefficient of the limit state obtained by the double reduction method in the literature is $\mathrm{SRF}^c=1.067$ and $\mathrm{SRF}^\varphi=2.985$, the reduction ratio is $k$ = 1.067/2.985 = 0.357. When the strength reduction is performed in this paper, it is assumed that the reduction coefficients of $c$ and $\varphi$ are the same for each reduction, and follow the exponential deterioration. Also, non-associated flow rule is adopted, and the failure criterion assumes the slope dissipated energy changes abruptly. The convergence condition is that the maximum unbalanced force ratio is less than $10^{-5}$. The results of the FOS are shown in Table 2, and the evolution curve of the strength parameter weight and FOS along with the whole process of reduction is shown in Fig.4.

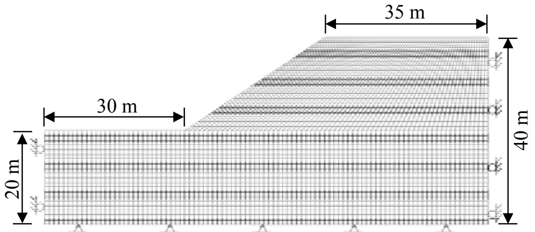


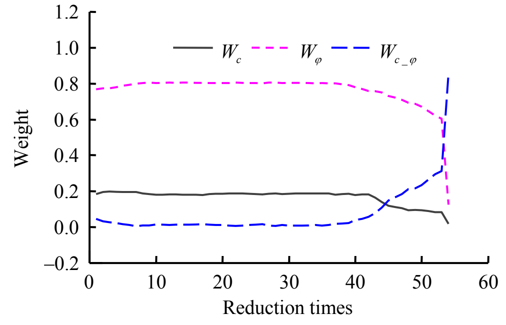
**Fig. 3 Numerical model of a homogenous slope (element: 3 200, node: 6 672)**

**Table 2 Comparison of the FOS of a homogeneous slope for different methods**

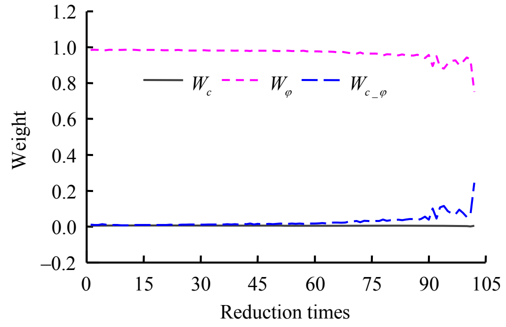
| Reduction method | Method | Initial FOS | Reduction times | Reduction ratio $k$ |
|---|---|---|---|---|
| Limit equilibrium method | Bishop method | 1.723 | — | — |
| Traditional strength reduction method (Proportion reduction) | Bai et al.[29] | 1.710 | — | — |
| | Method of this paper | 1.711 | 54 | 1.000 |
| Non-proportional reduction (Double reduction) | Bai et al. [29] | 1.239 | — | — |
| | Tang et al.[22] | 2.026 | — | — |
| | Yuan et al.[25] | 1.785 | — | — |
| | Yuan et al.[27] | 1.421 | — | — |
| | Isakov et al.[28] | 1.895 | — | — |
| | Method of this paper | 2.709 | 102 | 0.357 |
| Only the internal friction angle $\varphi$ is reduced | Tang et al.[22] | 1.994 | — | — |
| | Yuan et al.[25] | 1.728 | — | — |
| | Yuan et al.[27] | 1.341 | — | — |
| | Isakov et al.[28] | 1.888 | — | — |
| | Method of this paper | 2.988 | 110 | 0.335 |

Note: * For ease of comparison, in addition to the limit equilibrium method, control each reduction coefficient $\mathrm{SRF}^{\varphi} = 1.01$ of internal friction angle.

