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Abstract: This study focuses on tunneling under challenging conditions, particularly with regard to the stress distribution and deformation in the humidity stress field. The swelling phenomenon during tunneling has been treated as a coupled humidity–mechanics process, where the humidity diffusion and stress dilatancy are considered together to obtain stress and deformation fields for tunnels crossing the formations with high swelling potential. A solution to the nonstationary process of humidity transfer has been derived according to Fick’s second law. The swelling pressure has been included in the form of body force, and a non-associated flow rule has been adopted to obtain the analytical solutions. Next, considering the examples of rock tunnels that are excavated in two different quality rock mass, we have investigated the impact factors on stress and deformation in swelling surrounding rock. Numerical results show that the inclusion of the swelling stress increases the plastic zone of the surrounding rock and the maximum stress at the elastic-plastic boundary, whereas the stress convergence has been decreased. After a certain increase in swelling pressure, a tensile stress zone appears in the plastic circle. The deformation of surrounding rock caused by swelling pressure can be much more significant than that caused by in-situ stress. Furthermore, the effect of dilatancy on the deformation rock cannot be negligible especially when the support resistance is small. This paper presents a new possible workflow to quickly evaluate the elastic-plastic stress and deformation of tunnels in swelling surrounding rock.

Keywords: deep-buried tunnel; humidity stress field; swelling stress; dilatancy; elastoplastic solution

1 Introduction

Swelling soft rock expands in volume and changes its mechanical properties when being exposed to water. This property can cause serious damage to hydraulic structures, underground chambers, as well as buildings built in this type of rock mass^[1–2]. The problem can be severe especially when considering a tunnel passes through a deep-buried soft rock layer containing potential swelling clay minerals. The excavation process prompts the rapid release of the original rock energy storage, and the result is two-fold. On the one hand, it causes the redistribution of the stress of the surrounding rock, and the change of the surrounding rock further induces the swelling effect^[3]. On the other hand, the strain of swelling can then develop its direction and space due to the appearance of the unloading surface. As the water from excavation continues to penetrate and diffuse into the surrounding rock, the inward displacement of the tunnel wall and the floor swelling become more significant in the high-potential swelling area. If not maintained in time, the surrounding rock and support of the tunnel may be severely deformed and damaged.

According to the definition of The Commission on Swelling Rock of the International Society for Rock Mechanics, swelling soft rock refers to the type of rock with low strength and containing highly swelling

clay minerals, which undergoes significant deformation under low external stress (<25 MPa). Swelling soft rocks are widely distributed in China, and the related studies have been increasingly attractive. Chen et al.^[3] reviewed the problems of swelling rock and regarded the topic as a crucial one in soft rock mechanics and engineering development. The research on the swelling of soft rock in contact with water has a history of many years. The swelling constitutive model of swelling rock shows the stress–strain relationship during the swelling of rock in contact with water, which is fundamental for studying the mechanical response of tunnel excavation in the swelling host rock. Huder et al.^[4] first proposed a linear relationship between the axial strain and the logarithm of the swelling pressure. At present, the widely applied theories are based on specific experimental swelling models. Typical examples are given by Gysel’s one-dimensional swelling modelling theory^[5–6] and Wittke’s three-dimensional swelling modelling theory^[7]. Based on these, Zuo et al.^[8], Liu et al.^[9] and Ren et al.^[10], all have derived and demonstrated functions between swelling strain and stress based on swelling experiments.

However, the above swelling models are all derived by fitting to experimental data. The underlying assumptions include, for example, stress and strain obey a semilogarithmic relationship; water absorption and time obey an exponential relationship; and swelling force

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and water absorption obey an exponential relationship. They all have certain practical significance as simple mathematical models, but when considering constitutive models, they lack the foundation in physics. Moreover, the above theories are also limited by experiment. They assume that the stress field in the surrounding rock mass has no effect on the swelling part of the rock mass, which can be inconsistent with reality.

In fact, the extent and the significance of the water-contacted part of the rock mass can be variable. The swelling and softening of the rock mass after encountering water will cause changes in the stress and strain of the entire rock mass, and the stress field in the surrounding rock mass will also react against the water-contacted part of the rock mass. Based on this, Miao et al.^[11–12] proposed a preliminary framework for the theory of humidity stress field, which actually considered the changes in stress and strain caused by the internal and external constraints of the swelling surrounding rock. The theory has been verified and applied as shown by many researchers in theory, experiments and simulations. Bai et al.^[13] showed a rigorous proof of the humidity stress field theory, and analyzed the required conditions and mechanical significance of the theory. Zhu et al.^[14] proposed an elastoplastic constitutive model of swelling rock based on the theory of humidity stress field, considering the changes in the elastic modulus, Poisson's ratio and yield limit of swelling rock caused by the change of water content. Ji et al.^[15–17] presented the tests of lime mudstone with free swelling and verified that the results are consistent with the humidity field theory. Wang et al.^[18] proposed an elastoplastic constitutive model of the humidity stress field of swelling rock based on the incremental theory and the swelling deformation mechanism, and showed the secondary development of the constitutive model based on FLAC 3D. Zhang et al.^[19] assumed a hyperbolic relationship between swelling pressure and water content, and explained the physical meaning of the humidity swelling coefficient: it reflects the expansion characteristics of different swelling rocks after physical and chemical reactions caused by water seepage, and is also a measure of the potential of closed strain development. These studies have improved the humidity stress field theory to a certain extent, and also form the fundamentals for applying the theory to tunnel excavation in high-potential swelling areas.

In recent years, some researchers have derived the analytical solutions for tunnel excavation in weak swelling surrounding rock under the impact of humidity based on the theory of humidity stress field. Miao et al.^[20–21] assumed that the circular chamber is located in an infinite space, and is subjected to a uniform ground stress field, based on which they derived the stress distribution of the circular chamber considering both water and ground stress. Lu et al.^[22] aimed at the problem of water shrinkage of boreholes through swelling rock in coal mines, and introduced a humidity correction parameter. They derive an analytical solution for the radial displacement of the borehole considering the

water softening and swelling characteristics of surrounding rock based on the humidity stress field theory. Zhang et al.^[19] derived a viscoelastic-plastic creep solution of swelling rock tunnels based on the theory of humidity stress field. However, they made a few simplifications through the derivation. For example, the humidity of the surrounding rock was assumed to be constant and the swelling pressure did not affect the stress distribution of the surrounding rock. The boundary condition of the stress field was simply the swelling stress plus the initial ground stress of the surrounding rock. In order to study the displacement of the tunnel surrounding rock of anhydrite under the swelling action, Ren et al.^[23] proposed a tunnel elastic-expansion analytical model based on the humidity stress field theory, considering the time-dependent swelling of anhydrite. They discussed two working conditions: 1) the surrounding rock absorbs water uniformly; and 2) the water absorption of the surrounding rock decreases with the increase of the distance from the open surface. The analytical solution of the stress field and displacement field of the tunnel surrounding rock was then derived accordingly. However, the studies^[20–23] all adopt the assumptions that the distribution law of the humidity field satisfies $W(r) = (w_{\max} - w_0) \cdot R_0 / r$ or $W(r) = (w_{\max} - w_0) \cdot R_0^2 / r^2$, which make Ren's work controversial and lack support in theory. It also has certain limitations. The review of literatures reveals that a key problem has not been fully solved to apply the humidity stress field theory to analyze the deformation of swelling rock tunnels. That is, the humidity distribution of the surrounding rock has not yet been solved analytically when it encounters water.

