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Establishment and application of parametric geometrical damage model of rocks

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Abstract: The geometrical damage model of rocks is an important basis for the establishment of statistical damage constitutive model. Based on the mechanical properties of rock deformation, the existing geometrical damage models of rock are reviewed and these models have difficulties in considering the initial damage and post peak deformation failure characteristics. The rock mass is composed of the undamaged part, the initial damage part and the subsequent damage part, and a geometrical damage model of rocks considering the initial damage is proposed in this paper. The parametric geometric damage model of rocks is established by studying the influence of Weibull distribution parameters m and F_0 on the variation of damage variable. Furthermore, a statistical damage constitutive model of rocks characterized by strain softening is established and modified, and the determination method of model parameters is given. The model verification and parameter analysis show that the modified model can better simulate the whole process of rock deformation and failure. The influences of parameters λ and η on damage variable are equivalent to m and F_0, which solves the common problems of current rock geometric damage models. It demonstrates that the model and method in this paper are reasonable and feasible.
Keywords: rocks; geometric damage model; initial damage; post peak deformation; model parameters

1 Introduction

The determination of parameters in constitutive model of rock that is characterized as natural geological material, multiple components, heterogeneity and multi-scale complex structure, is always the concern of various engineering fields[1−2].

The classical continuum theory can reflect the apparent mechanical characteristics of rock due to the rigor system and it is the early theoretical basis of rock constitutive model, which is widely applied in the studies of rock constitutive models. However, in the continuum theory, it is to describe the main mechanical property of rocks by means of representation analogy[3−4], which means that the ideal continuum media with the same mechanical property is used to replace the natural rocks. The rock is assumed to be homogeneous. Therefore, the continuum medium model has deterministic and beautified mechanical properties. But it is difficult to fully reflect and predict the mechanical response of rocks, especially the initial damage characteristics of rocks and the deformation and failure after peaks. In fact, rock is made up of many components with different physical and mechanical properties, making the rock has a strong non-uniformity. It is found that the nonlinear rock deformation and rupture process is derived from the non-uniformity of rock based on indoor mechanical experiments. If the rock is regarded as uniform medium by ignoring its non-uniformity, the essential characteristics of rock deformation and failure process is unrevealed. Therefore, the relationship between macroscopic nonlinear deformation and failure behavior and microscopic heterogeneity of rocks is established by describing the heterogeneity of rocks with specific probability distribution based on statistical damage mechanical theory[5−10]. It focuses on the failure criteria of element strength and distribution types[11−13], effects of stress state and damage threshold[14−16], determination methods of model parameters[17−19] etc., making it has a good application effect in simulating the rock deformation and failure process. However, they still can not accurately describe the complex mechanical properties of rocks[17−20]. The reason for this is that the existing rock geometry damage model has defects or limitations in the following aspects: (i) Natural defects such as cracks and joints inevitably exist in the rock, so that there is always initial damage. However, the current damage model often ignores natural defects and the initial damage value is assumed to 0. (ii) The elastic modulus depends on the stress state of rock mass, while the existing damage model often ignores the relationship between the pre-peak mechanical characteristics and the pore closure effect, and the elastic modulus of rock is default set to the constant; (iii)The post-peak characteristics of rock mass have strong uncertainty. However, the statistical damage constitutive model based on the existing damage model can only reflect the rock mechanical characteristics under the specific confining

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pressure condition. In addition, the actual stress−strain state and the experimental stress−strain state are different. The theoretical curve of the constitutive model will be strongly deviated from the experimental curve by ignoring the difference, thus it is urgent to improve the physical concept of the constitutive model. Therefore, the key points of this paper are to explore the rock deformation and failure process under different stress conditions. Besides, a new rock geometry damage model, which has the advantages of existing damage models but overcomes the defects of current damage models, is established in this paper.

In this study, the rock deformation characteristics is analyzed based on the whole process of rock deformation and failure in the triaxial compression test. The rock damage models are reviewed and the parametric rock geometric damage model is established. Furthermore, the statistical damage theory is introduced to establish and verify the statistical damage constitutive model. The rationality and feasibility of the constitutive model and method in this paper are verified by comparing with the experimental results from triaxial compression tests.

2 Rock deformation and failure process and rock damage model analysis

2.1 Mechanical characteristics of rock deformation under triaxial compression

The triaxial compression tests are widely used to measure the rock deformation characteristics, and the experimental results are the important basis for verifying the rationality and feasibility of the constitutive model. Figure 1 shows the complete stress−strain curves of red sandstone under different confining pressure conditions measured by the MTS815.03 electro-hydraulic servo control rigid press[21]. The deformation mechanical characteristics of red sandstone are briefly analyzed as follows:

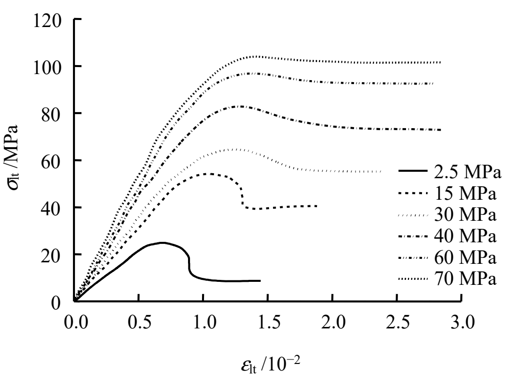


Fig. 1 Stress−strain curves of red sandstone under triaxial compression

(1) The stress characteristic values (i.e. yield stress, peak stress and residual stress) of red sandstone, will be increased with the increase of confining pressure, meanwhile the strain corresponding to the stress characteristic value also increases. The elastic modulus of red sandstone is not a constant, but it increases with the increase of confining pressure until a stable state, which is related to the initial damage of rock.