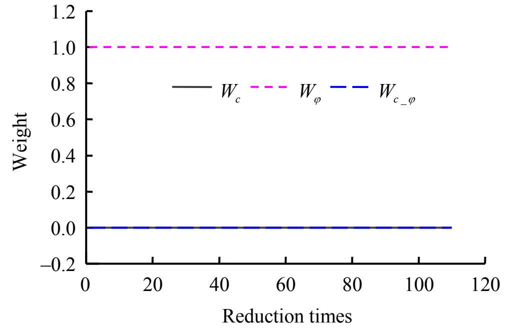
Table 2 shows that for proportional reduction, the initial FOS calculated by this method is close to that calculated by other methods, indicating that this method can be used in the case of proportional reduction. It can be seen from Fig.4(a) that in the case of proportional reduction, at the initial stage of reduction, the weight of the internal friction angle $\varphi$ is the largest, the weight of cohesion $c$ is the second, and the weight of the joint reduction is the smallest. With the increase of reduction times, the weight values of $\varphi$ and $c$ decrease, and the weight value of the joint reduction increases, indicating that the nonlinearity of the slope is more and more significant in the later period of reduction. For non-proportional strength reduction, it can be seen from Fig.4(b) that before the slope is reduced to failure, the weight of $\varphi$ is always close to 1.0, that is, under this reduction path ($k = 0.357$), the contribution to maintaining slope stability is always greater than that of $c$, and the stability of the slope is mainly dominated by the internal friction angle $\varphi$. In addition, the FOS of non-proportional strength reduction is 2.709, which is close to 2.988 under the condition of only reducing the internal friction angle, mainly because the reduction ratio of non-proportional strength reduction ($k = 0.357$) is close to the reduction ratio of only reducing the internal friction angle ($k = 1.000/2.987=0.335$). Therefore, under this reduction path, the stability of the slope is mainly contributed by the internal friction angle, and the cohesion has little effect on the stability under such a slight reduction (see Fig.4(c)). It is worth noting that the initial FOS calculated by the method in this paper is larger than that calculated by other methods. The main reason is that other methods do not consider the contribution of different mechanical parameters in the reduction process, which is different and dynamic, and the comprehensive FOS is calculated according to the



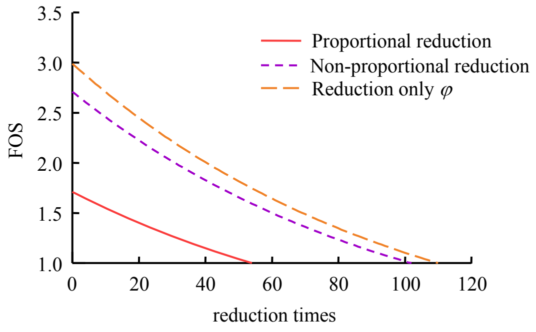
(a) Proportional reduction



(b) Non-proportional reduction



(c) Reduction only $\varphi$



(d) FOS evolution curves

**Fig. 4 Evolution of weight values and FOS in the shear strength reduction process (Example-1)**

formula in Table 1 based only on the total reduction factor ( $SRF^c$ and $SRF^{\varphi}$ ). It can be seen from Fig.4(d) that under these 3 reduction methods, the FOS of the slope decreases nonlinearly with the reduction times. The reduction times of proportion reduction, non-equal proportion reduction and only reduction of internal friction angle are 54, 102 and 110, respectively. The reduction times of proportion reduction are the least, indicating that when the reduction coefficient is the same, the times of reductions will be the least. To sum up, the FOS solution proposed in this paper is reasonable, and the weight value of different mechanical parameters to maintain slope stability and the evolution curve of FOS with reduction times can be obtained in order to better reflect the failure process.

### 3.2 Example 2—Slope supported by anti-slide piles

This example is a classic example of slope with pile support[5], with slope height of 10 m, slope ratio of 1: 1.5, and pile spacing of 2*D* (*D* is pile diameter, *D* = 0.8 m). The slope material is an ideal elastic-plastic material which obeys the Mohr-Coulomb strength criterion. The associated flow rule is adopted. The mechanical parameters of soil and anti-slide pile are shown in Table 3.The FOS of slope without pile is 1.184. The geometric size and grid division of the slope are shown in Fig.5. When strength is reduced, it is assumed that the reduction coefficients of cohesion and internal friction angle of soil and pile are the same for each reduction, and they all follow an exponential deterioration law. To facilitate calculation and comparison, the cohesion and internal friction angle of each material (soil and concrete pile) are set to be reduced proportionally. In this example, five working conditions are set according to the different degradation rates of soil and pile. In order to facilitate comparison, it is assumed that the reduction coefficients of soil in each of the five working conditions are the same ( $SRF_{soil}=1.02$ ). The failure criterion is that the dissipation energy of slope changes abruptly, and the convergence condition is that the maximum unbalanced force ratio is less than $10^{-5}$ . Fig.6(a)–6(e) are the weight evolution curves of different mechanical parameters in working conditions 1–5, respectively. Fig.6(f) is the whole process evolution curve of FOS with reduction times. The results are shown in Table 4.