Chen^[24] discussed the main reasons for the deformation and damage of surrounding rock when tunnels were excavated in swelling rock. The rock swells with water and expands under deviatoric stress, which forms a mutual effect and leads to a coupled, longterm development. This paper focuses on the deep-buried tunnel with swelling surrounding rock. We derive the analytical solution for the humidity distribution in the deep-buried swelling surrounding rock. Based on the Mohr-Coulomb strength criterion, we consider the effect of the swelling stress and dilatancy of the surrounding rock, and show the stress and deformation of surrounding rock after tunnel excavation with the proposed humidity field. We then investigate and discuss the impact of swelling stress and dilatancy on the stress and displacement of the surrounding rock.

2 Elastoplastic solution considering swelling stress

Considering deep-buried circular tunnels, the surrounding rock can be assumed to satisfy the hydrostatic condition if the tunnel is deep enough. The rock mass is assumed to be uniform, continuous and isotropic. The humidity distribution inside the rock mass is assumed to be axisymmetric. Through our derivation the compressive stress is positive and the tensile stress is negative. The model configuration is illustrated in Fig.1,

where R_0 and R_p denote the radius of the tunnel and the radius of the plastic zone, respectively. The hydrostatic pressure is p_0 , and the support resistance is p_1 .

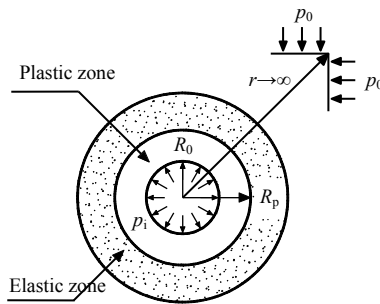


Fig. 1 Circular opening in an infinite medium

For the plane strain problem, the balance equation in polar coordinates is

$$r \frac{d\sigma_r}{dr} + (\sigma_r - \sigma_\theta) + r f_\rho = 0 \tag{1}$$

where f_ρ is the body force, and in this case it is the swelling stress regarding to the swelling rock tunnel; σ_r and σ_θ are the radial and tangential stress of the surrounding rock, respectively; r is the distance from a point in the surrounding rock to the center of the tunnel. Previous studies [15, 18] have shown the swelling stress of swelling rock can be derived as

$$f_\rho = P_s = \frac{E}{1-2\nu} \alpha \Delta w \tag{2}$$

where P_s is the swelling stress; α is the humidity swelling coefficient. However, the change of water content Δw at different positions in the swelling surrounding rock can be spatially dependent, and we treat it as a function of space and time, instead of a fixed value. Therefore, Eq. (1) can be modified to

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} - \frac{d}{dr} \left[\frac{E\alpha W(r)}{1-2\nu} \right] = 0 \tag{3}$$

where $W(r)$ is the function denoting the change of humidity with the radius of the swelling surrounding rock. The value of $W(r)$ is the difference of water content between the current status and the initial status.

2.1 Elastic zone

The swelling stress satisfies the physical relations of

$$\left. \begin{aligned} \varepsilon_r &= \frac{1-\nu^2}{E} \left[\sigma_r - \frac{\nu}{1-\nu} \sigma_\theta \right] + (1+\nu) \alpha W(r) \\ \varepsilon_\theta &= \frac{1-\nu^2}{E} \left[\sigma_\theta - \frac{\nu}{1-\nu} \sigma_r \right] + (1+\nu) \alpha W(r) \end{aligned} \right\} \tag{4}$$

and the swelling stress also stratifies the geometric relations of

$$\left. \begin{aligned} \varepsilon_r &= \frac{du}{dr} \\ \varepsilon_\theta &= \frac{u}{r} \end{aligned} \right\} \tag{5}$$

Combining Eqs. (1) – (5), we have

$$\frac{d^2 u}{dr^2} + \left[\frac{1}{E} \frac{dE}{dr} + \frac{2\nu(2-\nu)}{(1-\nu^2)(1-2\nu)} \frac{d\nu}{dr} + \frac{1}{r} \right] \frac{dv}{dr} +$$

$$\left[\frac{\nu}{E(1-\nu)} \frac{dE}{dr} + \frac{1+2\nu^2}{(1-\nu^2)(1-2\nu)} \frac{d\nu}{dr} \right] \frac{u}{r} - \frac{u}{r^2} - \frac{2(1+\nu)}{1-\nu} \frac{d(\alpha W)}{dr} - \frac{1+\nu}{(1-\nu)(1-2\nu)} \alpha W \frac{dv}{dr} = 0 \tag{6}$$

where $\frac{dE}{dr} = \frac{dE}{dW} \frac{dW}{dr}$, and $\frac{dv}{dr} = \frac{dv}{dW} \frac{dW}{dr}$.

Equation (6) is the governing equation to solve the problem based on displacement, which has included the softening effect of swelling rock in contact with water. However, it is not straightforward to obtain an explicit solution. To reduce the complexity in math, we assume that the mechanical properties of swelling rock do not change within a certain range of water content, i.e., E and μ do not change with w , and α does not change with w . Therefore, Eq. (6) can be simplified as

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} - \frac{2(1+\nu)}{1-\nu} \alpha \frac{dW(r)}{dr} = 0 \tag{7}$$

that is

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (ru) \right] = \frac{2(1+\nu)}{1-\nu} \alpha \frac{dW(r)}{dr} \tag{8}$$

with the boundary conditions defined as

$$\left. \begin{aligned} \sigma_r|_{r=R_0} &= p_1, & w|_{r=R_0} &= w_{\max} \\ \sigma_r|_{r=R_p} &= \sigma_{Rp}, & w|_{r=R_p} &= w_{Rp} \\ \sigma_r|_{r=\infty} &= p_0, & w|_{r=\infty} &= w_0 \end{aligned} \right\} \tag{9}$$

where w_0 is the initial water content. We assume the water content of the surrounding rock has the largest at the tunnel wall, so w_{\max} is the water content at $r = R_0$. σ_{Rp} is the stress at the elastic-plastic interface; and w_{Rp} is the water content of the surrounding rock.

