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Explicit solution of horizontal infiltration equation in unsaturated soils

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Abstract: When solving the water infiltration problem in unsaturated soils, the hydraulic function is a function of water content or suction, which makes the equation governed by hydraulic function exhibits strong nonlinear characteristics resulting in the difficulty of solving the horizontal infiltration problem. Based on the assumption that the water flow in soil medium follows the path of time - consuming extreme value, a time functional was introduced, and the horizontal infiltration problem was transformed into a functional extreme value problem based on a variational principle. By solving Euler-Lagrange equation and combining with the boundary conditions, the explicit analytical solution of the nonlinear transient horizontal infiltration problem was obtained. Combined with the Brooks-Corey hydraulic function, the distribution of volume water content of this type of soils in the unsaturated state was explicitly solved. By calculating the water horizontal infiltration laws of four different types of soil samples through theoretical and numerical methods, the results obtained by the solution matched well with the existing results and numerical results, verifying the effectiveness of the method. The results show that the distribution of volume water content has a power function against location distance and wetting front distance ratio, and the power exponent depends on the shape parameter of soil water characteristic curve. Initial conditions and boundary conditions had different effects on the distribution of volumetric water content.

Keywords: horizontal infiltration; variational method; Euler-Lagrange equation; Brooks-Corey model

1 Introduction

The process of water infiltration in unsaturated soils plays an important role not only in understanding many engineering problems such as landslides induced by rainfall[1−2], but also in the hydrological cycle. At present, the partial differential equations presented by Richard[3] is a relatively widely used mathematical model to describe the process of water infiltration. However, the complicated relationship between infiltration variables and hydraulic parameters results in the strong nonlinear characteristic of the equation. Besides, due to the existence of this nonlinear characteristic, the numerical method is mostly used for obtaining the solutions to the problems of water flow in unsaturated soils [4−7].

Although the numerical method shows excellent performance to solve the problems of water infiltration process under complex conditions, it often has the disadvantages of the instability of solutions and insufficient numerical oscillation, as well as the need for complex solution process. Therefore, many researchers have spared no effort to seek effective mathematical analytical methods to solve Richard's partial differential equations. However, most of these solutions are based on harsh assumptions, which weakens the nonlinear characteristics of the governing equations and greatly reduces the difficulty of solving them. For example, in the solution of steady-state flow equation[8−11], it contains the assumption that the change rate of volumetric water content or suction with time is zero, but the actual situation is mostly transient flow. Another common solution is to assume the hydraulic function as an exponential function or a linear function. By using the characteristics of exponential function, the governing equation can be easily transformed into a linear partial differential equation, and then the analytical solution of water infiltration process can be obtained[12−14]. Moreover, by introducing Feature Transformation[15] or Boltzmann Transformation[16], the partial differential equation is transformed into an ordinary differential equation, thus reducing the difficulty of solving the equation. But the physical significance of the introduced transformation remains to be further explored, and it is also equally complex and difficult to solve the ordinary differential equation after transformation. Based on the least action principle and the variation principle, Su et al. [17−18] put forward a new idea to solve the water infiltration process. However, since their solution is based on the mixed type governing equation with suction

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head and volumetric water content as dependent variables, this method is limited to the solution of the saturated volumetric water content at the infiltration boundary.

In this paper, based on the variation principle, in combination with understanding of the path dependence of water movement in soil, it was assumed that the water movement is along the optimal path of time consumption, then the distribution of soil volumetric water content at the infiltration boundary was solved explicitly. Combined with the classical Brooks-Corey hydraulic function, the explicit expression of soil volumetric water content distribution was derived. Finally, the results of the explicit solution were compared with the existing theoretical and numerical results.

2 Model

2.1 Fundamental equations

In order to simplify the problem, the effect of gravity is ignored in Richard's equations, that is, only one-dimensional horizontal adsorption equations or diffusion equations in unsaturated soils are considered [19]:

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial x}\left(D(\theta)\frac{\partial\theta}{\partial x}\right) \tag{1}$$

where θ is the soil volumetric water content; t is the time variable; x is the horizontal one-dimensional coordinate; and $D(\bullet)$ is the diffusion function.

