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Experimental investigation on the seepage flow through a single fracture in rocks based on the disc fracture model

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Abstract: The fracture parallel plate seepage model has been widely applied to describe the processes and characteristics of fluid flow through fractures in rocks, while the actual flow field of the fracture seepage in rocks could be far more complicated than the assumption of rock matrix to be parallel plates. As a result, it draws lots of attentions on investigations on suitability and corrections of the fracture parallel plate seepage model. In this study, by taking the disc fracture model which is closer to the actual fracture shape than the parallel plate model, a physical model of a single disc fracture was constructed in the laboratory. By changing the aperture of the disc fracture in rocks and the sizes of the water inlets and outlets, the seepage experiments under different pressure gradients were conducted to investigate the laws of fluid flow in fractures. The experiment results show that the Forchheimer model fitted the relationship between the seepage flow and pressure gradient inside the disc fracture relatively well while the Darcy law model presents also a good fitting for the Non-Darcy flow regimes. It should be noticed that the calculation formulas of the parameters *A* and *B* in the Forchheimer model need to be modified as to the Forchheimer model. Furthermore, an approach has been recommended for those modifications and the corresponding modified calculation formulas with certain reliability have been given based on the experimental results. This study is expected to provide in certain degree theoretical foundations and methods for further investigation of fluid flow through a single fracture in rocks and the complex fracture networks.

Keywords: seepage through fractures in rocks; disc fracture model; seepage flow experiment; seepage flow law; model modifications

1 Introduction

The study of fluid movement in underground rock matrix fractures is essentially the study of fluid flow in the entire fracture network models in which seepage characteristics are of great significance for guiding engineering designs. The fracture network is composed of numerous single fractures and network nodes. The permeability of a single fracture is not only related to the development and location characteristics of the three-dimensional fracture network, but also closely related to its own fracture width, shape and occurrence. Therefore, in order to better study the seepage problem of complex rock matrix fracture network, it is a pre-requisite to carry out a single fracture seepage test study. In 1856, French engineer Darcy proposed a linear relationship between seepage velocity and hydraulic gradient, which has become the most basic law of fluid movements of underground water[1]. It has been widely used in various numerical model assumptions and developed a large number of fluid flow numerical simulations tool. However, the roughness of underground fractures [2] and the state of high seepage velocity of water flow, local turbulence caused by the change of flow path direction or the continuous increase of velocity will cause non-negligible loss of inertia. Therefore, Darcy's law is not applicable in this case. At present, Bear[3], Qian et al.[4], Zhang et al.[5], Liu et al. [6], Yan et al.[7] and others have conducted research in this area, and found that the penetration velocity and hydraulic gradient satisfy the quadratic polynomial relationship (Forchheimer formula), which can be represented by the non-Darcy influence coefficient[8]. At the critical point of the two fittings, Zeng et al.[9] used the Forchheimer number to define the boundary of Darcy and non-Darcy effects; in addition, Chen et al.[10] and Li et al.[11] used power functions (Izbash equation) to describe the relationship between the two, but it was not as widely used as quadratic polynomial.

In terms of model materials currently used, there are smooth plexiglass parallel plate models, marble plate cracks, 3D printing technology, groove flow model[12], etc., but they are all based on parallel plate models where it assumes the crack widths are equal, which is in contradiction with the actual cracks whose shape is irregular, and the length of the intersection line between the cracks is also different. Therefore, the assumption that the width of the cracks is equal is quite different from the actual situation. In the study of karst fractures, the most widely used disc fracture network model is to generalize irregular fractures into circles, because of the evidence that the spatial lengths of the fractures in the strike and tendency

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are approximately equal [13–15].

In order to investigate the characteristics of seepage in a single fracture. The authors based on the previous results of generalizing the shape of a single fracture into a parallel plate of equal width, and generalizing the shape of the fracture into a disk based on the assumption of the disk model, constructed a physical model of a single disk fracture, and carried out seepage tests under different fracture geometric parameters and different hydraulic conditions. The experimental results are further analyzed to obtain the corresponding seepage law, which provides reference and correction basis for the numerical simulation of the three-dimensional fracture network.