(2) If the triaxial compression tests are repeated under the same confining pressure, the heterogeneity of red sandstone will inevitably lead to different stress−strain test curves, especially after the peak failure process. The stress−strain test curve of granite samples under the same confining pressure (e.g., 0 MPa or 5 MPa) is not consistent[22].

(3) Red sandstone under different confining pressure conditions shows different deformation stages. It is generally believed that the complete stress−strain curves of rock at low stress state is divided into linear elastic deformation stage, yield hardening deformation stage, strain softening deformation stage and residual strength deformation stage, etc. If the statistical damage constitutive model can accurately simulate the whole process, it is necessary to establish a geometric damage model that can fully reflect the deformation characteristics of rocks.

2.2 Damage model analysis

Rock damage model is one of the key components to establish the statistical damage constitutive model. At present, the typical rock geometric damage model that is widely used is based on the strain equivalent hypothesis[23]. As shown in Fig. 2, the total area of the loaded microelements and the nominal stresses are A_t and σ_i, respectively. The effective stress of undamaged part and the area of undamaged microelement are σ_i' and A_u, respectively. The effective stress of failure part and the area of failure microelement are 0 and A_n, respectively.

A typical rock damage model can be established by microscopic force analysis methods:

$$\sigma_i = \sigma_i'(1-D) \tag{1}$$

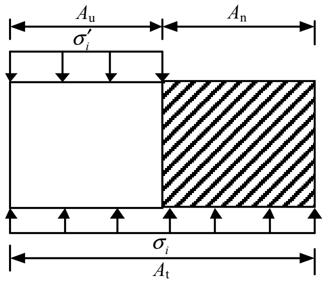


Fig. 2 Typical geometric damage model for rocks

where $D = A_n/A_t$ is the damage variable. When the rock is totally damaged, D is 1 and the theoretical bearing capacity σ_i is 0. In fact, the rock still has residual bearing capacity after the damage. It means that the statistical damage constitutive model based on Eq. (1) can reflect the strain softening stage of rocks, but it cannot represent the residual strength deformation stage. Therefore, Shen[24] and Cao et al.[17] thought that σ_i was shared by both the undamaged and damaged rock material in the improved rock geometric damage model, see in Fig. 3, and σ_i'' is the effective stress of the damaged part.

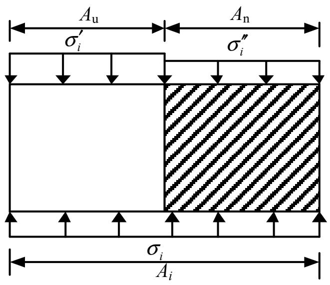


Fig. 3 Improved geometric damage model for rocks

An improved rock damage model can be established by microscopic force analysis methods:

$$\sigma_i = \sigma_i'(1-D) + \sigma_i''D \quad (2)$$

where the effective stresses of rock damage part (σ_1'' and σ_3'', i = 1,3) in triaxial stress state can be written as

$$\sigma_1'' = \sigma_3''\tan^2\beta + 2c\tan\beta \quad (3)$$

$$\sigma_3'' = \frac{(1+\mu)\sigma_c^2\cot\varphi}{3(1+\tan\beta)E\varepsilon_1} - \frac{c(\cot\varphi + 2\sin\beta)}{\sin\beta(1+\tan\beta)} \quad (4)$$

where $\beta = \varphi/2 + \pi/4$; c and φ are the rock cohesion and internal friction angle; E and σ_c are the elastic modulus and uniaxial compressive strength, respectively. The Eq. (2) seems to be more reasonable than Eq. (1) because the damaged part of rock can bear the load σ_i''. In fact, σ_i'' decreases with the increase of axial deformation, and the bearing capacity of damaged part gradually disappears with the increase of deformation. It is predicted that the σ_i'' tends to 0 when the deformation is large enough, and the theoretical bearing capacity σ_i finally turns to 0. Therefore, Eq. (2) can only approximately describe the residual deformation within a limited deformation range. Cao et al.[16] thought that the effective stress undertaken by the damage part was residual stress σ_r. The geometric damage model is shown in Fig. 4.

An improved rock damage model considering the residual strength of rocks can be established by microscopic force analysis methods:

$$\sigma_i = \sigma_i'(1-D) + \sigma_r D \quad (5)$$

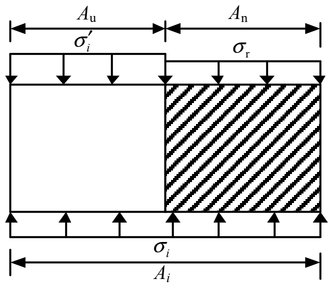


Fig. 4 Geometric damage model for rocks considering residual strength characteristics

where σ_r does not change with the increase of rock deformation. The statistical damage model based on Eq. (5) not only has the advantages of previous two models, but also can better reflect the rock characteristics at the residual strength deformation stage. Although the existing rock damage models can describe the deformation characteristics of rocks at different stages, however, it is found that they cannot reflect the initial rock damage characteristics based on the mechanical analysis of rock deformation characteristics.