**Table 3 Material properties for example 2**

| Material type | $\gamma$ /(kN • m$^{-3}$) | *E* /MPa | $\nu$ | *c* /kPa | $\varphi$ /(° ) |
|---|---|---|---|---|---|
| Soil mass[5] | 20 | 14 | 0.3 | 20 | 20 |
| Concrete pile [34] | 23 | 30 000 | 0.2 | 7 000 | 40 |

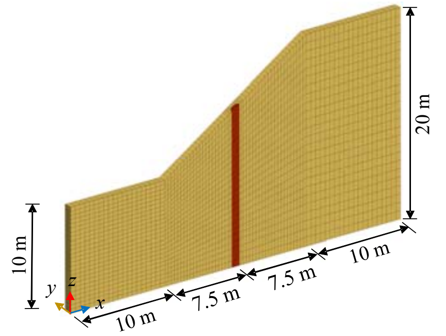
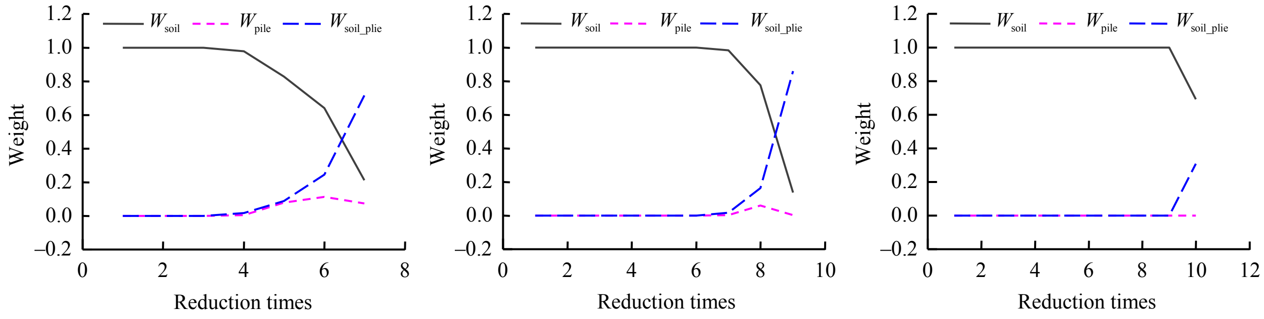
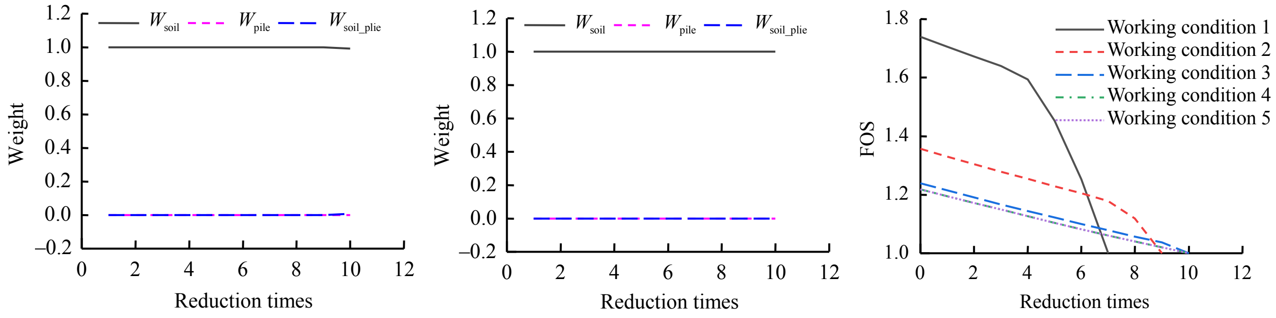


