Calculation of mean intergranular stress and its application in unsaturated soil

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Abstract: The physicochemical interactions between different phases of the soil have an important impact on the geotechnical engineering related to environment and energy. The principle of effective stress is always the most important and effective theory to solve the relevant problems. However, the change of inter-particle stress caused by physicochemical effects cannot be accurately described by current effective stress equation. In response to the above problems, a mean intergranular stress expression that uniformly describe capillary, adsorption and osmosis effects between soil particles has recently been proposed. The purpose of this research is to realize the quantitative calculation of the mean intergranular stress and verify its stability and effectiveness. Firstly, the physical meaning of each part in the expression of mean intergranular stress is analyzed. Through analyzing the parameter in the expression, the variation rule of each part of mean intergranular stress with the water content and the concentration of pore water are obtained. Then, the formula characterizing the solid-liquid interface interaction for the surface force potential is derived, which can be used for the quantitative calculation of the mean intergranular stress equation. Finally, the mean intergranular stress of unsaturated soil in the critical state is calculated, and applied to simulate the coupled chemical-mechanical loading test of unsaturated soil. The calculated results show that there is a unique relationship between mean intergranular stress and shear strength in the critical state, and the chemical-mechanical calculation results give good agreement with the experimental results, verifying the stability and effectiveness of the mean intergranular stress.

Keywords: unsaturated soil; physicochemical effect; mean intergranular stress; surface force potential

1 Introduction

The physicochemical interactions widely exist in natural deposits or artificial soils. With the development of society and economy, geotechnical engineering problems related to environment and energy are becoming increasingly important[1–2], for example, deep disposal of highly radioactive nuclear waste[3], treatment of contaminated sites, solid waste landfill and oil and gas exploitation, in which the physicochemical interactions between different phases of the soil are involved[4–5]. Moreover, the physicochemical interactions also have a significant effect on the soil landslide induced by rainfall[6] and the engineering characteristics of marine soft soils [7–8].

The principle of effective stress is the current most important and effective theory to solve the geotechnical engineering problems described above. Terzaghi[9] proposed the effective stress calculation formula for saturated soils and defined the effective stress as the difference between the total stress and pore water pressure. On the basis of this study, Bishop[10] put forward the effective stress formula for unsaturated soils in order to ensure that the effective stress principle can be applied to describe the unsaturated soil. This formula consists of two parts: the net stress $\bar{\sigma}_{ij}$ and the product of the effective stress parameter χ and the matric suction s, as shown in the following formula:

$$\sigma'_{ij} = \bar{\sigma}_{ij} + \chi s \delta_{ij} \qquad (1)$$

Likewise, Aitchison and Donald [11], Jennings [12] and Blight [13] presented similar formula to express the effective stress of unsaturated soils. It should be noted that the effective stress of unsaturated soils does not always increase with the increase of capillarity. When the matric suction exceeds a certain value, capillarity has little effect on the effective stress of unsaturated soils with the change of matric suction. Gallipoli et al.[14] introduced some variables related to the gas-liquid interface action to explain this fact. Khalili et al.[15] established and verified the formula for calculating the effective stress of unsaturated soils through the shear test at the critical state and the drying-shrinkage test.

Domestically, Chen et al.[16] established the effective stress expressions for both isotropic unsaturated soils and anisotropic elastic porous-fluid media. Based on continuum mechanics, Chen et al.[17] derived a series of stress state variables by starting from the stress equilibrium equation of unsaturated soil skeleton. Zhao et al.[18] carried out the mechanical equilibrium analysis of the representative body elements of three-phase (solid, liquid and gas) soils, and obtained the expression for the mean soil skeleton stress of unsaturated soils. Shao and Guo[19] and Shao et al.[19–20]

also derived the effective stress formula of unsaturated soils. Duan and Zeng[21] divided the unsaturated soil into six phases: particle skeleton, colloid, solid pore water, liquid pore water, shrinkage film and pore gas. They also analysed the bearing structure of unsaturated soils but did not give the effective stress formula that could be used for quantitative calculation.