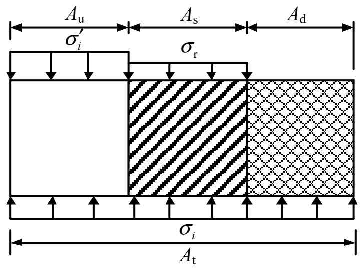


Fig. 5 Geometric damage model considering the initial damage

The initial damage is caused by geological process and human activities, so that the rock specimens in the laboratory experiment inevitably contain initial damages, which are manifested in the multi-scale inter-connected complex crack network with diverse geometries and strong random distribution characteristics. These fracture networks are unable to share the load on the rock, which will be destroyed with continuous deformation and failure behavior under continuous stress loading. Thus, the rock geometric damage model considering the initial damage is proposed, as shown in Fig. 5. The effective stress of subsequent damaged part and the microelement area are σ_r and A_s, respectively. The effective stress of initial damage part and its microelement area are 0 and A_d, respectively. The rock damage model considering the initial damage is established by the microscopic force analysis method:

$$\sigma_i = \sigma_i'(1-D_t) + \sigma_r D_s \quad (6)$$

Here,

$$D_t = D_d + D_s \quad (7)$$

where D_t, D_d and D_s are the total damage variable, initial damage variable and subsequent damage variable, respectively. When $D_d = 0$, Eq. (5) is the special example of Eq. (6) with $D_t = D_s$.

3 Establishment of parametric geometric damage model

Combining the continuum damage theory and statistical strength theory, the statistical damage constitutive model is established based on the effective stress and strain equivalence hypothesis. At present, there are different types of microelement strength distribution, such as Weibull distribution, logarithmic normal distribution, normal distribution and power function distribution, among which Weibull distribution has been widely applied. Therefore, the determination of distribution parameters m and F_0 is the key for the application of the statistical damage constitutive model.

The distribution parameters m and F_0 can be determined mainly by linear fitting method[25], inversion analysis method[26], and the peak point method[27] etc. Among them, the discrete points in experiments can be fitted by linear fitting method to obtain the distribution parameters, and the processing is simple but there is no physical significance. The inversion analysis method can get more accurate distribution parameters, but it requires a large number of experimental data. The physical significance of peak point method is clear and the processing is simple, making it widely applied. It requires the statistical damage model to meet the following two geometric governing equations.

$$\sigma_1(\varepsilon_1)\Big|_{\varepsilon_1=\varepsilon_{1c}} = \sigma_{1c} \quad (8)$$

$$\left.\frac{\partial \sigma_1}{\partial \varepsilon_1}\right|_{\sigma_1=\sigma_{1c},\, \varepsilon_1=\varepsilon_{1c}} = 0 \quad (9)$$

where, σ_{1c} and ε_{1c} are the peak stress and the corresponding strain, respectively. Thus, the Eqs. (8) and (9) require the theoretical curve passes the peak point of the experimental curve and satisfies the extreme value characteristics to solve the distribution parameters m and F_0 by combining the governing equations. It is worth noting that only a set of m and F_0 can be obtained under a specific confining pressure, indicating that there is only one theoretical curve of the existing constitutive model under the same confining pressure, for example, the theoretical curve of the statistical damage model based on Eq. (5), as shown in Fig. 6.

However, the above governing equations cannot become any constraints on the deformation and rupture process of rock after peak. It shows that the distribution parameter values calculated by the peak point method have strong randomness in the simulation of rock deformation after peak. It can neither be used for the quantitative simulation, nor can explain the characteristics of different rock samples under the same confining pressure condition, which is the common problem for the existing statistical damage constitutive model. This paper establishes the parametric rock geometric damage model on the basis of the damage model, and applies it in the statistical damage constitutive model, which can reflect the initial damage characteristics and quantitatively describe the deformation and rupture process of the rock after peak.

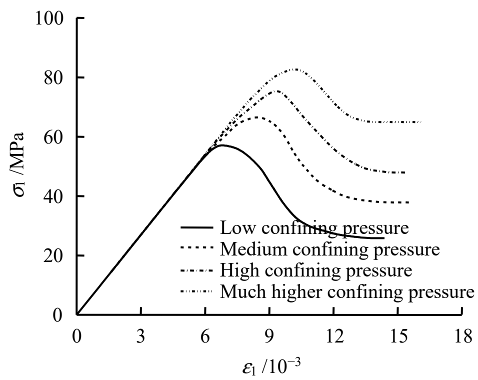


Fig. 6 Theoretical curves of statistical damage constitutive model of rocks

The microelement strength F obeys the Weibull distribution, thus the damage variable D can be expressed as

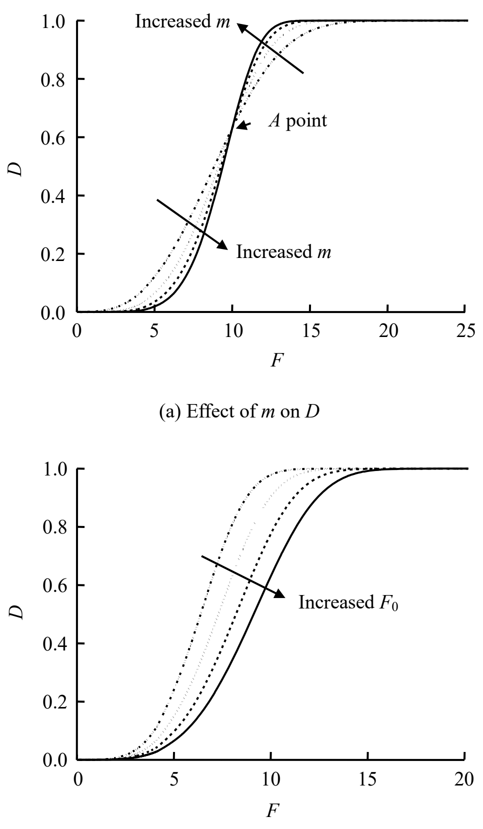
$$D = 1 - \exp[-(F / F_0)^m] \quad (10)$$