**Fig. 5 Numerical model for example 2**



(a) Working condition 1 (b) Working condition 2 (c) Working condition 3



(d) Working condition 4 (e) Working condition 5 (f) FOS evolution curve

**Fig. 6 Evolution of weight values and FOS in the shear strength reduction process (Example-2)**

Table 4 shows that the initial FOS of the slope after anti-slide pile reinforcement is 1.219 (working condition 5), which is 2.96% higher than that of the slope without reinforcement (1.184), indicating that the anti-slide pile reinforcement can improve the overall stability of the slope. The reduction coefficients of each soil in working conditions 1–5 are the same. With increase of the reduction ratio, the initial FOS decreases gradually and converge to 1.219. It can be seen from condition 5 that the initial FOS of the slope is equal to the reduction coefficients of the soil without considering the reduction of the pile, because the comprehensive FOS of the slope is contributed by the reduction coefficients of the soil in the reduction process. It can also be seen from Fig.6(e) that the weight value of soil in the reduction process has been 1.0, indicating that the increment of dissipation energy of slope system in the reduction process is contributed by soil reduction. With the decrease of the reduction ratio, that is, the reduction coefficient of each soil is the same. When the reduction coefficient of the pile increases, the initial FOS increases, which is mainly due to the strength reserve of the pile in the reduction process. It can be seen from Fig.6(f) that when the reduction coefficient of the pile is much larger than that of the soil (working condition 1), the FOS first decreases slowly with the reduction times, which is mainly because at the beginning of the reduction, the pile has sufficient strength, and its reduction has little effect on the stability of the slope system. When the pile is reduced to a certain strength, the FOS begins to decrease rapidly. This is mainly due to the rapid decline in the strength of the pile, and its bearing capacity can no longer maintain the stability of the slope well. It can also be seen from the corresponding weight change curve that at this time, the weight value of the soil reduced separately begins to decrease, but the weight when the pile and soil are reduced together begins to increase gradually, indicating that the harm of the reduction of pile strength to slope stability is greater than that caused by soil reduction.

**Table 4 Results of example 2**

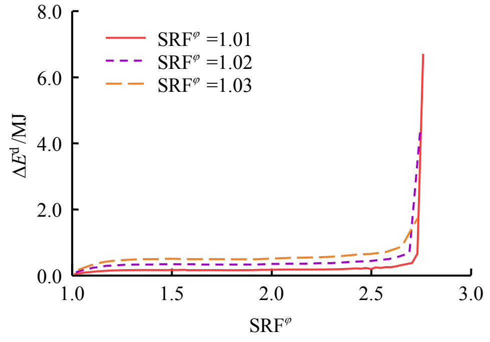
| Working condition | Soil pile reduction ratio $r=\mathrm{SRF}_{\mathrm{soil}}/\mathrm{SRF}_{\mathrm{pile}}$ | Soil reduction coefficient ( $\mathrm{SRF}_{\mathrm{soil}}^{\mathrm{total}}$ ) | Pile reduction coefficient ( $\mathrm{SRF}_{\mathrm{pile}}^{\mathrm{total}}$ ) | Reduction times | Initial FOS |
|---|---|---|---|---|---|
| 1 | 0.6 | 1.149 | 41.034 | 7 | 1.740 |
| 2 | 0.8 | 1.195 | 8.904 | 9 | 1.357 |
| 3 | 0.9 | 1.219 | 3.496 | 10 | 1.239 |
| 4 | 1.0 | 1.219 | 1.219 | 10 | 1.219 |
| 5 | — | 1.219 | 1.000 | 10 | 1.219 |

Note: $\mathrm{SRF}_{\mathrm{soil}}$ and $\mathrm{SRF}_{\mathrm{pile}}$ are the reduction coefficients of each soil and pile in the reduction process; $\mathrm{SRF}_{\mathrm{soil}}^{\mathrm{total}}$ and $\mathrm{SRF}_{\mathrm{pile}}^{\mathrm{total}}$ are the total reduction coefficients of soil and pile when they are reduced to the limit state, respectively.