Satisfying the initial conditions:

$$\theta(x,0)=\theta_{\mathrm{i}},0\leqslant x\leqslant x_{\mathrm{f}} \tag{2}$$

where x_{f} is the distance of wetting front and θ_{i} is the initial water content.

Satisfying the boundary conditions:

$$\theta(0,t)=\theta_0,0<t<\infty \tag{3}$$

$$\theta(x_{\mathrm{f}},t)=\theta_{\mathrm{i}},0<t<\infty \tag{4}$$

where θ_0 is the given boundary water content.

D is generally considered to be a function of soil water content or pressure head. For this reason, the Equation (1) presents strong nonlinear characteristics, which makes the solution of the equation extremely difficult.

2.2 Variational technique

In fact, the flow of water in unsaturated soils must follow a certain path and be driven by different energies. Mathematically, there are several possible paths for water flow from point A to point B, as shown in Fig. 1. The choice of different paths corresponds to different solutions to the governing equation of water movement. For classical Wave solutions and Boltzmann solutions, the essence of them is to introduce path variables $\varphi=x-vt$ (v is the velocity of pore water) and $\varphi=x/\sqrt{t}$, respectively, based on which Eq. (1) is solved. From a mathematical point of view, it is to introduce some new variables to simplify the problem. Although the problem has been simplified, the introduction of new variables also makes the solving process become relatively cumbersome, and the physical basis for the introduction of such paths is not exactly clear. In addition, when solving practical problems, it often has singularity. Therefore, the scope of above transformation method for solving problems is limited.

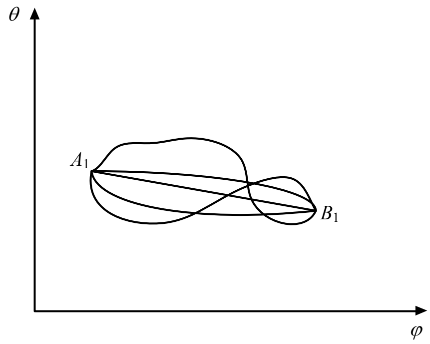


Fig. 1 Possible paths of water flow

To further solve the above problems, based on the principle of least action, among many possible paths, the water movement follows the path of the extreme value of a certain action [17−18]. For this reason, this paper introduces the following hypotheses: the water flow in soil follows the path of time-consuming extremum, and the corresponding argument is the change path of soil volumetric water content, that is, the change path of soil volumetric water content is regarded as a functional of time $T[\theta(x)]$, which is expressed as

$$T\left[\theta(x)\right]=\int_0^{x_{\mathrm{f}}}\frac{\mathrm{d}x}{v} \tag{5}$$

It should be noted that although Eq.(5) is established based on hypothetical conditions, no new variables are introduced here compared to the classical path, and each variable has a clear physical meaning. In addition, the principle of action in physics refers to the comparison of all possible experiences or movements of the object from a certain specific point of view. It is believed that the actual experience or movement of the object can be obtained by taking the extremum of a certain action, and the experience or movement of taking the extremum of the action is exactly the actual experience of the object. The idea of this action principle comes from all kinds of extremum thoughts in life and all kinds of extreme phenomena in nature. For example: light propagates according to the principle of the shortest time or optical path; the entropy value is the largest when the thermodynamic system is in equilibrium; for objects of the same volume, the spherical surface area is the largest, so the water droplets can remain perfectly spherical in the weightless air. Note that the explanations for different physical phenomena are derived by the choice of different actions.

With regard to the action of the water flow phenomenon in the soil medium, from the functional point of view, energy or energy loss is often selected as the evaluation standard because energy reflects the common measure of the mutual transformation between various movements. However, this approach will bring inconvenience to the solution of the problem, and eventually tend to rely on the numerical method. The purpose of this paper is to explicitly solve the distribution of volumetric water content in the process of water flowing in soil, which is extremely difficult to realize according to the traditional point of view. Based on the fact that the action reflects the common properties that various movements processes must satisfy, and the action is the basic cognition of the unity of energy and time, the flow of water in soil is analogous to the propagation of light and time is chosen as the action in this paper. According to the principle of action, when the first order variation of time to the path is zero (taking the extreme value), the actual flow trajectory of water can be described.