The variation of D is only related to the distribution parameters m and F_0, as shown in Fig. 7. When D increases from 0 to 1, the D curve presents the S-shape. With the increase of m, the shape of D curve remains unchanged and it rotates counterclockwise around point A. The variation rate of damage increases, indicating that the rate of stress drop after peak accelerates. With the increase of F_0, the shape of D curve remains the same and the position moves to right, and the damage variation rate decreases, indicating that the stress drop rate decreases after rock peak. Therefore, according to the existing statistical damage model, it can be known that the theoretical curve variation of the model is not only affected by the parameters of rock deformation, but also affected by D that is controlled by m and F_0. However, the parameter determined by the peak point method is unique and D does not change with parameters of m and F_0. Therefore, the existing statistical damage constitutive model cannot quantitatively describe the deformation and failure process of rock after peak.

Therefore, there are two main ways to establish geometric damage models that can accurately reflect characteristics variation of rock after peak: (i) the damage model considers the effect of particle composition and mesoscopic structure distribution on the actual physical and mechanical field of the rock; (ii) the existing geometric damage model can be improved, and its model parameters can play the same role as the distribution parameters m and F_0 in the post-peak characteristics of rock. The former is more complex and needs to be analyzed for specific problems, which is not discussed here. In this paper, we mainly works on the latter method.

In order to quantitatively analyze the indoor test results that are the overall mechanical response of the rock under external loading, the parametric geometric damage model of rock, which introduces the parameters λ and η to improve Eq. (6) and considers the initial damage, is established:

$$\sigma_i = \sigma_i'(1 - D_d - \lambda D_s) + \eta \sigma_r D_s \tag{11}$$



(a) Effect of m on D

(b) Effect of F_0 on D

Fig. 7 Impacts of distribution parameters on the damage variable *D*

When $\lambda = \eta = 1$, the Eq. (11) can be rewrite into Eq. (6). Equation (11) keeps consistent with the existing damage model, making it not only inherit the advantages of the existing damage model, but also benefits for mutual analysis, comparison and redevelopment of software. Whether the statistical damage constitutive model that considers the number of parameters and the establishment method of the damage model can reflect the initial damage characteristics and quantitatively describe the deformation and failure process after the peak will be analyzed in Section 5.

4 Statistical damage simulation method of rock strain softening

4.1 Statistical damage constitutive model

Based on the damage mechanics theory, when it does not attain the subsequent damage threshold under stress, the rock deformation can be fully restored. The deformation mechanics characteristics of the undamaged material belong to the elastic category and it obeys the generalized Hook Law[28],

$$\sigma_i' = E_u \varepsilon_i + \mu(\sigma_j' + \sigma_k') \tag{12}$$

where E_u and μ are the elastic modulus and Poisson's ratio of undamaged material; σ'_j and σ'_k are the effective stress of undamaged material in the j and k directions, respectively. According to the rock deformation and rupture process under triaxial compression, the rock resists load in the axial direction and deformation and fracture occurs. The rock sample deformation is constrained in the lateral direction and the bearing capacity is improved. It is generally believed that the rock damage only occurs in the axial direction and obeys

$$\sigma_j' = \sigma_j \tag{13}$$

$$\sigma_k' = \sigma_k \tag{14}$$

where σ_j and σ_k are nominal stresses of the rock in the j and k directions, respectively. In order to reflect the microscopic heterogeneity caused by the cracks and voids that represent the difference in microscopic composition of rocks, the microelements with damage are used for analysis. Because the nature of rock under load is the strength failure, this paper gives the mechanical strength of microelement F and considers the effects of stress state and subsequent damage threshold, the unified failure criterion is used for all microelements strength failure[9].

$$F = E_d \varepsilon_1 + (2\mu - \alpha)\sigma_3 - \kappa \tag{15}$$

Here,

$$\alpha = (1 + \sin\varphi_y) / (1 - \sin\varphi_y) \tag{16}$$

$$\kappa = 2c_y \cos\varphi_y / (1 - \sin\varphi_y) \tag{17}$$

where E_d and ε_1 are the elastic modulus and strain of rock; φ_y and c_y are the internal friction angle and cohesion after subsequent failure of rock. In order to simulate the rock deformation and rupture process, the probability description of microelement strength is also given to form the microscopic probability element. Thus, the heterogeneous microstructure properties are transformed into macroscopic material properties. The strength of microelement based on Weibull distribution is discretized.