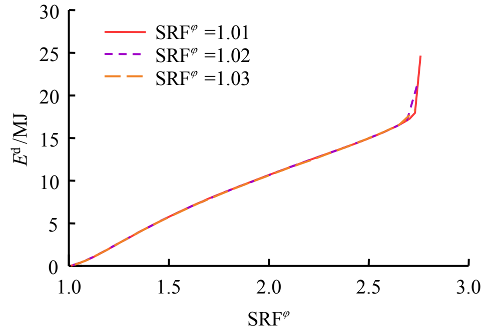
## 4 Discussions

### 4.1 Influence of the reduction step

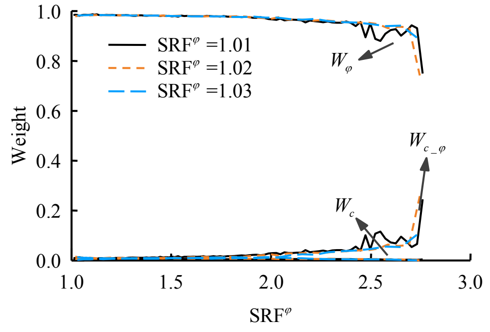
In order to study the influence of the reduction step on the energy evolution and FOS of the slope, a further study was carried out using the example in section 3.1. The mechanical parameters and calculation methods are consistent with the example 1. We keep the same reduction ratio $k$ for cohesion and internal friction angle of each reduction (i.e., the reduction path is the same), and set three calculation conditions with different reduction steps of internal friction angle ( $\mathrm{SRF}^{\varphi}=1.01$、 1.02、1.03). The evolution curves of energy and weight for internal friction angle during the reduction process are shown in Fig. 7. The results of FOS for the three conditions are listed in Table 5.



(a) Evolution curve of $\Delta E^{\mathrm{d}}$



(b) Evolution curve of $E^{\mathrm{d}}$



(c) Evolution curve of weight

**Fig. 7 Evolution of energy and weight values in the reduction process for $\varphi$**

Figure 7(a) shows the evolution curve of the total energy dissipation increment with respect to the reduction coefficient for each reduction. It can be seen that the total energy dissipation increment in different calculation conditions increases with the reduction coefficient, which is mainly due to the increase of the degree for the slope mechanical parameters reduction. However, since the reduction path is the same, the evolution trend of the total dissipated energy increment after each reduction is the same. Figure 7(b) shows the evolution curve of the total energy dissipation with the reduction coefficient. It can be observed that there is no significant difference in the total energy dissipation for different calculation conditions with the increase of the reduction coefficient, which is mainly because the reduction path for cohesion and internal friction angle between different calculation conditions are the same. As shown in Figure 7(c), with the increase of the reduction coefficient, there is not much difference in the weight indices. In the later stage of the reduction, the position of the inflection point of the curve has slight fluctuations. This is because the size of the reduction step value (representing the degree of reduction) will affect whether the last step of reduction can be performed when the slope reaches a critical state. Therefore, the reduction step should not be too large in the reduction process, otherwise the accuracy of FOS will be insufficient, but also should not be too small, otherwise the energy increment during the reduction process is too small, which will also affect the calculation accuracy.

It can be seen from Table 5 that there is not much difference in the total reduction factor of the final cohesion and internal friction angle of the 3 calculation conditions, and the initial FOS values are also very close, indicating that the change of reduction step length will not significantly affect the initial FOS. On the other hand, with increase of the step length of each reduction, the total times of reduction decreases when the slope is reduced to failure. This is mainly due to the fact that under a given reduction path and the same initially FOS of the slope, the larger the step length of each reduction (i.e., the larger the degree of each reduction), the smaller the number of reductions. This is consistent with the actual project that the greater degree of degradation of the slope in the same time period, the shorter its life.

In general, for a given slope with a certain reduction path, as the reduction coefficient increases, the difference in reduction step will affect the value of the incremental change in each reduced dissipation energy and reduction times, but does not affect the change of the total energy dissipation of the slope, the evolution of the energy weight index value and the initial FOS.