As for the rationality of the analogy, firstly, it can be seen from the results obtained in the following that taking time as action to get the extreme value can express the distribution of volumetric water content explicitly, and the results are consistent with the existing conclusions. Secondly, as shown in Fig. 2, when water moves from *A* to *B* without resistance, it is obviously the best choice to move along the *AB* straight line. However, due to the limitation of pore distribution, viscosity, solid particles and other factors in the soil, the movement from *A* to *B* will inevitably cause the loss of velocity, which in turn leads to loss of distance. The losses of L_1 segment and L_2 segment are equivalent to that of *AD* segment and *DB* segment, respectively. Here, the loss coefficient was set as N_n, the length of *AD* segment was N_nL_1, and the length of *DB* segment was N_nL_2. Such equivalence is analogous to the refraction of light. The detailed proof of the principle of the shortest time of light can be found in Feynman et al.[20]. Therefore, it is reasonable to choose time as the action of water flow in soil. Further experiments need be designed to verify its correctness and effectiveness. In addition, driven by external energy, water flows in the soil medium as a dissipative system. The specific physical explanation of this analogy hypothesis needs further study and exploration. This paper only uses it as a hypothesis, and this selection seems to be feasible to some extent based on the results of solution below.

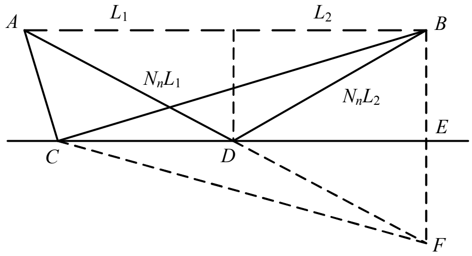


Fig. 2 Analogy of the shortest time principle

According to Eq. (1), we can know

$$v=\frac{\partial x}{\partial t}=\frac{\partial}{\partial\theta}\left(D(\theta)\theta'\right) \tag{6}$$

where $\theta'=\dfrac{\partial\theta}{\partial x}$.

Substituting Eq. (6) into Eq. (5), we can get

$$T\left[\theta(x)\right]=\int_0^{x_\mathrm{f}}\left[\frac{\partial}{\partial\theta}\left(D(\theta)\theta'\right)\right]^{-1}\mathrm{d}x \tag{7}$$

Based on the variational principle, the optimal time-consuming path needs to solve the following problems:

$$\text{Min}\ \ T\left[\theta(x)\right]=\int_0^{x_\mathrm{f}}\left[\frac{\partial}{\partial\theta}\left(D(\theta)\theta'\right)\right]^{-1}\mathrm{d}x \tag{8}$$

Satisfying the constraint conditions and corresponding to the boundary conditions of Eq.(1), namely Eqs. (3) and (4).

If

$$F(\theta,\theta')=\left[\frac{\partial}{\partial\theta}\left(D(\theta)\theta'\right)\right]^{-1} \tag{9}$$

To obtain the extreme value of the functional, the Euler– Lagrange Equation [21] must be satisfied:

$$F-\theta' F_{\theta'}=C \tag{10}$$

where $F=F(\theta,\theta')$; C is a constant; and $F_{\theta'}$ is

$$F_{\theta'}=\frac{\partial F}{\partial\theta'} \tag{11}$$

Obtaining:

$$C\frac{\mathrm{d}D}{\mathrm{d}\theta}\mathrm{d}\theta=2\mathrm{d}x \tag{12}$$

From this, we can get

$$D(\theta)=\frac{2x-C_0}{C} \tag{13}$$

where C_0 is the integral constant.

According to the boundary conditions, the solution can be obtained:

$$C=\frac{2x_\mathrm{f}}{D(\theta_i)-D(\theta_0)} \tag{14}$$

$$C_0=-\frac{2x_\mathrm{f}D(\theta_0)}{D(\theta_\mathrm{i})-D(\theta_0)} \tag{15}$$

Then there is

$$D(\theta)=D(\theta_0)+\left[D(\theta_i)-D(\theta_0)\right]\frac{x}{x_f} \tag{16}$$

Namely:

$$\frac{D(\theta)-D(\theta_0)}{D(\theta_i)-D(\theta_0)}=\frac{x}{x_f} \tag{17}$$

Based on Eqs. (16) and (17), it can be seen that the diffusion function is related to the ratio of soil horizontal distance and wetting front, initial conditions and boundary conditions. Given initial and boundary conditions, the diffusion function is linearly dependent on the ratio of spatial distance and the distance of wetting front. On the contrary, if the diffusion function is given, the distribution of volumetric water content with spatial location can also be calculated based on Eq.(17). As some integrals need to be solved numerically in the process of obtaining the final solution based on Boltzmann transform, the solving process of the above method is much simpler compared to that based on Boltzmann transform.