The number of damaged microelements during continuous deformation and rupture process under load increases, and the subsequent damage variable D_s gradually increases. The total damage of rock includes initial damage and subsequent damage, so the upper limit of D_s is less than 1. The evolution of subsequent damage model based on the Eq. (11) is

$$D_s = \begin{cases} 0 & F < 0 \\ 1 - \exp[-(F/F_0)^m] & 0 \leqslant F < F_s \\ (1 - D_d)/\lambda & F_s \leqslant F \end{cases} \quad (18)$$

where F_s is the microelement strength of D_s reaching the upper limit. Since rock deformation and rupture is a continuous damage process, F_s can be determined by the continuity condition of Eq. (18), namely,

$$F_s = F_0 \left(-\ln \frac{D_d + \lambda - 1}{\lambda} \right)^{1/m} \quad (19)$$

The rock statistical damage model under triaxial compression conditions can be obtained by combing Eqs. (11) to (14) and Eq. (18), namely,

$$\sigma_1 = \begin{cases} (1 - D_d)(E_u \varepsilon_1 + 2\mu\sigma_3) & F < 0 \\ (1 - D_t) E_u \varepsilon_1 + \xi D_s + 2\mu\sigma_3 (1 - D_d) & 0 \leqslant F < F_s \\ (1 - D_d)\eta\sigma_r / \lambda & F_s \leqslant F \end{cases} \quad (20)$$

Here,

$$\xi = \eta\sigma_r - 2\lambda\mu\sigma_3 \quad (21)$$

It is worth noting that the result of triaxial compression test is the axial stress and axial strain (σ_{1t}-ε_{1t}) curve, the measured apparent confining pressure σ_3 can be used directly, but the axial stress σ_{1t} needs to be processed. When $\sigma_3 = 0$, the measured σ_{1t}-ε_{1t} curve is the real σ_1-ε_1 curve. When $\sigma_3 \neq 0$, the measured σ_{1t} is σ_1 minus σ_3, namely,

$$\sigma_1 = \sigma_{1t} + \sigma_3 \quad (22)$$

Since the hydrostatic pressure does not cause subsequent damage of rock, i.e., $D_s = 0$. The initial axial strain ε_0 is available by using the first equation in Eq. (20):

$$\varepsilon_0 = \frac{[1 - 2\mu(1 - D_d)]\sigma_3}{E_u (1 - D_d)} \quad (23)$$

Therefore, the measured axial strain ε_{1t} can be written as

$$\varepsilon_1 = \varepsilon_{1t} + \varepsilon_0 \quad (24)$$

4.2 Determination method of model parameters

It can be seen in Eq. (20) that the determination of D_d, E_u, m, F_0, λ and η is the key for the establishment of statistical damage constitutive mode. It should be noted that the determination methods will be introduced later as the role of λ and η is unknown.

4.2.1 Determination method of D_d

Because the fracture network weakens the mechanical properties of rock deformation, the macroscopic mechanical response is the decrease of rock strength and elastic modulus. D_d can represent the deterioration degree of damaged rock elastic modulus E_d to the intact rock elastic modulus E_u[29], namely,

$$D_d = 1 - E_d / E_u \quad (25)$$

4.2.2 Determination method of E_u

There are different scales of micro-cracks and pores in the rock. The complete closure of pores requires extremely high compressive stress, and it is difficult to obtain the elastic modulus E_u of intact rock. The normalization of triaxial compression experimental results is a simple and effective method by using the statistical method, and the relationship between E_d and σ_3 is established[30] as follows:

$$E_d / E_c = a / \{1 + b\exp[-k(\sigma_3 / \sigma_c)]\} \quad (26)$$

where E_c is the elastic modulus of rock under uniaxial compression; a, b and k are the experimental constants. It can be found that the normalized elastic modulus increases nonlinearly with the increase of standardized confining pressure. When the confining pressure tends to be extremely large, the initial defects in the interior of rock are eliminated:

$$\lim_{\sigma_3 \to \infty} \frac{a}{1 + b\exp[-k(\sigma_3 / \sigma_c)]} = a \quad (27)$$

Therefore, the E_u can be determined by

$$E_u = aE_c \quad (28)$$

4.2.3 Determination method of m and F_0

The commonly used peak point method is employed in the determination of m and F_0. The analytical expression of m and F_0 can be obtained by substituting Eq. (20) into Eq. (8) and Eq. (9), respectively:

$$m = \frac{F_c (1 - D_d - \lambda D_{sc})}{(1 - D_d)(1 - D_{sc})(\xi - \lambda E_u \varepsilon_{1c}) \ln(1 - D_{sc})} \quad (29)$$

$$F_0 = F_c [-\ln(1 - D_{sc})]^{-1/m} \quad (30)$$

where

$$F_c = E_d \varepsilon_{1c} + (2\mu - \alpha)\sigma_3 - \kappa \tag{31}$$

$$D_{sc} = \frac{(1 - D_d)(E_u \varepsilon_{1c} + 2\mu\sigma_3) - \sigma_{1c}}{\lambda E_u \varepsilon_{1c} - \xi} \tag{32}$$

5 Validation of parametric damage model and parameter analysis

Except λ and η, the determination method of other model parameters has been introduced. In order to prove that the parametric damage model can effectively solve the defects and shortcomings of the existing rock damage model, it is now verified step by step based on the model establishment process. First, the rock geometric damage model (i.e., Eq. (6)) that considers the initial damage has been verified to discuss the simulation effect. The parameters λ and η are then analyzed in the parametric rock geometric damage model (i.e., Eq. (11)). The determination method has been proposed and the Eq. (11) has been verified based on their effects to show the rationality and feasibility of the model in this paper.

5.1 Validation of geometric damage model considering the initial damage

Based on the experimental data of red sandstone[21], E_c = 4.1 GPa, μ = 0.25, σ_c = 19.8 MPa, c_y = 7.22 MPa, φ_y = 20°. Based on Eq. (26), the elastic modulus of red sandstone under different confining pressures is normalized, obtaining the test constant a = 2.477, b = 1.385, k = 0.776 and R^2 = 0.97, as shown in Fig. 8. Therefore E_u = 10.156 GPa is obtained based on Eq. (28).