**Table 5 Influence of reduction factor of every reduction step on calculation results**

| Calculation conditions | $k$ | SRF$^c$ | SRF$^\varphi$ | Reduction times | Initial FOS |
|---|---|---|---|---|---|
| 1 | 0.357 | 1.062 | 2.759 | 102 | 2.709 |
| 2 | 0.357 | 1.062 | 2.745 | 51 | 2.696 |
| 3 | 0.357 | 1.061 | 2.732 | 34 | 2.684 |

## 4.2 The influence of reduction path

According to the literature research, the deterioration forms of rock and soil mechanical parameters are mainly linear[35–36], exponential[15, 36–37], and logarithmic[18-19, 38] or their combination in the research

$$S = S_0 - \beta\psi \tag{18}$$

$$S = S_0\exp(-\beta\psi) \tag{19}$$

$$S = S_0 - \beta\ln(\psi + 1) \tag{20}$$

where $S$ is the mechanical parameters after deterioration, such as cohesion, internal friction angle and elastic modulus; $S_0$ is the mechanical parameter of the initial state; $\beta$ is the parameter that affects the degradation rate; $\psi$ is a variable, which can be time, strain, moisture content, dry wet cycle times and freeze-thaw cycle times.

In order to study the influence of the reduction path on the calculation results, the above three different degradation forms are used as the reduction path. The case 1 in Section 3.1 is still used here. The mechanical parameters and calculation method of the slope are the same as that of example 1. In order to study the influence of the change of the reduction path form, the exponential type is taken as the reference condition. It is assumed that the cohesion and internal friction angle of the slope in the initial state and failure state are consistent under different conditions, and the total reduction times are 102 times (that is, the reduction step is the same). The parameter setting and calculation results of numerical calculation are shown in Table 6 and Fig. 8.

**Table 6 Influence of reduction path on FOS**

| Reduction path type | Initial state | | Failure state | | Reduction times | Initial FOS |
|---|---|---|---|---|---|---|
| | $c$ /kPa | $\varphi$/(°) | $c$ /kPa | $\varphi$/(°) | | |
| Linear | 42.0 | 27.0 | 39.547 | 10.463 | 102 | 2.715 |
| Exponential | | | | | | 2.716 |
| Logarithmic | | | | | | 2.708 |

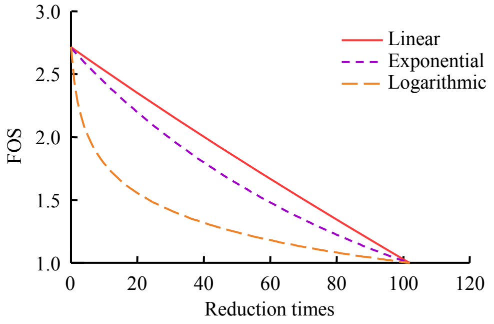


**Fig. 8 Evolution of FOS with different reduction paths**

It can be seen from Table 6 and Fig. 8 that there is almost no difference in the initial FOSs under different reduction paths, and the final FOSs are all 1.0. This is because the initial state and failure state of the slope are the same. It can be seen from Eq. (8) that the FOSs will be the same in theory. In fact, FOS is a relative concept, which reflects the safety reserve of the research object in a certain state compared with the reference state [20]. The FOS is directly related to the studied state and the corresponding reference state. However, due to different reduction paths, the intermediate states in the reduction process are different, and there is process correlation among the states in the reduction process. Therefore, the evolution of FOS in the whole reduction process is different, and the selection of reduction path will affect the evolution curve of FOS in the whole reduction process.

## 5 Conclusion

(1) In this study, we discussed the results and shortcomings of the current non-proportional reduction method, and proposed that when using SRM for stability analysis, it is not only necessary to consider the non-proportional reduction for different mechanical parameters of the same

material, nor the materials with different strengths, but also necessary to understand the evolution of FOS in the whole process of slope reduction.

(2) Based on the energy evolution theory, a new method to calculate the FOS with multiparameter non-proportional reduction of shear strength parameters was developed in conducting strength reduction by referring to the definition of safety factor of reference slope and taking the contribution of different mechanical parameters to dissipated energy evolution as the weight.

(3) Two examples were used to verify the applicability of the proposed method in the calculation of non-proportional reduction for different parameters of the same material and materials with different strengths.