2 Methodology

2.1 Single disk fracture physical model description

Part of the rock itself is impermeable, so that groundwater can only flow through the cracks in it. In view of the above situation, this test only considers the permeability of the fractures in the rock matrix. Furthermore, according to the actual shape of the natural fractures tend to be round or elliptical, it is generalized into a disc shape, which is more morphological than the equal width parallel plate model. Therefore, the movement of water in them can be closer to reality in nature fractures. The difference between the disc and the parallel plate is that the width of the cracks in each place is not completely the same. This article is to simulate the situation that the length of the line of intersection when the cracks intersect in the field is obviously less than the maximum width of the crack. When the length of the intersection is very large, it can be approximated as a parallel plate model. The generalized diagram of the physical model of a single disc fracture is shown in Fig.1. Zhou[16] classified the fracture aperture, among which cracks with medium width (slit width of 1 to 5 mm) are more common. This test device was made of plexiglass with a thickness of 2 cm, which was used to simulate the seepage properties of fractures with medium width, and the fracture width can be adjusted. A total of two sets of tests were completed. The first set was the main test. In the test, the diameter of the disc was 50 cm, the fracture aperture was set to be 2.3, 3.1, and 3.8 mm, and the entry and exit intersection lengths were set to 2, 4, 6, 8, and 10 cm, in which it can simulate the water movement of different intersection lengths in the fractures. The second set was a validation test. In the test, the diameter of the disc was 40 cm, the fracture aperture was set to 3.0 mm, and the intersection lengths of the entry and exit were 4, 6, and 8 cm. In both sets of tests, a gasket of the same thickness was fixed on the edge of the bottom plate of the lower disc, the upper disc plate was embedded, and the edges of the two discs were sealed with glass glue to ensure airtightness. Each gasket was accurately measured with a vernier caliper. In addition, the thickness of the plexiglass was 2 cm, which was not easy to deform, so it ensured that the internal gap of the disc remained the same. The intersection length was variable. It should be noted that, in order to compare with the traditional parallel plates of equal width, the length of the inlet and the outlet were kept the same in the experiment, and only the difference in the form of the middle seepage area was compared. The entire test system consisted of a water supply system, a buffer tank, a single disc fracture physical model and a stopwatch. The physical model parameters of the first set of test discs are shown in Table 1, and the test system is shown in Fig. 2. Figure 3 is a physical diagram of the physical model of a single disc fracture.

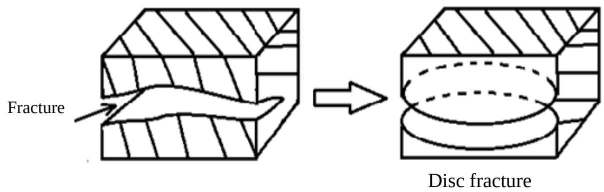

Fig. 1 Schematic diagram of a single disc fracture physical model

Table 1 Single disk physical model parameter values

Fracture aperture e / mm	Disc diameter / cm	Intersection length L / cm	Inlet number	Exit number	Device length / cm	Device width / cm	Device height / cm
2.3, 3.1, 3.8	50	2, 4, 6, 8, 10	1	1	80	52	15

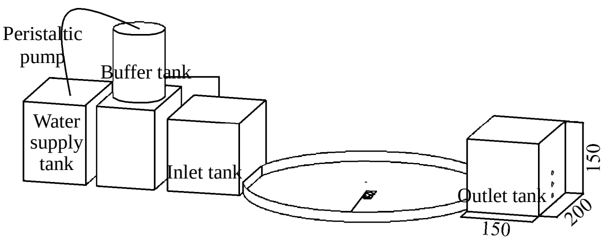

Fig. 2 Schematic diagram of the experimental system (unit: mm)

Fig. 3 Image of the single disk fracture physical model

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2.2 Experimental method

In this first set of tests, 105 sets of seepage tests were carried out. Before the beginning of the test, the device was adjusted to make it level. The pipes, the inlet tank, the inside of the fractures, and the outlet tank were filled with tap water and air was discharged to make the entire system saturated with water to meet the saturated flow conditions. A peristaltic pump is used to pump water from the water supply tank into the buffer tank; water enters the inlet tank through the buffer tank adjustment switch; the length and width of the fracture intersection line are adjusted; the water level in the inlet and outlet tanks is recorded when it is table, and then the flow rate at the outlet is measured 3 times, the average value is taken. Moreover, 7 pressure gradients are considered for each set of fracture parameters. Finally, the fracture parameters and pump speed are adjusted in turn, and the above procedure is repeated. The length of the intersection is the length of the connection between the water tank and the fracture. The number of fracture parameter groups and the corresponding pressure gradient changes are listed in Table 2.