Since Eq. (6) is the special form of Eq. (11), when $\lambda = \eta = 1$, Eq. (20) is the statistical damage constitutive model based on Eq. (6). The model curve σ_1-ε_1 is compared with the red sandstone test curve σ_{1t}-ε_{1t}. Meanwhile, the model axial stress and strain are corrected according to Eqs. (22)−(24) to obtain the model curve σ_{1t}-ε_{1t}. Besides, the total damage curve D_t-ε_{1t} and subsequent damage curve D_s-ε_{1t} are obtained by Eq. (9) and Eq. (18), see Fig. 9.

(1) Model curve σ_1-ε_1 can qualitatively reflect the deformation characteristics of red sandstone at different stages, i.e., linear elasticity, yield hardening, strain softening and residual strength. The deviation between the analytical results in model and experimental results increases with the increase of the confining pressure, making it difficult be neglected. The modified model curve σ_{1t}-ε_{1t} can eliminate the data deviation at pre-peak and quantitatively describe the test data of red sandstone at pre-peak under different confining pressure conditions, but the data deviation at post-peak still exists.

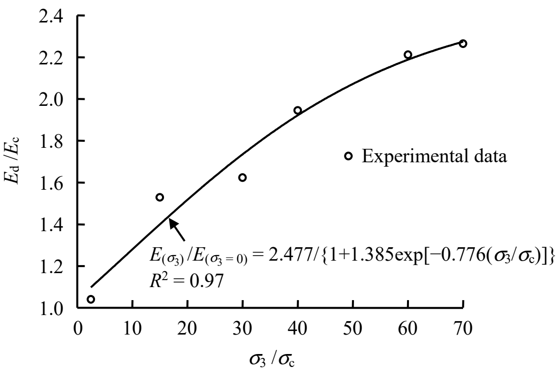
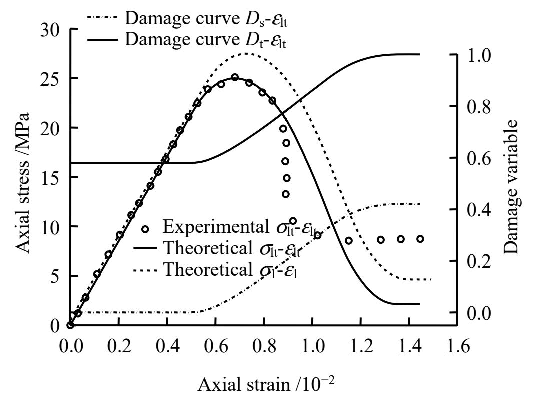
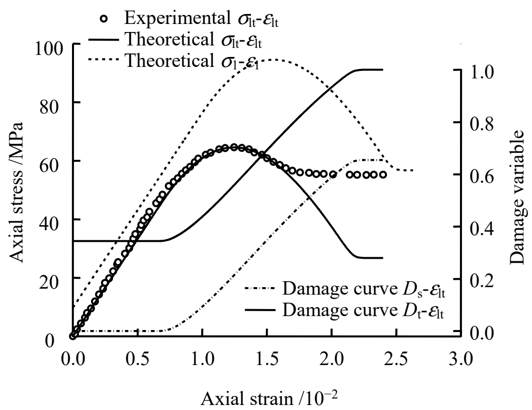


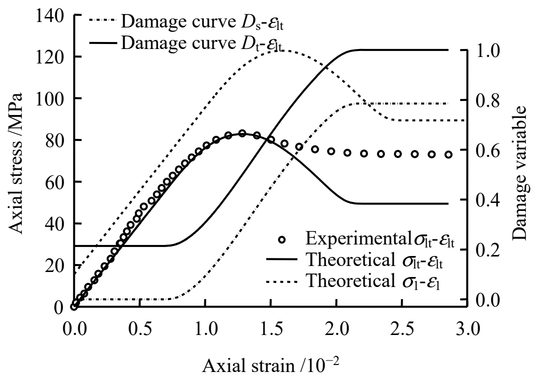
Fig. 8 Relationship between E_d/E_c and σ_3/σ_c