(4) For a given slope, when the reduction path is fixed, the difference in reduction step will affect the increment of dissipated energy in each slope reduction, but will not affect the change of the total dissipated energy of the slope, the evolution of the energy weight values, and initial FOS. When the initial state and the limit state are the same, different reduction paths almost do not affect the initial FOS, but it will affect the evolution of FOS in the whole process of slope reduction.

## References

[1] ZIENKIEWICZ O C, HUMPHESON C, LEWIS R W. Associated and non-associated visco-plasticity and plasticity in soil mechanics[J]. Géotechnique, 1975, 25(4): 671–689.

[2] MATSUI T, SAN K. Finite element slope stability analysis by shear strength reduction technique[J]. Soils and Foundations, 1992, 32(1): 59–70.

[3] GRIFFITHS D V, LANE P A. Slope stability analysis by finite elements[J]. Géotechnique, 1999, 49(3): 387–403.

[4] DAWSON E M, ROTH W H, DRESCHER A, et al. Slope stability analysis by strength reduction[J]. Géotechnique, 1999, 49(6): 835–840.

[5] WEI W B, CHENG Y M. Strength reduction analysis for slope reinforced with one row of piles[J]. Computers and Geotechnics, 2009, 36(7): 1176–1185.

[6] ZHENG Ying-ren, ZHAO Shang-yi. Application of strength reduction FEM in soil and rock slope[J]. Chinese Journal of Rock Mechanics and Engineering, 2004, 23(19): 3381–3388.

[7] ZHENG Ying-ren, ZHAO Shang-yi. Calculation of inner force of support structure for landslide/slope by using strength reduction FEM[J]. Chinese Journal of Rock Mechanics and Engineering, 2004, 23(20): 3552–3558.

[8] ZHENG Hong, TIAN Bin, LIU De-fu, et al. On definitions of safety factor of slope stability analysis with finite element method[J]. Chinese Journal of Rock Mechanics and Engineering, 2005, 24(13): 2225–2230.

[9] LU Feng. Research on safety degree of existing tunnel based on strength reduction method[D]. Chengdu: Southwest Jiaotong University, 2014.

[10] DENG D P, LI L, ZHAO L H, et al. Limit equilibrium method (LEM) of slope stability and calculation of comprehensive factor of safety with double strength-reduction technique[J]. Journal of Mountain Science, 2017, 14(11): 2311–2324.

[11] DENG D P, LI L. Limit equilibrium analysis of slope stability with coupling nonlinear strength criterion and double-strength reduction technique[J]. International Journal of Geomechanics, 2019, 19(6): 04019052.

[12] BARTON N, PANDEY S K. Numerical modelling of two stoping methods in two Indian mines using degradation of $c$ and mobilization of $\varphi$ based on Q-parameters[J]. International Journal of Rock Mechanics and Mining Sciences, 2011, 48(7): 1095–1112.

[13] HAJIABDOLMAJID V, KAISER P K, MARTIN C D, et al. Modelling brittle failure of rock[J]. International Journal of Rock Mechanics and Mining Sciences, 2002, 39(6): 731–741.

[14] LIU Xin-rong, LI Dong-liang, ZHANG Liang, et al. Influence of wetting-drying cycles on mechanical properties and microstructure of shaly sandstone[J]. Chinese Journal of Geotechnical Engineering, 2016, 38(7): 1291–1300.

[15] PINEDA J A, ROMERO E, GRACIA M D, et al. Shear strength degradation in claystones due to environmental effects[J]. Géotechnique, 2014, 64(6): 493–501.

[16] SKEMPTON A W, LAROCHELLE P. The Bradwell Slip: a short-term failure in London clay[J]. Géotechnique, 1965, 15(3): 221–242.

[17] XIE S B, QU J J, LAI Y M, et al. Effects of freeze–thaw cycles on soil mechanical and physical properties in the Qinghai–Tibet Plateau[J]. Journal of Mountain Science, 2015, 12(4): 999–1009.

[18] YUAN W, LIU X R, FU Y, et al. Study on deterioration of strength parameters of sandstone under the action of dry–wet cycles in acid and alkaline environment[J]. Arabian Journal for Science and Engineering, 2018, 43(1): 335–348.