Table 2 Number of fracture parameter groups and applied pressure gradients for first experimental set

e / mm	*L* / cm	Number of pressure gradients
2.3	2	7
2.3	4	7
2.3	6	7
2.3	8	7
2.3	10	7
3.1	2	7
3.1	4	7
3.1	6	7
3.1	8	7
3.1	10	7
3.8	2	7
3.8	4	7
3.8	6	7
3.8	8	7
3.8	10	7

2.3 Fracture parallel plate seepage model

2.3.1 Cubic law

When the fluid velocity inside a fractured rock matrix is small, the simplest fluid model for passing through a single rock fracture is the parallel plate model, which generalizes the fracture into two smooth parallel plates separated by a narrow gap. In the linear Darcy flow zone, the cubic law shown in the following equation can be used to calculate the fluid flow inside the fracture.

$$Q = -\frac{w_0 e^3}{12\mu}\nabla P \tag{1}$$

where Q is the permeation flow rate (m^3/s); w_0 is the crack width (cm); e is the fracture aperture (mm); μ is the dynamic viscosity coefficient (Pa·s); and ∇P is the pressure gradient (Pa/m).

2.3.2 Forchheimer law

For the parallel plate fracture model, when the flow velocity inside the fracture is small, it can often be calculated using cubic or Darcy's law. But as the flow velocity increases, the flow rate and pressure gradient are no longer linearly related, and the cubic or Darcy's law no longer applies. Bear[3], Chen et al.[10] proposed to use the Forchheimer law shown below to express this nonlinear relationship, namely:

$$-\nabla P = AQ + BQ^2 \tag{2}$$

$$A = \frac{12\mu}{w_0 e^3},\ B = \frac{\rho\beta}{w_0^{\,2} e^2} \tag{3}$$

where A is the linear coefficient (Pa·s/m^4); B is the non-linear coefficient (Pa·s^2/m^7); ρ is the density (kg/m^3); β is the non-Darcy coefficient (m^{-1}). This article uses A and B to characterize the parameter changes of Forchheimer's formula.

3 Seepage characteristics within disc fracture

3.1 Reynolds number range within the fractures

In order to study the characteristics of water flow inside the plexiglass disc, this study conducted seepage tests as mentioned above with fracture apertures of 2.3, 3.1, and 3.8 mm. Each fracture aperture is divided into 5 types of intersection length, i.e., 2, 4 , 6, 8, and 10 cm. Moreover, 7 groups of different pressure gradient percolation tests were carried out under each intersection length condition. On the meantime the flow rate under each group of pressure gradient was measured. In total there are 105 groups. Usually, people first select the experimental seepage calculation formula according to the flow state of the fluid[17–18]. Reynolds number is an important parameter representing the flow state and it is the ratio of the fluid inertial force to the viscous force, which fully reflects the fluid density and viscosity coefficient. Therefore, we first calculates the Reynolds number to determine the fluid flow state under all test conditions. The formula for calculating Reynolds number Re is as follows:

$$Re = \frac{\rho v d}{\mu} \tag{4}$$

where d is the characteristic length (cm); when in a non-circular pipe, d is the equivalent diameter, which is equal to 4 times the area divided by the circumference:

$$d = 4\frac{e w_0}{2(e + w_0)} \tag{5}$$

The formula for calculating Reynolds number R_e is as follows:

$$Re = \frac{\rho v d}{\mu} = \frac{\rho v}{\mu}\cdot 4\frac{e w_0}{2(e + w_0)} = \frac{2\rho v e w_0}{e + w_0} \tag{6}$$

Different from the parallel plate model with equal width, the width of the cross section along the flow direction in the disc fracture model is different, in which the length of the permeation path is different, resulting in different average hydraulic gradients of each permeation path. In order to solve the equation, it is necessary to determine the equivalent fracture width w and the equivalent penetration path length. For the disc fracture model, the equivalent fracture width w

can be estimated by $(2R-L)/\ln(2R/L)$[14], where R is the fracture radius. Considering that the perpendicular distance between inlet intersection line and outlet intersection line is the shortest penetration path, and the penetration path near the arc is the longest. This paper chooses one of the paths to estimate the equivalent penetration path length, as shown in Fig. 4.

It can be seen from Table 3 that the Reynolds number range under all test conditions is between 307 and 1594, which is laminar flow domain from the average flow velocity of the water in the disc [19].