However, all the above effective stress formulas of unsaturated soils cannot describe the effects of physicochemical interactions between soil particles. Mitchell and Soga[22] proposed an effective stress formula of unsaturated soils considering the physicochemical interactions between soil particles: $\sigma' = \sigma - p_w^l + (A - R)$, where R and A respectively represent the repulsion and attraction between soil particles, but the calculation method is not clear. Xu[23] established an effective stress formula for bentonite on the premise of considering the influence of osmotic suction and verified the formula.

In addition to capillary effect and osmotic action, adsorption also affects the mechanical behavior of unsaturated soils. Obviously, a uniform variable is required to express these effects. The conventional effective stress expression does not solve these problems. Therefore, the concept of mean intergranular stress of unsaturated soils has been introduced. In recent years, Wei[24] proposed the expression of mean intergranular stress, which can uniformly describe the capillary, adsorption and osmotic effects. However, the calculation method of the surface force potential representing the solid liquid interface interaction in this formula is not clear yet, and the accuracy of the mean intergranular stress obtained by this formula has not been verified experimentally.

Due to the fundamentality and importance of the effective stress of unsaturated soils and the need for engineering practice, it is necessary to quantify the mean intergranular stress of unsaturated soils proposed by Wei[24] and verify its reliability and accuracy. The mean intergranular stress obtained by quantitative calculation will provide an important theoretical basis for solving practical engineering problems involving the physicochemical interactions between different phases of the soil and provide an important theoretical guidance for the geotechnical engineering related to environment and energy.

2   Mean intergranular stress of unsaturated soils

Either the external forces at the boundary or the physicochemical interactions between the soil skeleton and pore water (electrostatic interaction, Van der Waals' force, hydrogen bond and double layer interaction) can cause the deformation of soil skeleton only by transmission through the soil skeleton or soil particles. The shear strength of unsaturated soils is also determined by the intergranular stress. For unsaturated soils, the mean intergranular stress can be expressed as[24]

$$\boldsymbol{\sigma}'' = \boldsymbol{\sigma} - p^g\mathbf{I} + n^l(s-\Pi)\mathbf{I} \qquad (2)$$

where $\boldsymbol{\sigma}''$ is the mean intergranular stress tensor; $\boldsymbol{I}$ is the unit vector; $\boldsymbol{\sigma}$ is the total stress tensor; p^g is the gas phase pressure; n^l is the volume fraction of the liquid phase; and $n^l(s-\Pi)$ is the stress caused by the unsaturated action and physicochemical interactions, and can be called as the suction stress[25]. The physical definition of each part of the mean intergranular stress expression is introduced below.

s represents the matric suction and can be expressed as

$$s = p^g - p_w^l \qquad (3)$$

where p_w^l is the pore water pressure measured by the pore pressure sensor. Matric suction is a function of water content (degree of saturation), and the relationship between them is the equation of soil–water characteristic curve (SWCC), represented by the VG model[26]

$$S_r = \left[\frac{1}{1+(s/\alpha)^n}\right]^{1-(1/n)} \qquad (4)$$

where S_r is the degree of saturation; and α and n are parameters for the SWCC model.

Π is the generalized osmotic pressure, which can macroscopically represent the internal micro physicochemical interactions of soils. It can be explained by the difference between the measured pore water pressure and the real pore water pressure in the soil: when measuring the pore water pressure, the water in the soil is connected with the water in the measuring vessel (equilibrium solution); When the equilibrium state is reached, they have the same potential energy. Wei[24] deduced the relationship between the real pore water pressure in soils, p^l, and the measured pore water pressure, p_w^l, based on the principle of potential energy equality