[19] YUAN W, LIU X R, FU Y, et al. Chemical thermodynamics and chemical kinetics analysis of sandstone dissolution under the action of dry–wet cycles in acid and alkaline environments[J]. Bulletin of Engineering Geology and the Environment, 2019, 78(2): 793–801.

[20] BAI Bing, YUAN Wei, SHI Lu, et al. Comparing a new double reduction method to classic strength reduction method for slope stability analysis[J]. Rock and Soil Mechanics, 2015, 36(5): 1275–1281.

[21] TAYLOR D W. Fundamentals of soil mechanics[M]. New York: John Wiley and Sons, 1948: 441–465.

[22] TANG Fen, ZHENG Ying-ren, ZHAO Shang-yi. Discussion on two safety factors for progressive failure of soil slope[J]. Chinese Journal of Rock Mechanics and Engineering, 2007, 26(7): 1402–1407.

[23] JIANG X Y, WANG Z G, LIU L Y, et al. The determination of reduction ratio factor in homogeneous soil-slope with finite element double strength reduction method[J]. Open Civil Engineering Journal, 2013, 7(1): 205–209.

[24] WU S C, XIONG L F, ZHANG S H, et al. Strength reduction method for slope stability analysis based on a dual factoring strategy[J]. International Journal of Geomechanics, 2018, 18(10): 04018123.

[25] YUAN Wei, LI Xiao-chun, WANG Wei, et al. Study on strength reduction method based on double reduction parameters[J]. Rock and Soil Mechanics, 2016, 37(8): 2222–2230.

[26] ZHU Yan-peng, YANG Xiao-yu, MA Xiao-rui, et al. Several questions of double reduction method for slope stability analysis[J]. Rock and Soil Mechanics, 2018, 39(1): 331–338, 348.

[27] YUAN W, BAI B, LI X C, et al. A strength reduction method based on double reduction parameters and its application[J]. Journal of Central South University, 2013, 20(9): 2555–2562.

[28] ISAKOV A, MORYACHKOV Y. Estimation of slope stability using two-parameter criterion of stability[J]. International Journal of Geomechanics, 2014, 14(3): 06014004.

[29] BAI B, YUAN W, LI X C, et al. A new double reduction method for slope stability analysis[J]. Journal of Central South University, 2014, 21(3): 1158–1164.

[30] TU Y L, LIU X R, ZHONG Z L, et al. New criteria for defining slope failure using the strength reduction method[J]. Engineering Geology, 2016, 212: 63–71.

[31] HUANG L C, HUANG S, LAI Z S, et al. On an energy-based criterion for defining slope failure considering spatially varying soil properties[J]. Engineering Geology, 2020, 264: 105323.

[32] CHAI Hong-bao, CAO Ping, LIN Hang, et al. Criteria of elastic strain energy in slope stability analysis using strength reduction method[J]. Journal of Central South University (Science and Technology), 2009, 40(4): 1054–1058.

[33] ZHAO L H, YANG F, ZHANG Y B, et al. Effects of shear strength reduction strategies on safety factor of homogeneous slope based on a general nonlinear failure criterion[J]. Computers and Geotechnics, 2015, 63: 215–228.

[34] GUAN Z C, JIANG Y J, TANABASHI Y, et al. A new rheological model and its application in mountain tunnelling[J]. Tunnelling and Underground Space Technology, 2008, 23(3): 292–299.

[35] SHIMAMOTO K, YASHIRO K, KOJIMA Y, et al. Prediction method of tunnel deformation using time-dependent ground deterioration model[J]. Quarterly Report of Rtri, 2009, 50(2): 81–88.

[36] MOMENI A, HASHEMI S S, KHANLARI G R, et al. The effect of weathering on durability and deformability properties of granitoid rocks[J]. Bulletin of Engineering Geology and the Environment, 2017, 76(3): 1037–1049.

[37] PINEDA J A, ALONSO E E, ROMERO E, et al. Environmental degradation of claystones[J]. Géotechnique, 2014, 64(1): 64–82.

[38] LIU X R, JIN M H, LI D L, et al. Strength deterioration of a Shaly sandstone under dry–wet cycles: a case study from the Three Gorges Reservoir in China[J]. Bulletin of Engineering Geology and the Environment, 2018, 77(4): 1607–1621.