The Brooks-Corey soil−water characteristic curve model[22] is often used to study the hydraulic properties of unsaturated soils, and it is expressed as

$$S_e=\begin{cases}\left(\dfrac{h_b}{h}\right)^n, h>h_b\\ 1,\ h\leqslant h_b\end{cases} \tag{18}$$

where $S_e=\dfrac{\theta-\theta_r}{\theta_s-\theta_r}$ is the effective saturation; h is the suction head; h_b is the air-entry pressure head ; n is the shape parameter; θ_s is the saturated volumetric water content and θ_r is residual water content.

The corresponding permeability function K is

$$K=\begin{cases}K_s\left(\dfrac{h_b}{h}\right)^m, h>h_b\\ K_s,\ h\leqslant h_b\end{cases} \tag{19}$$

where $m=3n+2$; K_s is the saturated infiltration coefficient.

According to Eqs. (18) and (19), we can get

$$D(\theta)=K\frac{dh}{d\theta}=D_0S_e^{\frac{2n+1}{n}} \tag{20}$$

where $D_0=-\dfrac{K_sh_b}{n(\theta_s-\theta_r)}$.

Substituting Eq. (20) into Eq. (17), we can get

$$\frac{(\theta-\theta_r)^{\frac{2n+1}{n}}-(\theta_0-\theta_r)^{\frac{2n+1}{n}}}{(\theta_i-\theta_r)^{\frac{2n+1}{n}}-(\theta_0-\theta_r)^{\frac{2n+1}{n}}}=\frac{x}{x_f} \tag{21}$$

Namely:

$$\frac{S_e^{\frac{2n+1}{n}}-S_0^{\frac{2n+1}{n}}}{S_i^{\frac{2n+1}{n}}-S_0^{\frac{2n+1}{n}}}=\frac{x}{x_f} \tag{22}$$

where

$$S_0=\frac{\theta_0-\theta_r}{\theta_s-\theta_r},\ S_i=\frac{\theta_i-\theta_r}{\theta_s-\theta_r} \tag{23}$$

So, according to Eq. (22), the distribution of effective saturation can be obtained:

$$S_e=\left[\frac{x}{x_f}\left(S_i^{\frac{2n+1}{n}}-S_0^{\frac{2n+1}{n}}\right)+S_0^{\frac{2n+1}{n}}\right]^{\frac{n}{2n+1}} \tag{24}$$

Based on Eq.(24), it can be found that the volumetric water content presents a power function distribution with the change of spatial position and the exponent depends on the shape parameter n. Next, the relationship between the distance of wetting front and the consumption time of infiltration will be deduced based on the relationship between infiltration rate and cumulative infiltration volume.

The infiltration rate is

$$q=D(\theta)\frac{\partial\theta}{\partial x}\Big|_{x=0}=aD(\theta_0)S_0(\theta_s-\theta_r) \tag{25}$$

where q is the infiltration rate.

$$a=\left(S_i^{\frac{2n+1}{n}}-S_0^{\frac{2n+1}{n}}\right)\Big/x_f \tag{26}$$

The relationship between the distance of wetting front and the time consumption of infiltration is as follow[23]:

$$x_f^2=At \tag{27}$$

where A is a constant.

The cumulative infiltration volume Q is

$$Q=\int_0^{x_f}(\theta-\theta_i)dx=\int_0^t q dt \tag{28}$$

Substituting Eqs.(24), (25) and (27) into Eq. (28) yields:

$$A=\frac{2(\beta+1)(f-b)^2b^{\frac{1}{\beta}}D_0}{\beta^2\left(f^{\frac{\beta+1}{\beta}}-b^{\frac{\beta+1}{\beta}}\right)-\beta(\beta+1)(f-b)S_i} \tag{29}$$

where $f=\dfrac{D(\theta_i)}{D_0}$, $b=\dfrac{D(\theta_0)}{D_0}$, $\beta=\dfrac{2n+1}{n}$.