$$p^l = p_w^l + \Pi \qquad (5)$$

The generalized osmotic pressure Π consists of two parts

$$\Pi = \Pi_D - \rho_\oplus^{l_{H_2O}}\Omega^l \qquad (6)$$

The first part is the Donnan osmotic pressure resulting from the fixed negative charge on the surface of soil particles. When soil particles meet water, the double electric layer occurs, which makes the charge distribution in the pore solution nonuniform and produces a certain concentration difference with the equilibrium solution, acting like a semi-permeable membrane. This results in the difference between the pressure of the pore solution and the pressure of the equilibrium solution. Donnan osmotic pressure is the macroscopic characterization of the effect of double electric layer. In addition to the physicochemical interactions induced by the fixed negative charge of soil particles, there are also some other interactions between the surface of the soil particles and the pore water, including Van der Waals' force, surface tension, and adsorption action between the liquid and solid

phases. The second part of generalized osmotic pressure characterizes these surface forces at the microscopic level, where $\rho_{\oplus}^{l_{H_2O}}$ is the density of pure water, and Ω^l represents the surface force potential.

1—Cation in the pore solution; 2—Clay layer; 3—Adsorption water; 4—Capillary water; 5—Pore air

(a) Composition of unsaturated soils

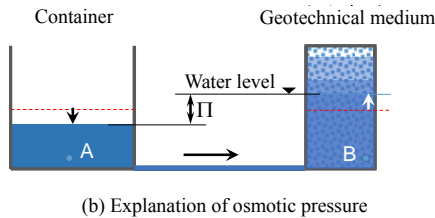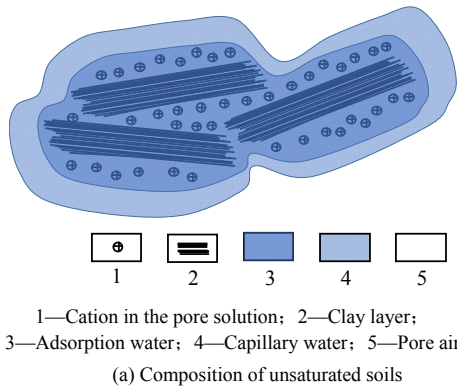(b) Explanation of osmotic pressure

Fig. 1 Schematic of three–phase composition of unsaturated soil

As the first part of the generalized osmotic pressure, the value of the Donnan osmotic pressure is related to the fixed negative charge density c_{fix}, the volume fraction of liquid phase n^l, and the concentration of equilibrium solution c_0

$$\Pi_D = \frac{RT\rho_{\oplus}^{l_{H_2O}}}{M_{H_2O}}\ln\left(\frac{a_R^{l_{H_2O}}}{a_P^{l_{H_2O}}}\right) \qquad (7)$$

where R is the universal gas constant with a value of 8.314 J/(mol·K); T represents the temperature; M_{H_2O} is the molar mass of water, the value of which is 18 g/mol; and $a_R^{l_{H_2O}}$ and $a_P^{l_{H_2O}}$ are respectively the activity of water (H_2O) in the equilibrium solution and the pore solution of soil. When the solution is an ideal solution, the activity of water is equal to the mole fraction of pore water. The detailed calculation process of Π_D has been given by Wei[24] and Ma et al.[4].

Parameter analysis of Donnan osmotic pressure Π_D is shown in Fig.2[24]. As can be seen in Fig.2, as the fixed negative charge density c_{fix} increases, Π_D tends to increase. c_{fix} is defined as the mole numbers of the fixed negative charge per unit volume of soil particles, which is closely related to the cationic exchange capacity (CEC) of clay. That is, with the increase of the cationic exchange capacity (CEC) of clay, the effect of Donnan gets enhanced. As the concentration of the equilibrium solution c_0 increases, the value of Π_D gradually decreases. When the salt solution is used to replace the pore water in saturated soils, the thickness of the double electric layer becomes thinner and the repulsion force decreases, resulting in the deform-