Integrate both sides of Eq. (8), it has

$$u^e = C_2 r + \frac{C_1}{r} + \frac{2\alpha(1+\nu)}{(1-\nu)r} \int_{R_p}^r W(r) r dr \tag{10}$$

whrer C_1 and C_2 are the constants obtained from integration.

Substitute the boundary conditions (Eq.(9)) into Eq. (10), the stress of surrounding rock in the elastic zone can be solved as

$$\left. \begin{aligned} \sigma_r^e &= p_0 - \frac{R_p^2}{r^2} \left[(p_0 - \sigma_{Rp}) + \frac{E\alpha\Delta w_{Rp}}{1-2\nu} \right] + \frac{E\alpha W(r)}{1-2\nu} + \frac{2E\alpha}{(1-\nu)r^2} \int_{R_p}^r W(r) r dr \\ \sigma_\theta^e &= p_0 + \frac{R_p^2}{r^2} \left[(p_0 - \sigma_{Rp}) + \frac{E\alpha\Delta w_{Rp}}{1-2\nu} \right] + \frac{(3\nu-1)E\alpha W(r)}{(1-\nu)(1-2\nu)} + \frac{2E\alpha}{(1-\nu)r^2} \int_{R_p}^r W(r) r dr \end{aligned} \right\} \tag{11}$$

2.2 Plastic zone

Considering the plastic zone (where $R_0 \leq r \leq R_p$), combining the balance Eq. (3) and Mohr-Coulomb strength criterion

$$\frac{\sigma_r^p + c \cot \varphi}{\sigma_\theta^p + c \cot \varphi} = \frac{1 - \sin \varphi}{1 + \sin \varphi} \tag{12}$$

it gives

$$\frac{d\sigma_r^p}{dr} - \frac{2 \sin \varphi}{(1 - \sin \varphi)r} \sigma_r^p = \frac{E\alpha}{1 - 2\nu} \frac{dW(r)}{dr} + \frac{2c \cos \varphi}{(1 - \sin \varphi)r} \tag{13}$$

Solve the differential equation and substitute the boundary conditions in, the stress of the surrounding rock in plastic zone can be derived as

$$\left. \begin{aligned} \sigma_r^p &= (p_i + c \cot \varphi) \left(\frac{r}{R_0} \right)^{\frac{2 \sin \varphi}{1 - \sin \varphi}} - c \cot \varphi + \\ &\quad \frac{E\alpha}{1 - 2\nu} r^{\frac{2 \sin \varphi}{1 - \sin \varphi}} \int_{R_0}^r r^{\frac{-2 \sin \varphi}{1 - \sin \varphi}} dW(r) \\ \sigma_\theta^p &= \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_r^p + \frac{2c \cos \varphi}{1 - \sin \varphi} \end{aligned} \right\} \tag{14}$$

Next is to determine the displacement distribution in the plastic zone. We assume the elastic strain is relatively small compared to the plastic strain, and the plastic strain follows the non-associated flow rule. We also assume that the radial plastic strain and the hoop strain satisfy the relation^[19, 25]

$$\varepsilon_r^p + \lambda \varepsilon_\theta^p = 0 \tag{15}$$

where ε_r^p and ε_θ^p are the radial strain and the hoop strain in the plastic zone, respectively; λ is the dilatancy coefficient, and $\lambda = 1$ means that the plastic volume strain is 0. The dilatancy coefficient λ and the dilatancy angle ψ satisfy

$$\lambda = \frac{1 + \sin \psi}{1 - \sin \psi} \tag{16}$$

Substitute the geometric relationship (Eq.(5)) in, it gives

$$\frac{\partial u^p}{\partial r} + \lambda \frac{u^p}{r} = 0 \tag{17}$$

According to the deformation compatibility conditions at the elastic-plastic interface, the radial displacement u^p of the surrounding rock in the plastic zone can be solved as

$$u^p = \frac{R_p^{\lambda+1}}{2Gr^\lambda} \left[(p_0 - \sigma_{Rp}) + \frac{E\alpha \Delta w_{Rp}}{1 - \nu} \right] \tag{18}$$

where Δw_{Rp} is the value of function $W(r)$ at the elastic-plastic interface, that is, $W(r)|_{r=R_p} = w_{Rp} - w_0$.

Combining the two equations in Eq. (11) yields

$$\sigma_r^e + \sigma_\theta^e = 2p_0 + \frac{2\nu}{1 - \nu} \frac{E\alpha W(r)}{1 - 2\nu} \tag{19}$$

According to the continuity of stress, at the elastic-plastic interface ($r = R_p$) it should satisfy the relation

$$\sigma_r^p + \sigma_\theta^p = 2p_0 + \frac{2\nu}{1 - \nu} \frac{E\alpha \Delta w_{Rp}}{1 - 2\nu} \tag{20}$$

By combining Eqs. (20) and (12), the stress at the elastic-plastic interface ($r = R_p$) in the surrounding rock can be solved as

$$\left. \begin{aligned} \sigma_r^p &= \left(p_0 + \frac{\nu}{1 - \nu} \frac{E\alpha \Delta w_{Rp}}{1 - 2\nu} \right) (1 - \sin \varphi) - c \cos \varphi = \sigma_{Rp} \\ \sigma_\theta^p &= \left(p_0 + \frac{\nu}{1 - \nu} \frac{E\alpha \Delta w_{Rp}}{1 - 2\nu} \right) (1 + \sin \varphi) + c \cos \varphi \end{aligned} \right\} \tag{21}$$