Based on Eqs. (24), (27) and (29), the distribution of volumetric water content with the change of spatial location and infiltration time is explicitly expressed.

In addition, when the infiltration boundary is saturated volumetric water content, Eq. (24) can be degenerated to the results of Su et al. [17].

$$\theta=(\theta_s-\theta_r)\left(1-a\frac{x}{x_f}\right)^{\frac{n}{2n+1}}+\theta_r \tag{30}$$

$$a=1-\left(\frac{\theta_i-\theta_r}{\theta_s-\theta_r}\right)^{\frac{2n+1}{n}} \tag{31}$$

When $\theta_i\to\theta_r$, i.e. the soil is in an extremely dry state and it is nearly saturated ($\theta_0 \to \theta_s$) at the infiltration boundary, Eq.(22) can be further simplified as

$$1-(S_e)^{\frac{2n+1}{n}}=\frac{x}{x_f} \tag{32}$$

Namely:

$$\theta=(\theta_s-\theta_r)\left(1-\frac{x}{x_f}\right)^{\frac{n}{2n+1}}+\theta_r \tag{33}$$

According to Eqs. (24), (27) and (29), the variation of volumetric water content with spatial location and infiltration time can be predicted in the process of water infiltration in the soil that have classical Brooks-Corey hydraulic function[23], satisfying Eq.(1) and initial boundary conditions (Eqs. (2)−(4)). In addition, the relationship obtained by this method can be expressed explicitly, which will be conducive to its practical application.

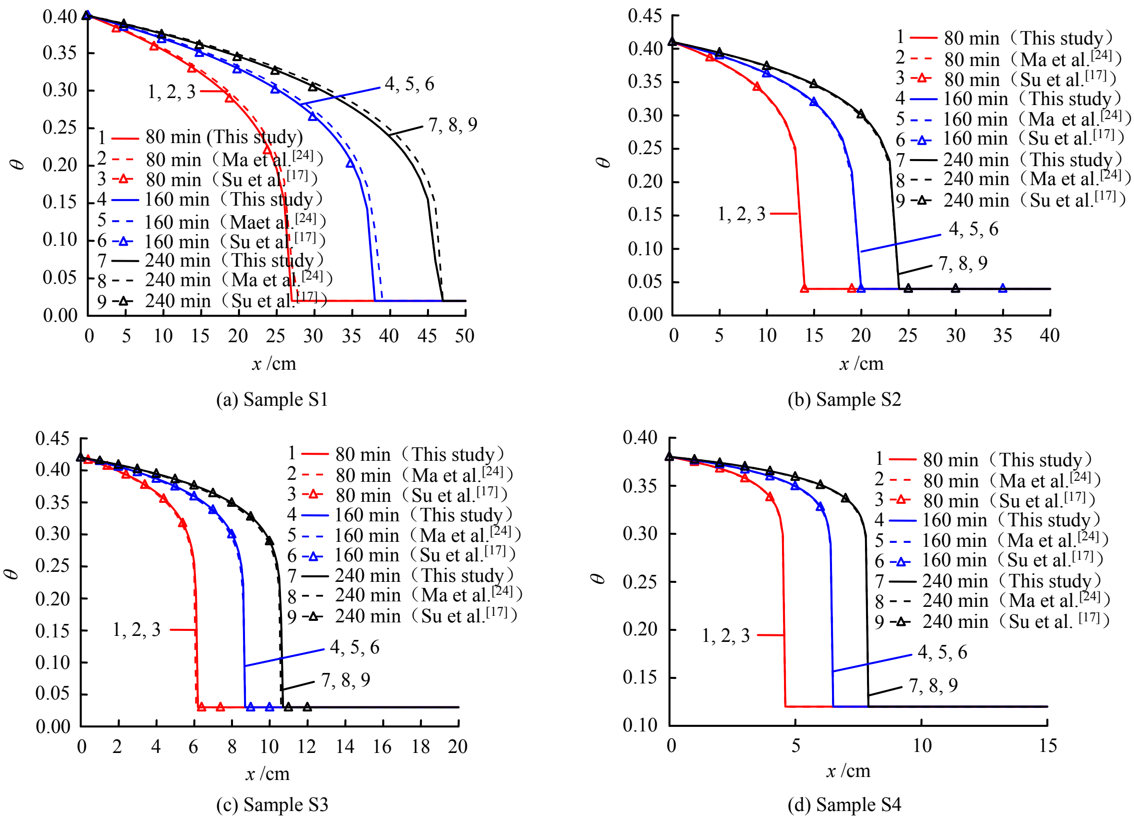
3 Numerical example

To verify the correctness and effectiveness of the explicit solution solved by the above method, the results was compared with the existing results. The hydraulic parameters of soils for calculation are given in Table 1.