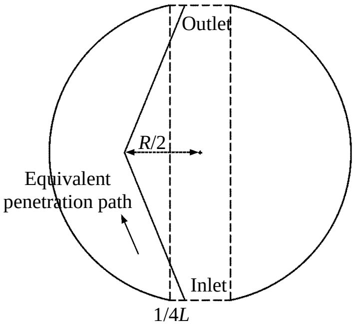


Fig. 4 A typical equivalent path method

Table 3 Reynolds numbers under various experimental conditions

L / cm	e=2.3 mm		e=3.1 mm		e=3.8 mm	
	∇P / (Pa • m^{-1})	Re	∇P / (Pa • m^{-1})	Re	∇P / (Pa • m^{-1})	Re
2	177	307	283	416	247	403
2	318	399	406	495	389	514
2	459	485	459	536	565	612
2	583	540	654	625	813	705
2	742	616	919	735	901	763
2	848	686	1 131	854	1 096	835
2	1 043	786	1 661	1 055	1 484	1 020
4	125	327	267	615	89	427
4	249	454	445	801	178	581
4	427	628	730	1 036	356	799
4	623	810	926	1 218	552	986
4	837	925	1 033	1 313	712	1 094
4	1 068	1 126	1 282	1 490	855	1 236
4	1 300	1 260	1 496	1 594	1 068	1 490
6	161	413	197	584	108	538
6	287	572	341	801	197	722
6	413	705	538	1 059	269	891
6	682	900	610	1 234	377	1 021
6	825	1 082	807	1 381	466	1 186
6	915	1 188	897	1 455	592	1 325
6	1 023	1 246	1 005	1 560	646	1 471
8	145	373	163	525	72	481
8	271	529	307	746	163	640
8	407	656	470	977	253	819
8	614	832	578	1 128	361	946
8	777	983	741	1 296	416	1 115
8	885	1 112	849	1 405	542	1 234
8	994	1 180	922	1 419	596	1 366
10	109	341	127	492	109	573
10	218	458	273	655	182	730
10	400	599	364	879	273	827
10	510	755	564	1 034	455	1 117
10	637	905	673	1 192	510	1 281
	783	1 012	783	1 282	582	1 342
	1 019	1 189	855	1 382	710	1 506

3.2 Analysis of ∇P -Q relationship

The section will demonstrate to use Darcy's law and Forchheimer formula to fit the relationship between seepage flow Q and pressure gradient ∇P .

Figure 5 shows the ∇P–Q relationship diagrams fitted by Darcy's law and Forchheimer's formula when the fracture aperture e is 2.3, 3.1 and 3.8 mm, respectively. Table 4 is a table of fitting parameters corresponding to the each intersection length under different fracture aperture. From the data in Table 4, the correlation coefficient R^2 fitted by Darcy's law formula is mostly between 0.86 and 0.93, which can still describe the statistical relationship between seepage flow and pressure gradient to a certain extent; whilst the R^2 fitted by Forchheimer formula fits is all above 0.99, indicating that the Forchheimer formula can better fit the relationship between the seepage flow and the pressure gradient in the disc fissure than Darcy's law in this test condition.

3.3 The relationship between parameters *A*, *B* and *e*, *w*, respectively

The difference between the Forchheimer formula and Darcy's law is that there is an additional quadratic term, which means that the head loss of non-Darcy seepage is determined by both the viscosity term and the inertia term[15]. The larger the inertia term is, the higher the degree of the deviation from Darcy flow will be. A represents the seepage capacity of the rock mass itself, while B reflects the non-Darcy effect of seepage. There are the four statistical relationship diagrams (see Fig.6) and statistical relationship expressions (see Table 5) of A–e, A–w, B–e and B–w through the test data below.