(a) σ_3 = 2.5 MPa



(b) σ_3 = 30.0 MPa

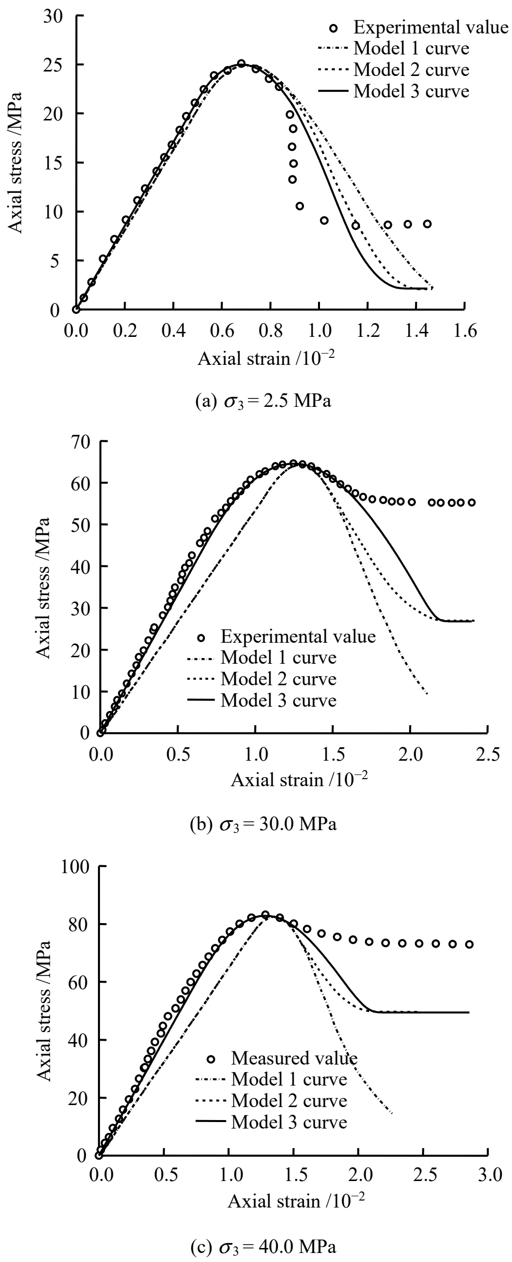


(c) σ_3 = 40.0 MPa

Fig. 9 Statistical damage constitutive model curves of red sandstone under the consideration of initial damage

(2) The modified model curve σ_{1t}-ε_{1t} can reflect the initial damage characteristics of red sandstone, and E_d increases with the increase of confining pressure, but D_d decreases with the increase of confining pressure. The variation laws of D_t-ε_{1t} and D_s-ε_{1t} are shown as S-type, and the distance between these two damage curves is D_d. It is shown from the rock damage mechanics theory that the variation law can explain the whole deformation and rupture process of red sandstone.

In order to clearly show the defects of existing rock geometric damage model, the corresponding statistical damage constitutive model is established and verified based on Eqs. (1), (5) and (6), respectively. The constitutive models after revised are called Model 1, Model 2 and Model 3. Comparing the model curves with the experimental data of red sandstone, it can be observed from Fig. 10 that:



(a) $\sigma_3 = 2.5$ MPa

(b) $\sigma_3 = 30.0$ MPa

(c) $\sigma_3 = 40.0$ MPa

Fig. 10 Comparison of three geometric damage models of rocks

Model 1 and Model 2 can reflect the linear elasticity and yield hardening deformation of red sandstone, but they cannot describe the variation characteristics of elastic modulus. This is because that the initial state of rock is not damaged and the elastic modulus remains the same. The Model 3 established in this paper can effectively solve the problem.

Models 1, 2 and 3 can qualitatively reflect the deformation characteristics of red sandstone at post-peak, but they are seriously deviated from the experimental data. It can be seen that this is a common problem in the non-parametric rock geometric damage model, and the reasons are explained in Section 3.

5.2 Parameter analysis and validation of parametric geometrical damage model

To solve the common problems shown in the existing rock geometric damage model, the parametric geometrical damage model of rock, i.e. Eq. (11), is established on the basis of Eq. (6). The rationality and feasibility of introduced parameters and model construction methods are discussed, λ and η are set to be 1 by controlling the single-parameter variable. By analyzing the impact of λ and η variation on the theoretical curve of constitutive model and damage variable, it can be seen in Fig. 11 that:

(1) The variations of λ and η have no effect on the deformation characteristics of the model curve under different confining pressure at the pre-peak. But it has significant impact on the decrease rate of stress and residual strength characteristics of model curve after peak.

(2) With the increase of λ, the decrease rate of stress after the peak of the model curve increases, the residual strength and the upper limit of D_s ceaselessly reduces, and the shape of D_s curve remains unchanged and rotates approximately counterclockwise around a certain point, indicating that the effect of λ on damage variable is equivalent to that of m on damage variable.

(3) With the increase of η, the decrease rate of stress after the peak of the model curve decreases, the residual strength gradually increases, the upper limit of D_s remains unchanged, and the shape of D_s curve remains unchanged and its position moves to the direction of axial strain decreasing, indicating that the effect of η on damage variable is equivalent to that of m on damage variable.

From the above analysis, it can be concluded that the effects of λ and η are equivalent to that of m and F_0 on the damage variables. It indicates that m and F_0 determined by the peak point method can influence the variation law of D_s by changing λ and η values while keeping the values unchanged, so that the model can accurately describe the post-peak deformation and failure process of rock.

Based on the brittleness index of red sandstone and the residual strength, λ and η can be determined, as shown in Table 1.

The values of λ and η in Table 1 are substituted into the parametric statistical damage constitutive model, and the theoretical curve of the model under different confining pressures can be obtained as shown in Fig. 12. The plot illustrates that the model can accurately capture the initial damage and deformation damage characteristics of rock at post-peak, and the defects of above-mentioned rock damage models have been solved, implying that the model proposed in this paper has a certain degree of rationality and feasibility.