Table 1 Hydraulic parameters of soils for calculation

Sample number	Saturated volumetric water content θ_s	Residual volumetric water content θ_r	Volumetric water content at the boundary θ_0	Initial volumetric water content θ_i	Saturated permeability coefficient K_s/(cm • min^{-1})	Infiltration time t /min	Shape parameter n	Air-entry pressure head h_b /cm
S1	0.40	0.02	0.40	0.02	0.40	80, 160, 240	0.6	7.25
S2	0.41	0.04	0.41	0.04	0.04	80, 160, 240	0.3	14.60
S3	0.42	0.03	0.42	0.03	0.01	80, 160, 240	0.2	11.20
S4	0.38	0.12	0.38	0.12	0.01	80, 160, 240	0.1	37.30

Figure 3 shows the distribution curves of the volumetric water content with the horizontal position of four different soils within the infiltration time of 80, 160 and 240 minutes. Results show that the distributions of volumetric water content in this paper are consistent with those of Su et al. [17] and Ma et al. [24]. Note that Su et al. and Ma et al. also studied the same infiltration problem, and they also compared the experimental results to the numerical results. However, the research work of Su et al. [17] focused on solving the mixed type Richard's governing equations, in which the relationship between the volumetric water content and the pressure head needed to be further processed. The solving method of Ma et al. [24] was Boltzmann transform, but the distribution of volumetric water content should be assumed in advance, and multiple groups of parameters were needed to be introduced.



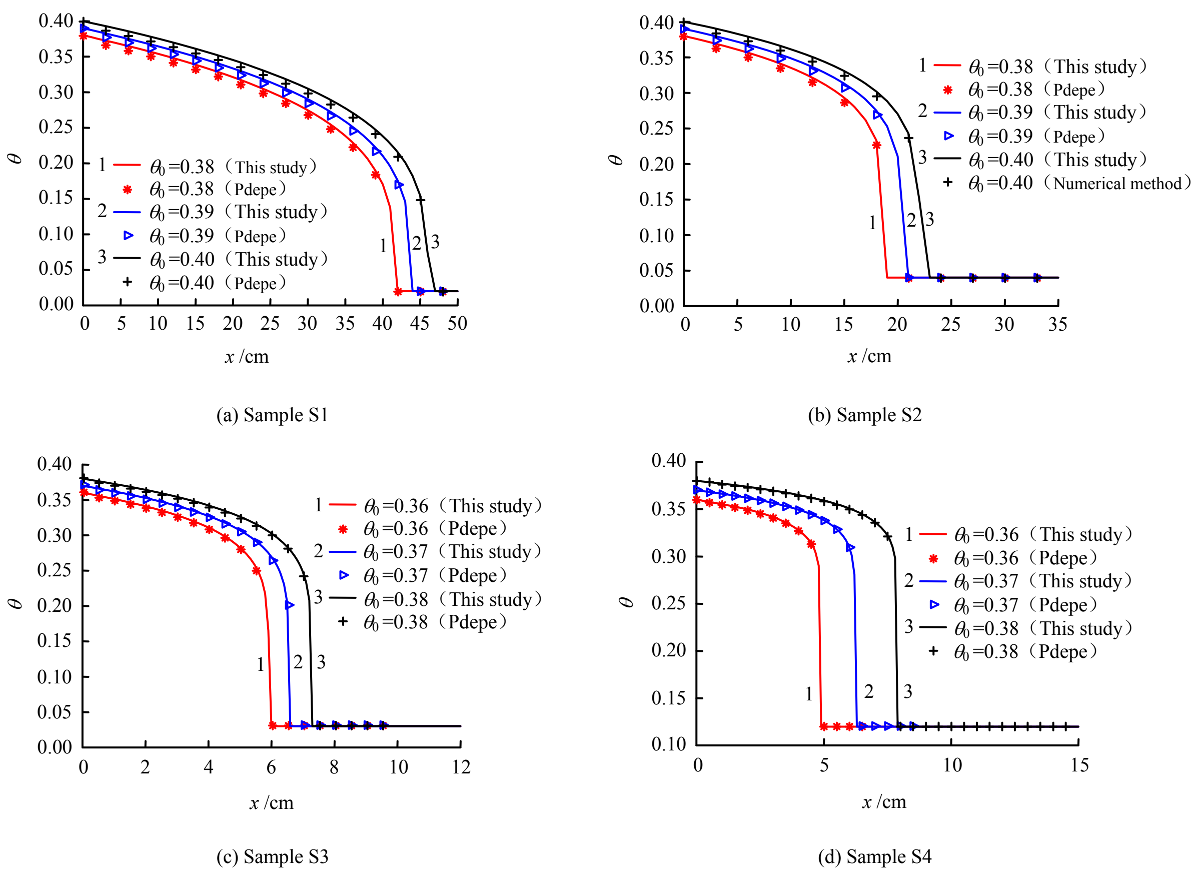
(a) Sample S1 (b) Sample S2
(c) Sample S3 (d) Sample S4