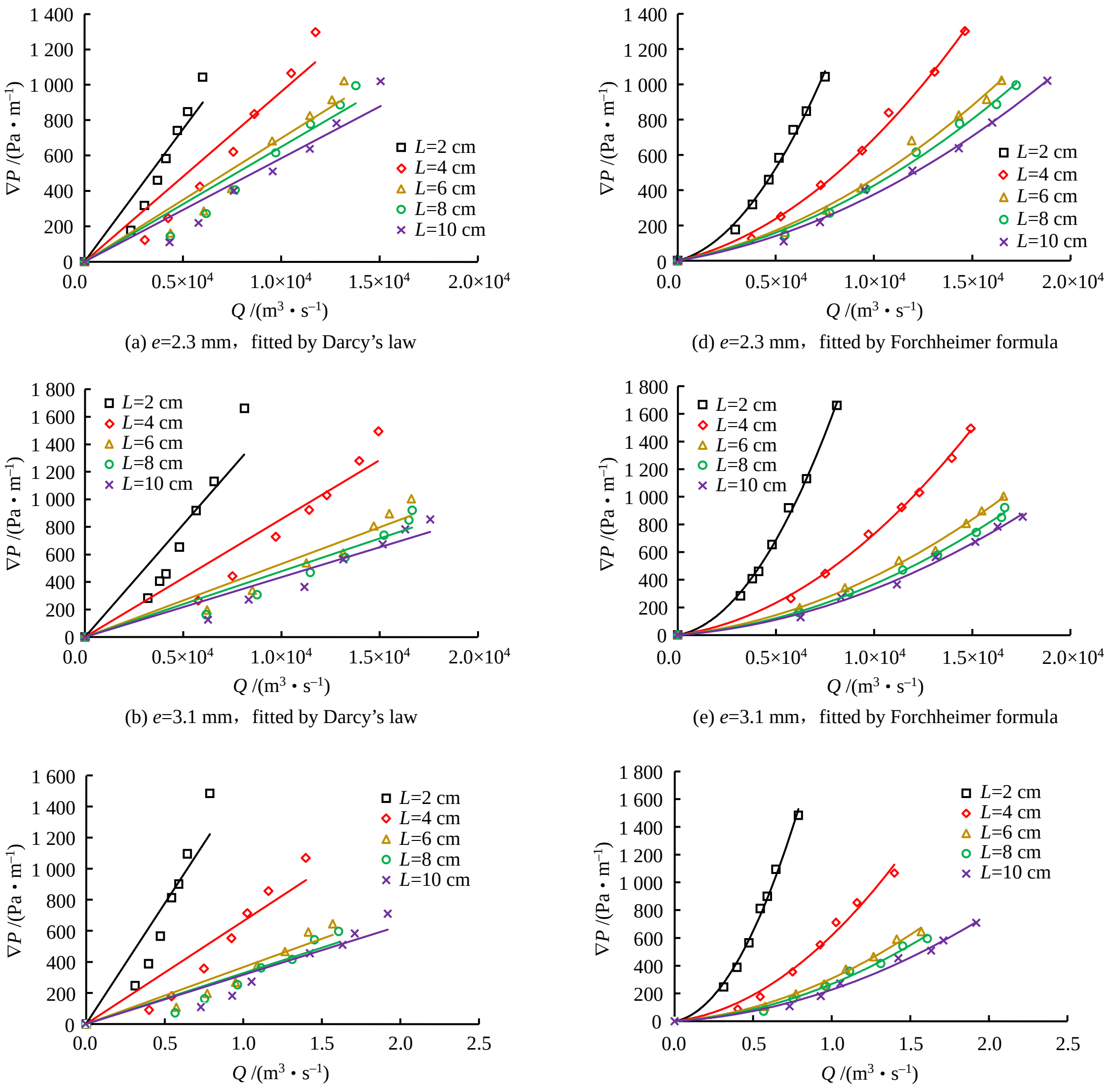
It can be seen from Fig. 6 and Table 5 that A decreases with the increase of e and w, and there is a power function decreasing relationship between A and e and w, respectively. At the same time, B also decreases with the increase of the fracture aperture e and the equivalent fracture width w, which also shows a power function decreasing relationship. Therefore, the Forchheimer model for the seepage through the disc fracture model under the experimental conditions is similar to the changes of A and B in the seepage through the parallel plate fracture model. However, there are differences where one of them is the power exponent.

3.4 Modification of the expression of *A* and *B* in the Forchheimer seepage model for disc fracture

As shown above, the biggest difference between a disc fracture and a parallel plate model is that the cross section of the disc is constantly changing along the direction of the water flow, and the lengths of the penetration path of different streamlines are different, which results in the flow velocity of each point inside the disc different. The test results show that if the aforementioned equivalent fracture width and equivalent penetration path length estimation methods are used, A and B in the Forchheimer seepage model of disc fractures are not suitable for Eq. (3). Table 5 illustrates that in different test scenarios. A, B and e, w have a good statistical relationship, respectively. Based on this, this paper proposes a new method for determining A and B. That is, through the seepage test under different typical conditions, the representative test results are obtained, and the multivariate statistical relations of power functions of A, B and e and w are established, respectively, and finally the modified expressions of A and B with certain credibility are obtained. According to the test results of this article, the following statistical relationships are obtained:

$$A = 10^{9.14} \times e^{-2.016} \times w^{-1.498} \quad R^2 = 0.96 \tag{7}$$

$$B = 10^{17.006} \times e^{-0.918} \times w^{-4.542} \quad R^2 = 0.91 \tag{8}$$



(a) e=2.3 mm，fitted by Darcy's law

(d) e=2.3 mm，fitted by Forchheimer formula

(b) e=3.1 mm，fitted by Darcy's law

(e) e=3.1 mm，fitted by Forchheimer formula

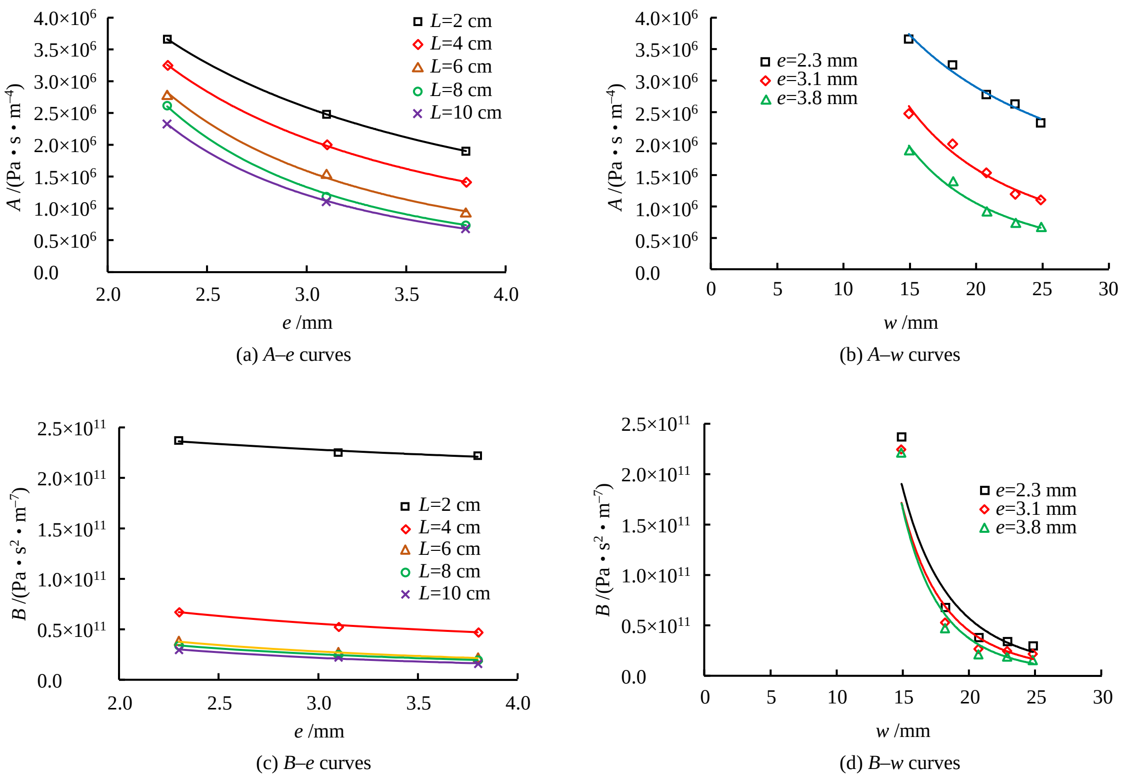
(c) e=3.8 mm，fitted by Darcy's law

(f) e=3.8 mm，fitted by Forchheimer formula

Fig. 5 ∇*P*-*Q* diagram of the Darcy law and the Forchheimer formula fitted

Table 4 Correlation coefficients of fitting results by two formulas for three apertures

Fracture aperture e /mm	L/cm	Darcy's law fitting results: K / (10^6 Pa • s • m^{-4})	R^2	Forchheimer formula fitting results: A / (10^6 Pa • s • m^{-4})	B / (10^{10} Pa • s^2 • m^{-7})	R^2
2.3	2	15.00	0.901	3.66	23.70	0.994
2.3	4	9.61	0.924	3.25	6.76	0.997
2.3	6	6.97	0.936	2.78	3.78	0.995
2.3	8	6.49	0.938	2.63	3.38	0.992
2.3	10	5.85	0.928	2.33	2.95	0.993
3.1	2	16.3	0.856	2.48	22.5	0.997
3.1	4	8.56	0.914	2.00	5.31	0.998
3.1	6	5.31	0.928	1.54	2.69	0.995
3.1	8	4.77	0.924	1.20	2.48	0.997
3.1	10	4.34	0.915	1.11	2.21	0.993
3.8	2	15.5	0.869	1.90	22.20	0.994
3.8	4	6.62	0.887	1.41	4.76	0.986
3.8	6	3.66	0.905	0.93	2.15	0.993
3.8	8	3.27	0.898	0.75	1.94	0.988
3.8	10	3.17	0.900	0.68	1.58	0.992