Substitute $r = R_p$ into Eq. (14), and with the use of Eq. (21), the relation between the support resistance p_i and the radius of the plastic zone R_p can be solved as

$$\left. \begin{aligned} p_i &= \left[\left(p_0 + c \cot \varphi + \frac{\nu}{1 - \nu} \frac{E\alpha \Delta w_{Rp}}{1 - 2\nu} \right) (1 - \sin \varphi) - \right. \\ &\quad \left. \frac{E\alpha}{1 - 2\nu} R_p^{\frac{2 \sin \varphi}{1 - \sin \varphi}} \int_{R_0}^{R_p} r^{\frac{-2 \sin \varphi}{1 - \sin \varphi}} dW(r) \right] \left(\frac{R_0}{R_p} \right)^{\frac{2 \sin \varphi}{1 - \sin \varphi}} - \\ &\quad c \cot \varphi \end{aligned} \right\} \tag{22}$$

and the radius of the plastic zone of the surrounding rock is solved as

$$\left. \begin{aligned} \frac{R_p}{R_0} &= (p_i + c \cot \varphi)^{\frac{\sin \varphi - 1}{2 \sin \varphi}} \cdot \\ &\quad \left[(1 - \sin \varphi) \left(p_0 + c \cot \varphi + \frac{\nu}{1 - \nu} \frac{E\alpha \Delta w_{Rp}}{1 - 2\nu} \right) - \right. \end{aligned} \right\} \tag{23}$$

$$\left. \begin{aligned} &\quad \left. \frac{E\alpha}{1 - 2\nu} R_p^{\frac{2 \sin \varphi}{1 - \sin \varphi}} \int_{R_0}^{R_p} r^{\frac{-2 \sin \varphi}{1 - \sin \varphi}} dW(r) \right]^{\frac{1 - \sin \varphi}{2 \sin \varphi}} \end{aligned} \right\}$$

Equations (22) and (23) show the relationship between the support resistance p_i and the radius of the plastic zone R_p when the swelling stress has been included. When the effect of swelling stress is excluded, Eqs. (22) and (23) then becomes the modified Fenner formula^[26]. Substituting the plastic displacement u_{R_0} when $r = R_0$ into Eq.(22), the supporting resistance p_i and the plastic displacement u_{R_0} of the surrounding rock can be related as

$$\left. \begin{aligned} p_i &= -c \cot \varphi + \left(\frac{p_0 \gamma R_0}{2Gu_{Rp}} \right)^{\frac{2 \sin \varphi}{(\lambda+1)(1-\sin \varphi)}} \cdot \\ &\quad \left[\left(p_0 + c \cot \varphi + \frac{\nu}{1 - \nu} \frac{E\alpha \Delta w_{Rp}}{1 - 2\nu} \right) (1 - \sin \varphi) - \right. \\ &\quad \left. \frac{E\alpha}{1 - 2\nu} R_p^{\frac{2 \sin \varphi}{1 - \sin \varphi}} \int_{R_0}^{R_p} r^{\frac{-2 \sin \varphi}{1 - \sin \varphi}} dW(r) \right] \end{aligned} \right\} \tag{24}$$

where

$$\gamma = 1 - \frac{\sigma_{Rp} - \frac{E\alpha \Delta w_{Rp}}{1 - \nu}}{p_0} \tag{25}$$

Equations (11), (14), (18), (24) and (25) show the stress and displacement distribution of the elastic and plastic zone of the swelling rock surrounding a tunnel, as well as the response from the surrounding rock. However, to obtain the complete explicit solution, the expression of the humidity field $W(r)$ needs to be found.

3 Solution to the humidity field

We consider the model configuration as a cylindrical array with an inner radius of R_0 and an outer radius of $R_\infty \approx 10R_0$. We define the humidity field as the sum of the instantaneous humidity of all the points in the internal space of the model^[27]. For the unsteady humidity field, it is a function of space and time, and it can be expressed as $w(r, t)$. To extend the use of our solution, we consider a general form of humidity through the derivation. The humidity can be defined by physical quantities such as water content, water head, pore pressure and even concentration. Nevertheless, the unit needs to be standardized before use.

The initial condition and boundary conditions are given as

$$\left. \begin{aligned} w(r, 0) &= w_0 \\ w_1(R_0, t) &= w_{\max} = \mu w_0 \\ w_1(R_\infty, t) &= w_0 \end{aligned} \right\} \quad (26)$$

where the scale factor $\mu \geq 0$. When $\mu < 1$, it means that the humidity of the cylinder increases from the inside to the outside. We mainly consider the case where $\mu > 1$, because it corresponds to the process where humidity transfers from the inside to the outside of the cylinder model.

For the plane strain problem, the unsteady state of humidity transfer can be expressed as

$$\frac{\partial w}{\partial t} = k_w \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) \quad (27)$$

where k_w is the hydraulic conductivity of the swelling rock.

To simplify the calculation, the variables are transformed to be dimensionless:

$$t = \frac{a^2}{k_w} \tau; \quad r = a\rho \quad (28)$$

which yields

$$\frac{\partial w}{\partial \tau} = \frac{\partial^2 w}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial w}{\partial \rho} \quad (29)$$

To derive the analytical solutions accurately, Eqs. (26) and (29) are decomposed into

$$w(\rho, \tau) = w_1(\rho) + w_2(\rho, \tau) \quad (30)$$

That is

$$\left. \begin{aligned} \frac{d^2 w_1}{d\rho^2} + \frac{1}{\rho} \frac{dw_1}{d\rho} &= 0 \\ w_1(\rho_a) &= w_{\max} \\ w_1(\rho_b) &= w_0 \end{aligned} \right\} \quad (31)$$

$$\left. \begin{aligned} \frac{\partial w_2}{\partial \tau} &= \frac{\partial^2 w_2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial w_2}{\partial \rho} \\ w_2(\rho, 0) &= w_0 \\ w_2(\rho_a, \tau) &= 0 \\ w_2(\rho_b, \tau) &= 0 \end{aligned} \right\} \quad (32)$$

where $\rho_a = 1$; and $\rho_b = R_\infty / R_0$.

The solution to Eq. (31) is

$$w_1(\rho) = \frac{w_b \ln(\rho / \rho_a) - w_a \ln(\rho / \rho_b)}{\ln(\rho_a / \rho_b)} \quad (33)$$

Solving Eq. (32) by the separation of variables, it yields

$$\left. \begin{aligned} w_2(\rho, \tau) &= R(\rho)T(\tau) \\ \frac{1}{T} \frac{\partial T}{\partial \tau} &= \frac{1}{R} \left(\frac{\partial^2 R}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial R}{\partial \rho} \right) = -k_n^2 = \text{const} \end{aligned} \right\} \quad (34)$$

By integrating both sides of Eq. (34), the function of T and R can be derived as

$$\left. \begin{aligned} T(\tau) &= Ae^{-k_n^2 \tau} \\ R(\rho) &= BJ_0(k_n \rho) + CY_0(k_n \rho) \end{aligned} \right\} \quad (35)$$

where J_0 and Y_0 are the first and second zero-order Bessel functions, respectively.