Fig. 3 Distribution of volumetric water content in different soils at different times

Furthermore, as can be seen from Fig. 3, when the saturated volumetric water content is at the infiltration boundary, the volumetric water content gradually decreases with the spatial location. At a certain position, the volumetric water content tends to be stable, and the volumetric water content increases with time at the same position.

Although the calculation results in this paper are basically consistent with those of Su et al. [17] and Ma et al. [24], it should be noted that the results from Su et al. [17] and Ma et al. [24] are only limited to the initial condition of residual volumetric water content in the initial state and the boundary condition of saturated volumetric water content at the infiltration boundary. Figures 4 and 5 show the distribution of volumetric water content in soil (240 min) with different volumetric water contents at the infiltration boundary and different initial volumetric water contents, respectively. The numerical solving method employed the Pdepe solver in Matlab to solve Eqs. (1)−(4). Pdepe is an effective numerical method, based on the line method to discretize space terms[25], and it has been used by most scholars for model calculation[26]. The parameters used in the process of solving are given in Table 2. Comparison results between Fig. 4 and Fig. 5 show that the solving results in this paper are basically consistent with the numerical simulation results, proving the rationality and effectiveness of the method of problem solving in this paper.

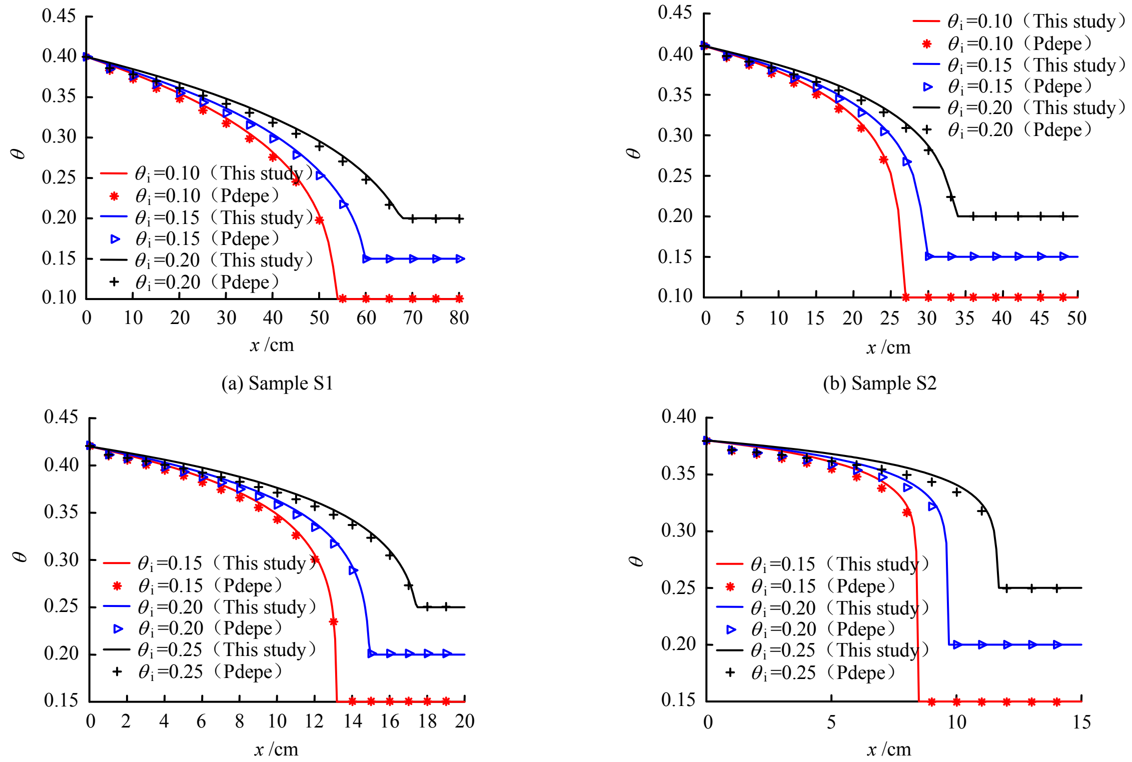
In addition, as can be seen from Fig.4, the volumetric water content gradually decreases with the spatial location until the water content is stable at a certain location, and different infiltration boundaries will cause obvious differences in the distribution of soil volumetric water content. Figure 5 shows that the distribution of volumetric water content is obviously different under different initial conditions. The influence caused by different initial conditions is not significant near the infiltration boundary, but the further away from the infiltration boundary, the more pronounced the influence is. Therefore, it can be found that the initial conditions and boundary conditions have different effects on the distribution of soil volumetric water content.