(a) A–e curves (b) A–w curves

(c) B–e curves (d) B–w curves

Fig. 6 Relationships between parameters *A*, *B* and *e*, *w*

Table 5 Formulas for fitting *A* and *B* to *e* and *w* respectively

Correlation variables	Fracture geometry parameters L/cm	e/mm	Fitting formula	R^2
A–e	2		$A = 1.2\times10^7 e^{-1.305}$	0.999
	4		$A = 1.44\times10^7 e^{-1.666}$	0.999
	6		$A = 1.57\times10^7 e^{-2.03}$	0.992
	8		$A = 2.18\times10^7 e^{-2.53}$	0.999
	10		$A = 1.93\times10^7 e^{-2.468}$	0.999
A–w		2.3	$A = 3.95\times10^7 w^{-0.873}$	0.976
		3.1	$A = 2.32\times10^8 w^{-1.66}$	0.976
		3.8	$A = 6.31\times10^8 w^{-0.98}$	0.980
B–e	2		$B = 2.64\times10^{11} e^{-0.133}$	0.947
	4		$B = 1.21\times10^{11} e^{-0.707}$	0.988
	6		$B = 9.64\times10^{10} e^{-1.12}$	0.987
	8		$B = 8.50\times10^{10} e^{-1.10}$	0.999
	10		$B = 8.35\times10^{10} e^{-1.22}$	0.979
B–w		2.3	$B = 1.22\times10^{16} w^{-4.1}$	0.926
		3.1	$B = 4.09\times10^{16} w^{-4.58}$	0.909
		3.8	$B = 2.15\times10^{17} w^{-5.2}$	0.930

From the fitting results, the statistical relationship is very relevant. It can be expected that, for example, for the study of seepage in a single disc fracture with similar actual conditions, the above formula can be used as an empirical formula to directly calculate the parameters A and B under different scenarios. Because this empirical formula is based on the seepage test data of a disc fracture model with a diameter of 50 cm and a fracture aperture in the range of 2.1 to 3.8 mm, the applicability of this formula needs to be further confirmed for different diameters, especially discs with a diameter closer to 50 cm. For this reason, a disc with a diameter of 40 cm was selected, the fracture aperture was 3 mm, and the intersection length was selected as 4, 6, and 8 cm (the upper and lower limits of the length of the intersection based on the statistical relationship above are taken), and 3 different pressure gradients, so as to obtain the corresponding seepage test data. Substituting the fracture parameters into Eqs. (7) and (8) to inversely calculate A and B, and then substituting A and B into Eq. (2) to obtain the flow rate Q, and calculate the relative error with the measured flow rate Q^* in the test. The specific values are shown in Table 6.

From the data in Table 6, the average value of the relative error in all test cases is 6%, which shows to a certain extent that the empirical formula proposed in this paper that includes the change of the disc diameter has good performance in characterizing the seepage law of a disc close to 50 cm in diameter. It should be pointed out that the parameters A and B mentioned in this article are only based on the case of a diameter of 50 cm. Although they have been well verified in a disk with a diameter of 40 cm, it is necessary to pass more radius seepage test data to further confirm.

In addition, for the value of A in the Forchheimer model (because B contains unknown parameters, only A is discussed here) can also be calculated according to Eq. (3). Then A can be corrected according to the size of the e and w to obtain the revised estimation of A closer to the actual value, which is recorded as A^*. According to the test results, the corrected relation expression is as follows:

$$A^* = A\times(10^{0.11}\times e^{0.982}\times w^{-0.538}),\quad R^2 = 0.83 \qquad (9)$$

R^2=0.83 implies the corrected relation expression has a strong correlation, that is related to both the fracture aperture e and the equivalent fracture width w.

Table 6 Validation on reliability of parameters *A* and *B* using the experimental results from the disc fracture model with a diameter of 40 cm

e=3 mm,*L*=4 cm				*e*=3 mm,*L*=6 cm				*e*=3 mm,*L*=8 cm			
∇P /(Pa • m^{-1})	Measured Q^* /(10^{-5}m^3 • s^{-1})	Calculated Q /(10^{-5}m^3 • s^{-1})	(Calculated Q–Measured Q^*)/Measured Q^* /%	∇P /(Pa • m^{-1})	Measured Q^* /(10^{-5}m^3 • s^{-1})	Calculated Q /(10^{-5}m^3 • s^{-1})	(Calculated Q–Measured Q^*)/Measured Q^* /%	∇P /(Pa • m^{-1})	Measured Q^* /(10^{-5}m^3 • s^{-1})	Calculated Q /(10^{-5}m^3 • s^{-1})	(Calculated Q–Measured Q^*)/Measured Q^* /%
114	2.2	2.1	4	199	3.7	4.0	6	167	3.9	4.4	15
423	4.9	4.7	4	280	5.2	4.9	5	239	5.4	5.5	3
594	6.2	5.7	7	421	6.7	6.3	6	335	7.0	6.8	3