The general solution of Eq. (34) for any k_n is

$$w_2(\rho_a, \tau) = [BJ_0(k_n \rho) + CY_0(k_n \rho)] \cdot e^{-k_n^2 \tau} \quad (36)$$

Besides, it satisfies $w_2(\rho_a, \tau) = 0$, and $w_2(\rho_b, \tau) = 0$, which leads to

$$J_0(k_n \rho_a)Y_0(k_n \rho_b) - J_0(k_n \rho_b)Y_0(k_n \rho_a) = 0 \quad (37)$$

Let $k_n \rho_a = x > 0$, and $k_n \rho_b = \lambda x (\lambda = \rho_b / \rho_a > 1)$, Eq. (37) is equivalent to

$$J_0(x)Y_0(\lambda x) - J_0(\lambda x)Y_0(x) = 0 \quad (38)$$

We define

$$f(x) = J_0(x)Y_0(\lambda x) - J_0(\lambda x)Y_0(x) \quad (39)$$

Based on the S-L theory regarding to the eigenvalue problem^[28], there are infinite zero-crossing points x_n ($n = 1, 2, 3, \dots$), and the eigenvalues can be solved as

$$k_n^2 = \left(\frac{x_n}{\rho_a} \right)^2, \quad (n = 1, 2, 3, \dots) \quad (40)$$

Therefore it gives

$$R_0(x_n \rho / \rho_a) = J_0(x_n \rho / \rho_a)Y_0(x_n) - J_0(x_n)Y_0(x_n \rho / \rho_a) \quad (41)$$

By superposition, the general solution of w_2 is derived as

$$w_2(\rho, \tau) = \sum_{n=1}^{\infty} A_n R_0(x_n \rho / \rho_a) \exp(-\tau x_n^2 / \rho_a^2) \quad (42)$$

Combining Eqs. (42), (30) and (33), it gives

$$w(\rho, \tau) = \frac{w_b \ln(\rho / \rho_a) - w_a \ln(\rho / \rho_b)}{\ln(\rho_b / \rho_a)} + \sum_{n=1}^{\infty} A_n R_0 (x_n \rho / \rho_a) \exp(-\tau x_n^2 / \rho_a^2) \quad (43)$$

where

$$A_n = \frac{\pi R_0 (k_n \rho) (w_{\max} - w_0)}{[J_0^2(k_n \rho_a) / J_0^2(k_n \rho_b) + 1]} \quad (44)$$

Therefore, after encountering water, the humidity distribution in the swelling surrounding rock is solved as

$$w(\rho, \tau) = \frac{w_b \ln(\rho / \rho_a) - w_a \ln(\rho / \rho_a)}{\ln(\rho / \rho_a)} + \pi (w_{\max} - w_0) \times \sum_{n=1}^{\infty} \left(\frac{J_0^2(\lambda x_n) R_0 (x_n \rho / \rho_a) \cdot \exp(-\tau x_n^2 / \rho_a^2)}{J_0^2(x_n) + J_0^2(\lambda x_n)} \right) \quad (45)$$

where

$$R_0 \left(\frac{x_n}{\rho_a} \rho \right) = J_0 \left(\frac{x_n}{\rho_a} \rho \right) Y_0(x_n) - J_0(x_n) Y_0 \left(\frac{x_n}{\rho_a} \rho \right) \quad (46)$$

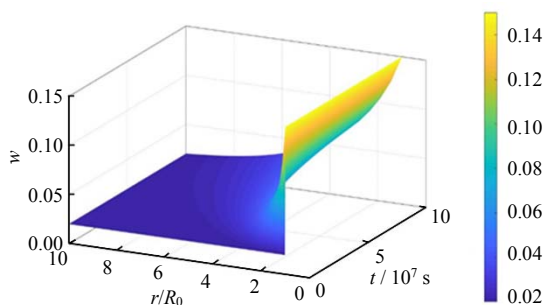
4 Numerical demonstration and analysis

4.1 Numerical demonstration

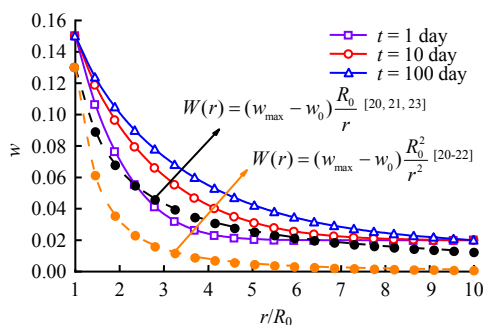
To demonstrate the correctness of the theory derived in this paper, we compare our solution to the ones from previous studies.

4.1.1 Comparison of the humidity field

We choose the hydraulic conductivity of swelling rock to be 2×10^{-10} cm/s [29–30], the maximum water content to be 15%, and the initial water content to be 2%. The humidity distribution in the surrounding rock is obtained from Eq. (45), and shown in Fig. 2.



(a) Distribution of unsteady humidity



(b) Comparison with other existing solutions

Fig. 2 Humidity distribution in the surrounding rock

Figure 2 shows the distribution of the humidity field in the swelling surrounding rock. With the increase of time, the humidity in the surrounding rock also increases. Centered on the tunnel, the humidity diffuses continuously into the surrounding rock. For a certain period of time, the water content of the surrounding rock $W(r)$ shows approximately a negative exponential relation with the radius r , which means the assumptions in literatures [20–23] are reliable to some extent. However, as shown in Fig. 2(b), the assumptions that $W(r)$ and r satisfying the relations $W(r) = (w_{\max} - w_0) R_0 / r$ or $W(r) = (w_{\max} - w_0) R_0^2 / r^2$ have obviously underestimated the distribution of humidity in the surrounding rock.

4.1.2 Comparison of displacement

Previous work [31] applies the Mohr-Coulomb strength criterion and the non-associated plastic flow rule to solve the displacement for the elasto-plastic deformation of axisymmetric tunnels. However, the swelling stress and dilatancy effects have not been considered, which can be seen as a special case ($W(r) = 0$, $\lambda = 1$) of the solution derived in this paper. We use the following parameters for the computational model: the hydrostatic pressure is $p_0 = 10$ MPa; roadway radius is $R_0 = 1.0$ m; no support is included; elastic modulus of surrounding rock is $E = 1$ GPa; Poisson’s ratio is $\nu = 0.3$; uniaxial compressive strength of surrounding rock is $\sigma_c = 6$ MPa; and the internal friction angle is $\varphi = 30^\circ$.