(a) Sample S1 (b) Sample S2

(c) Sample S3 (d) Sample S4

Fig. 4 Distributions of volumetric water content in different soils with different infiltration boundary conditions at 240 min



(a) Sample S1 (b) Sample S2

(c) Sample S3 (d) Sample S4

Fig. 5 Distributions of volumetric water content in different soils with different initial conditions at 240 min

Table 2 Hydraulic parameters of soils for calculation under different initial boundary conditions

Sample number	θ_0	Infiltration time /min	Initial volumetric water content θ_i	Infiltration time /min	Other parameters
S1	0.38, 0.39, 0.4	240	0.10, 0.15, 0.20	240	Same as Table 1
S2	0.38, 0.39, 0.4	240	0.10, 0.15, 0.20	240	Same as Table 1
S3	0.36, 0.37, 0.8	240	0.15, 0.20, 0.25	240	Same as Table 1
S4	0.36, 0.37, 0.38	240	0.15, 0.20, 0.25	240	Same as Table 1

4 Conclusions

Combined with the knowledge that the water movement follows the path of the extreme value of a certain action among many possible paths, the horizontal infiltration problem was transformed into the problem of the functional extremum of time by introducing the basic assumption that water flows along the shortest path in soil. Finally, the explicit analytical solution of nonlinear transient horizontal infiltration problem was obtained. The conclusions are drawn as follows:

(1) In the problems of horizontal infiltration or infiltration ignoring the influence of gravity in unsaturated soils, the diffusion function is related to the ratio of soil horizontal distance and wetting front. For constant initial and boundary conditions, the diffusion function is linearly related to the ratio of spatial distance and wetting front distance.

(2) Based on the given classical Brooks-Corey hydraulic function model, the distribution of the volumetric water content of unsaturated soils with the change of spatial location and infiltration time was explicitly solved. Results showed that the volumetric water content had a power function against the ratio of location distance and wetting front distance and the exponent depended on the shape parameter of the soil−water characteristic curve.

(3) By studying four different types of soils with different hydraulic parameters, the solving results obtained from the method proposed in this paper were compared with the existing results. Comparison results indicated that the trend of volumetric water content distribution here is consistent with that from existing results. Under different initial and boundary conditions, the solving results in this paper were compared to the numerical results. Comparison results showed that the solving results here were basically consistent with the numerical simulation results, proving the rationality of the method of problem solving in this paper. In addition, solving results showed the initial conditions and boundary conditions had different effects on the distribution of soil volumetric water content.

(4) The shortcoming of this paper is that the rationality of the introduced hypothesis needs further experimental verification and physical explanation, which will be carried out as the next work.

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