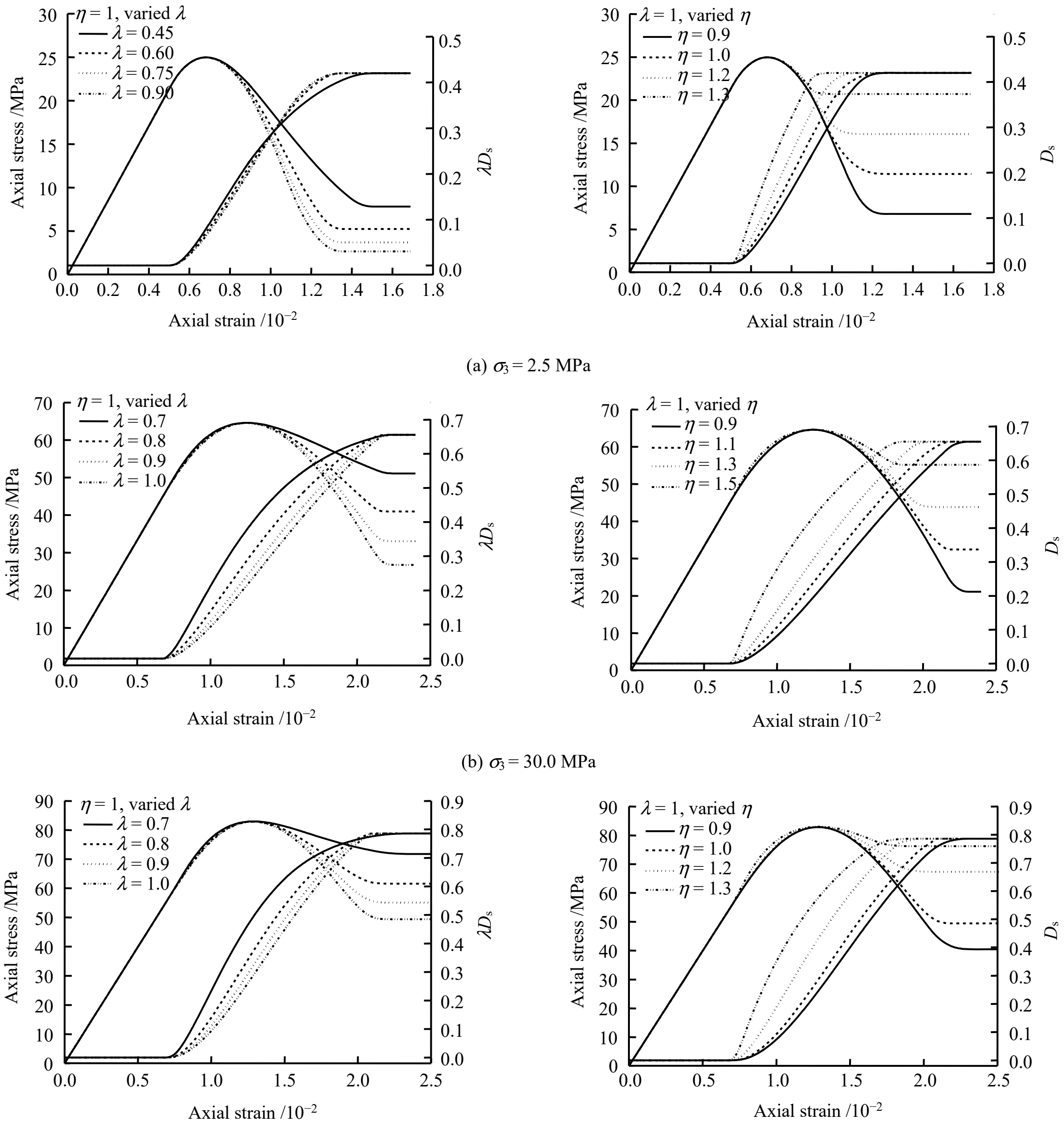


Fig. 11 Effects of λ and η on axial stress−strain curves and damage variable

Table 1 Parameters λ and η under different confining pressures

σ_3 /MPa	λ	η
2.5	1.00	2.40
30.0	0.82	1.22
40.0	0.79	1.00

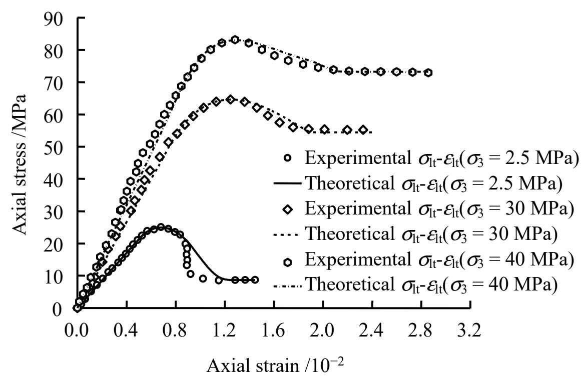


Fig. 12 Parametric statistical damage constitutive model curves of red sandstone

6 Conclusions

Based on the characteristics of rock deformation, this paper discusses the defects of existing rock geometric damage models, and puts forward parametric geometrical damage model of rock, which is applied in the statistical damage model of rock strain softening. Some conclusions can be drawn as follows:

(1) When the statistical damage model is used to simulate the stress−strain curves under rock triaxial compression tests, the axial stress and axial strain need to be calibrated. The actual axial stress equals to the addition of the measured axial stress and confining pressure. The actual axial strain is the sum of measured axial strain and initial strain.

(2) The model can simulate the deformation characteristics at the pre-peak by using the m and F_0 obtained by the peak point method, but it can't well simulate the deformation and failure characteristics at the post-peak. λ and η have no impact on the deformation characteristics at pre-peak, but they have obvious impact on the deformation and failure chaacteristics at post-peak, showing the same impact with m and F_0 on damage variables.

(3) By comparing the results calculated by the model proposed in this paper and the triaxial compression test results of red sandstone, it proves that the model can simulate the deformation characteristics of rocks at various stages, especially the deformation and failure characteristics at the post-peak stage. The existing rock geometric damage model is the particular case of the model in this paper, demonstrating that the model and method proposed in this paper is reasonable and feasible.

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