According to the solution given in literature [31], the radius of the plastic zone is calculated as 1.47 m, which is consistent with our result ($R_p = 1.47$ m) when the swelling stress effect has been turned off. Figure 3 shows the distribution of displacement in the plastic zone of surrounding rock under different dilatancy angles ψ of 0° , 10° , 20° , and 30° , respectively. Without the inclusion of swelling stress, the results in this paper are mostly consistent with the results given in [31], which further demonstrates the correctness of our solution.

4.2 Numerical examples and analysis

The theory derived in this paper includes the effect of swelling stress, and compares the results with the ones where swelling stress is absent. The comparison demonstrates in particular the importance of swelling stress. In this case we use the water content w to characterize the humidity of the surrounding rock. It is worth pointing out that, according to the swelling stress as given in Eq.(2), the strength of swelling stress depends on the change of the water content of the surrounding rock. That is, it is the current water content and the initial water content of the surrounding rock that both affect the stress. Based on Eq. (45), it only needs the knowledge of the initial water content w_0 and the maximum water content w_{\max} to determine the humidity distribution in the surrounding rock. Substituting the water content into the formula of the humidity field, the stress in the surrounding rock can therefore be obtained.

4.2.1 Numerical example 1

The first model considers a deep-buried tunnel with swelling surrounding rock that has the following parameters: the hydrostatic pressure is $p_0 = 1.5$ MPa; the supporting resistance is $p_i = 0.60$ MPa; the radius of tunnel excavation is $R_0 = 6$ m; the surrounding rock

has an elastic modulus of $E = 1$ GPa, with a Poisson's ratio of $\nu = 0.35$ and a cohesive force of $c = 0.3$ MPa. The friction angle is $\varphi = 20^\circ$. The humidity swelling coefficient of the surrounding rock is $\alpha = 0.03$. The maximum water content w_{max} is 3%, and the initial water content w_0 is 2%.

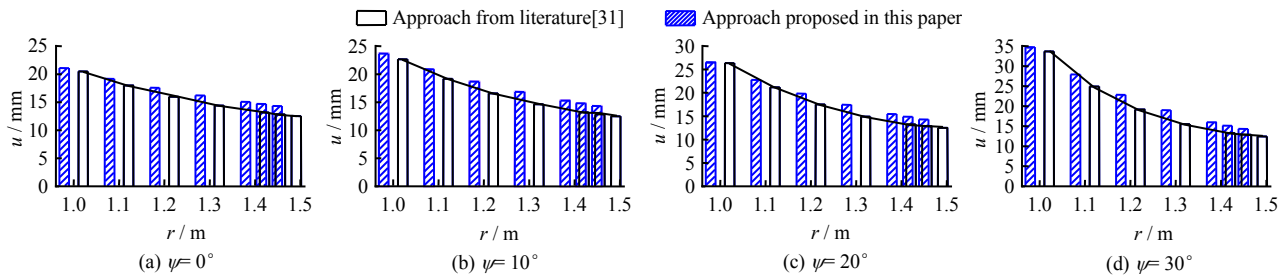


Fig. 3 Distribution of displacements in the plastic region of tunnel with different ψ

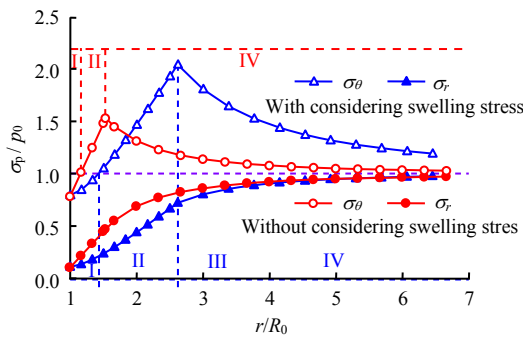


Fig. 4 Plots of stress of surrounding rock with (blue) and without (red) considering swelling stress

Figure 4 plots the variation of stress with and without considering the effect of humidity field on the surrounding rock. Literature [32] discusses the division of the impact of stress after tunnel excavation into 4 zones. The outer part of the plastic zone (zone II) is the area where the stress has increased compared with the initial stress. The zone can be combined with the area where the stress in the elastic zone of the surrounding rock has also increased (zone III), to together represent the part where the surrounding rock supports. In contrast, the inner part of the plastic zone (zone I) has the stress decreased comparing with the initial stress, and it can be characterized as the 'loosening' zone. The stress in zone IV is the area which has not been affected by excavation, and the rock mass remains the original stress status. As shown by Fig. 4, with the inclusion of the effect of swelling stress, the radius of plastic zone has increased. Given the same support resistance of $p_i = 0.16$ MPa, the radius of the plastic zone has increased by a factor of 1.7 when the swelling stress has been considered. As shown by Fig. 4, considering the swelling stress, the area of zone I expands, and the expansion means the thickness of the 'loosening' zone of the surrounding rock increases, and the stress in the 'loosening' zone decreases significantly at the same time. Such effect causes the cracks to expand and increase, and therefore more support are required to prevent the further development of cracks.

4.2.2 Numerical example 2

The second model considers a deep-buried tunnel with swelling surrounding rock that has the following parameters: the hydrostatic pressure is $p_0 = 7$ MPa; the supporting resistance is $p_i = 0.10$ MPa; the radius of tunnel excavation is $R_0 = 6$ m; the surrounding rock has an elastic modulus of $E = 1$ GPa, with a Poisson's ratio of $\nu = 0.3$ and a uniaxial compressive strength of $\sigma_c = 6$ MPa. The internal friction angle is $\varphi = 20^\circ$. The humidity swelling coefficient is $\alpha = 0.03$; $w_{max} = 3\%$; and $w_0 = 2\%$.