Table 7 Relative error between *A* values before and after correction and test results

e / mm	*w* / cm	(formula(3) calculated *A*-test *A*)/Test *A* / %	(formula(9) calculated *A*-test *A*)/Test *A* / %
2.3	14.91	83	24
2.3	18.21	68	3
2.3	20.75	73	1
2.3	22.92	65	11
2.3	24.85	72	11
3.1	14.91	10	1
3.1	18.21	12	8
3.1	20.75	27	3
3.1	22.92	48	7
3.1	24.85	47	2
3.8	14.91	22	13
3.8	18.21	14	14
3.8	20.75	14	7
3.8	22.92	29	14
3.8	24.85	30	11

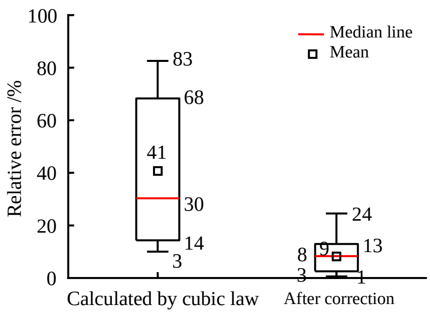


Fig. 7 Relative error between *A* values before and after correction and test results

In Section 3.2, the statistical relevance of the linear law is also strong, that is, it has good reliability to directly express it in a linear relationship. We verified the corrected expression proposed this paper by using test data, and used the test value as the standard to calculate the relative error of the calculated value before the correction (Eq. (3)) and after the correction (Eq. (9)) (see Table 7). A box plot was drawn (see Fig. 7). It can be found that the overall relative error after correction is greatly reduced, the average and median are 9% and 8%, respectively, which are 32% and 22% higher in accuracy than those before correction calculated by Eq. (3). Therefore, in the case of laminar flow in the range of Reynolds number in this experiment, if Eq. (3) given by the traditional parallel plate cube law is used to calculate *A*, the method is not reliable, whereas the proposed corrected the formula to calculate the value of *A* has higher reliability.

4 Conclusions

For the traditional non-Darcy flow with different Reynolds numbers in a certain range, the Darcy's law model can still describe the statistical relationship between seepage flow and pressure gradient to a certain extent, but the regression equation of the Forchheimer model has a higher correlation coefficient than Darcy's law. The model can better express the relationship between seepage flow and pressure gradient inside the disc fractures.

The test results show that the parameters *A* or *B* in the Forchheimer model expression of the disc fracture seepage law both decrease with the power function of the fracture aperture *e* and the equivalent fracture width *w*. However, it is worth noting that if the application of this paper with equivalent fracture width and equivalent penetration path length in the disc fracture Forchheimer seepage model, *A* and *B* should not be directly expressed by the *A* and *B* equations of the parallel plate fracture seepage, Instead *A* multivariate power function modification formula of the effective fracture width *w* needs be used that considers the larger difference including *e* and *w*. At the same time, it is also necessary to point out that it is valuable to conduct a seepage test including larger interval changes in fracture aperture and disc diameter in the subsequent in-depth study, in order to further confirm or modify the proposed formulas *A* and *B*.

This study also provides a widely applicable research method for disc fracture seepage law, that is, for non-Darcy seepage in disc fractures under a certain range of fracture geometric parameters, the Forchheimer seepage can be established by finite physical model simulation. In the model, the statistical relationship between *A*, *B* and *e*, *w* can be used to obtain the corresponding determination method of *A* and *B*. Specifically, through seepage tests under different test conditions, test results are obtained and the multivariate statistical relationships between *A*, *B* and *e*, *w* are established, respectively. Finally, a correction relationship of *A* and *B* can be obtained with a certain credibility. At the same time, the problem that the non-Darcy coefficient β in *B* is difficult to determine is solved.

This study is expected to provide a certain theoretical basis and method reference for the study of the seepage law of single fracture and complex fracture network in rock matrix and the engineering application of rock mass fracture hydrodynamics.

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