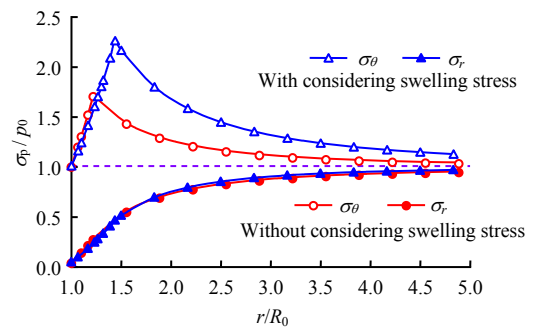


Fig. 5 Plots of stress of surrounding rock under the same supporting condition, with (blue) and without (red) considering swelling stress

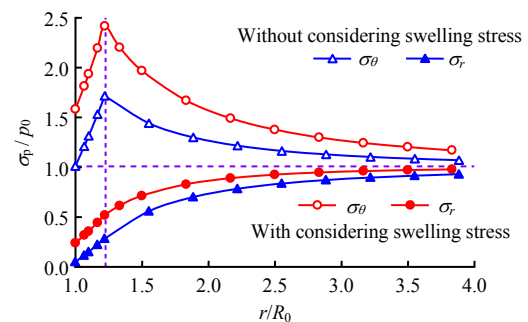


Fig. 6 Plots of stress of surrounding rock in the same plastic zone radius, with (red) and without (blue) considering swelling stress

Figures 5 and 6 show the calculated radial stress σ_r and hoop stress σ_θ with and without the inclusion of the swelling stress. When the surrounding rock turns into a plastic state, the spatial position where the maximum stress occurs moves from the tunnel to the elastic-plastic interface. Towards the internal of the rock mass, the stress of the surrounding rock gradually returns to the original state. As shown in Fig.5, given the same support resistance of $p_i = 0.05 p_0$, the radius of the plastic zone of the surrounding rock is $1.22 R_0$ if not considered the swelling stress, whereas the radius increases to $1.44 R_0$ when the swelling stress has been included, which is a factor of 1.17 compared with the former. Similarly, as shown in Fig. 6, given the same plastic zone radius of $R_p = 1.22 R_0$, the support reaction force is solve as $p_i = 0.24 p_0$ when the swelling stress has been considered, which is 4.8 times of that when it has been ignored.

As shown by the modelling results, a proper control of the radius of the plastic zone allows the surrounding rock to fully exert its self-supporting. If the external support can be applied after a specific amount of deformation of the tunnel, the burden on the support can be reduced efficiently.

5 Sensitivity analysis of parameters

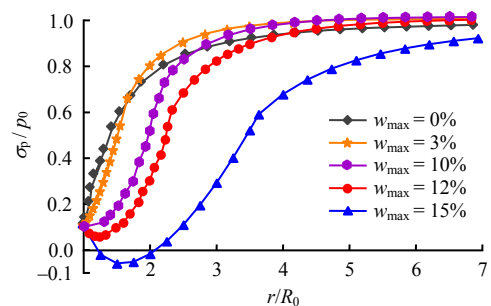
The elastoplastic solution given in this paper has included the effects of swelling stress and dilatancy. In this section, we analyze the sensitivity of the solution to the parameters such as the water content change Δw , the swelling coefficient of the surrounding rock α , and the dilatancy angle ψ , etc., to further investigate the effects of swelling stress and dilatancy on the stress and strain of the surrounding rock.

5.1 Effect of the water content change Δw

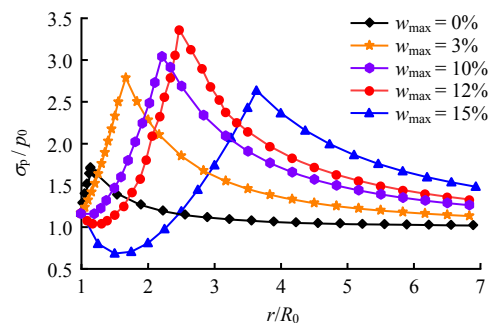
We first investigate the effect of water content change. The computational model uses the following parameters: hydrostatic pressure is $p_0 = 7$ MPa; support resistance is $p_i = 0.10$ MPa; tunnel excavation radius is $R_0 = 6$ m; the surrounding rock has an elastic modulus of $E = 1$ GPa, with a Poisson's ratio of $\nu = 0.3$, and a uniaxial compressive strength of $\sigma_c = 6$ MPa. The internal friction angle is $\varphi = 30^\circ$, and the humidity swelling coefficient is $\alpha = 0.03$. Since the swelling stress of the surrounding rock depends only on the difference between the current and the initial water content, we fix the initial water content to be zero ($w_0 = 0$), to simplify the problem. That is, we use the value of w_{\max} to represent the change of the water content of surrounding rock.

As shown from Fig.7, given the same supporting resistance p_i , the higher the maximum water content w_{\max} , the larger the radius of the plastic zone. Since $w_0 = 0$, the value of w_{\max} reflects the change of water content at each point in the surrounding rock. As the change of water content in the surrounding rock increases, the swelling stress also increases accordingly. It is worth noting that when the change of water content reaches to a certain amount, the tensile stress will appear in surrounding rock of the tunnel. As shown in

Fig. 4(a), a tensile stress zone appears in the plastic zone of the surrounding rock when $w_{\max} = 15\%$. As the support resistance is fixed, the plastic zone expands, and results in a decrease of the maximum stress on the elastic-plastic interface compared with the case when $w_{\max} = 12\%$. The emergence of tensile stress is unfavorable to the surrounding rock of the tunnel. For the swelling surrounding rock, the support should be closed immediately after the excavation to prevent it from absorbing water or soaking. Besides, waterproof and drainage measures are necessary to reduce the impact of water on the surrounding rock. On the one hand, it reduces the weakening effect of water on the surrounding rock. On the other hand, it reduces the swelling stress and therefore prevent the appearance of tensile stress zones in the surrounding rock.



(a) Radial stress



(b) Hoop stress f_l

Fig. 7 Plots of stress of surrounding rock with different w_{\max}

5.2 Effect of the humidity swelling coefficient α

Figure 8 shows the response of surrounding rock to different humidity swelling coefficient α . The computation uses the following parameters: hydrostatic pressure is $p_0 = 7$ MPa; support resistance is $p_i = 0.10$ MPa; tunnel excavation radius is $R_0 = 6$ m; the surrounding rock has an elastic modulus of $E = 1$ GPa, with a Poisson's ratio of $\nu = 0.3$ and a uniaxial compressive strength of $\sigma_c = 6$ MPa. The internal friction angle is 30° ; $w_{\max} = 4\%$, and $w_0 = 2\%$.

The straight line l shown in Fig. 8 is the radial displacement of the tunnel wall when the radius of the plastic zone is R_0 . This displacement is the minimum deformation of the surrounding rock, and the corresponding supporting resistance p_i is the maximum. As illustrated

from the figure, with the increase of the swelling coefficient of the surrounding rock, the minimum deformation increases linearly. The impact of the swelling coefficient is significant on the deformation of the surrounding rock. With the same supporting resistance, the deformation of the surrounding rock can be different by a few times. If no support are given ($p_i = 0$), the displacement at the tunnel wall is calculated as 6.1, 13.6, 16.3, 26.5, and 34.1 cm, corresponding to the swelling coefficients α of the surrounding rock chosen as 0.00, 0.03, 0.04, 0.08, and 0.10, respectively. As shown by this example, considering the water effect during tunnel excavation with highly swelling surrounding rock (e.g., $\alpha = 0.1$), the deformation of the surrounding rock caused by ground stress is 6.1 cm, whereas the deformation caused by swelling stress is 28 cm. The total deformation of the surrounding rock has increased 5.6 times, and the deformation caused by swelling stress is much more significant than the one caused by ground stress.

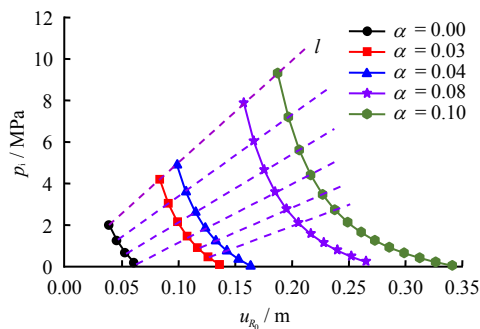


Fig. 8 Ground response of surrounding rock with different α

5.3 Effect of the dilatancy angle ψ

Figure 9 illustrates the response of surrounding rock to different dilatancy angle ψ . The computation uses the following parameters: hydrostatic pressure is $p_0 = 7$ MPa; support resistance is $p_i = 0.10$ MPa; tunnel excavation radius is $R_0 = 6$ m; the surrounding rock has a uniaxial compressive strength of $\sigma_c = 6$ MPa, with an elastic modulus of $E = 1$ GPa and a Poisson's ratio of $\nu = 0.3$. The internal friction angle is $\varphi = 30^\circ$. The humidity swelling coefficient is $\alpha = 0.03$; $w_{\max} = 4\%$, and $w_0 = 2\%$.

As shown by the figure, when the supporting pressure is high enough, the effect of dilatancy can be neglected due to the small deformation of the surrounding rock. With the decrease of the supporting pressure, the impact of the dilatancy angle becomes severe on the deformation of the surrounding rock. For example, if no support is given, the radial displacement of the rock around the tunnel is 13 cm if the dilatancy effect is not considered (that is, $\psi = 0^\circ$). In contrast, with $\psi = 30^\circ$, the calculated radial displacement of the rock is 20.7 cm around the tunnel. Compared with the simulated result without the dilatancy effect, the radial displacement of the surrounding rock has increased by 59%. From the perspective of the de-

formation mechanism of swelling rock, the observation is essentially the mutual product of swelling deformation and stress dilatancy caused by the swelling of surrounding rock in contact with water. Swelling soft rocks, especially the ones that are deeply buried, are not swellable before being disturbed, where the swelling pressure generated by water is less than the ground stress. The rock is in a latent plastic state^[19], with a high storage of the strain potential. As the tunnel excavation unloads, due to the low strength-to-stress ratio, the surrounding rock will rapidly form a plastic zone. The pressure in the surrounding rock will be changed from the initial hydrostatic pressure state, to a non-linear state that gradually increases from the inner rock to the rock-air interface. Non-linear swelling occurs. Shear dilatation then appears with the growth of secondary fractures, as well as the expansion and merging of various primary and newly generated fractures. The adjustments of structures of swelling soft rocks can strengthen the diffusion of humidity, which further promotes the process of swelling that had been constrained before. Therefore, the swelling of the surrounding rock to water and the stress dilatancy of the rock have resulted in a coupled process, which together contribute to the inward displacement of the tunnel wall. Thus, the dilatancy effect of the swelling surrounding rock cannot be negligible when considering the deformation of the surrounding rock of the tunnel.

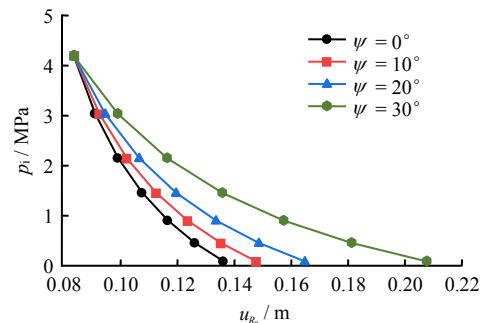


Fig. 9 Ground response of surrounding rock with different ψ

6 Discussion

This paper derives the solution to humidity distribution in porous media. The solution describes the humidity transfer from the inside to the outside of the tunnel under pressure. Therefore, the solution derived here that has considered the stress and strain of the swelling rock tunnel can be applicable to the case when the solid skeleton expands due to the effect of "adsorption".

To reduce the complexity in maths, the solution derived in this paper has made a few simplifications. For example, we assume that the mechanical properties of swelling rock remain unchanged within a certain range of water content, and we have not included the effect of the intermediate principal stress and dilatancy angle into the solution of stress, as well as the softening of strain, etc. These problems will be further studied in our future research based on the Hoek-Brown failure

criterion^[33–34] and the use of a semi-analytic method^[33].

7 Conclusion

Based on the theory of humidity stress field and with the use of the non-associated flow rule, we have derived an elastoplastic solution that considers the swelling stress and dilatancy effect to a deep-buried circular tunnel. We present a few numerical examples to demonstrate our theory and to investigate the effects of the parameters on the stress and the deformation of the surrounding rock, including the change of water content Δw , the humidity coefficient α and the dilatancy angle ψ , etc. The key findings are revisited and concluded as follows:

(1) The inclusion of swelling stress, comparing with the one that not considering it, has the plastic zone expanded and the thickness of the loosening zone increased given the same supporting resistance. Given the same the radius of the plastic zone, the required support resistance appears to be higher when considering the swell stress. The burden on the support can be reduced efficiently, if the radius of the plastic zone can be controlled appropriately to fully exert the self-supporting of the rocks, and if the support can be applied after allowing a specific amount of deformation of the tunnel.

(2) The impact of the change of water content is obvious on the stress and deformation of the surrounding rock. The higher the w_{\max} , the larger the plastic zone of the surrounding rock. When the water content changes to a certain extent, tensile stress appears in part of the plastic zone of the surrounding rock. With the increase of the humidity swelling coefficient α , the deformation of the surrounding rock increases linearly. Considering the tunnel excavation with high swelling surrounding rock that in contact with water, the deformation caused by the swelling stress can be much more significant than that caused by the ground stress.

(3) The impact of stress dilatancy of swelling surrounding rock cannot be negligible when considering the deformation of the tunnel surrounding rock. In particular, when the supporting resistance is relatively small, the radial displacement of the surrounding rock at the tunnel wall can be increased significantly if the dilatancy effect has been considered.

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