ation of soils accordingly. Π_D can quantitatively characterize the above phenomena from a macro perspective. Π_D also decreases with the increase of the volume fraction of liquid phase n^l, that is, when the water content decreases, the physicochemical interactions between soils become more obvious.
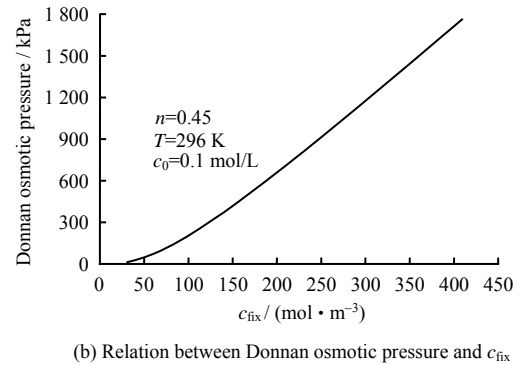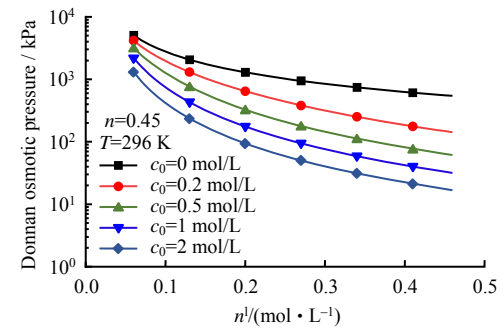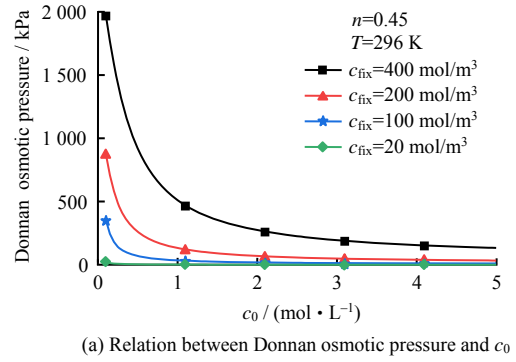

(a) Relation between Donnan osmotic pressure and c_0

(b) Relation between Donnan osmotic pressure and n^l

(b) Relation between Donnan osmotic pressure and c_{fix}

Fig. 2 Parameter analysis of Donnan osmotic pressure

The second part of the generalized osmotic pressure is expressed as

$$n^l\rho^l\Omega^l(T,n^l) = n_0^l\rho^l\Omega_0^l(T,n_0^l) + \int_{n^l}^{n_0^l}(s-\Pi_D)dn^l \qquad (8)$$

where ρ^l is the density of water; n_0^l is the volume fraction of water in saturated soils; and Ω_0^l is the surface force potential of saturated soils. The surface force potential above is closely related to water content and temperature. Under the isothermal condition and when Ω_0^l is equal to zero, the parameter analysis of Eq. (8) was carried out, and the relationship between surface force potential and degree of saturation is shown in Fig.3. As the degree of saturation decreases,

the surface force potential increases, indicating that the pore water is in a more stable state and more energy is required to move the pore water per unit mass.
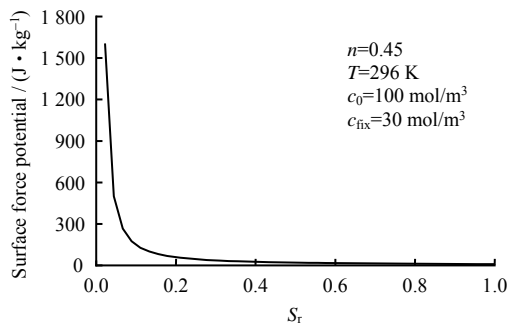

Fig. 3　Relationship between surface force potential and degree of saturation

By calculating Eqs. (4), (6) and (7), the variation curves of s, Π_D and Π with the degree of saturation are obtained, as shown in Fig.4[24]. With the decrease of water content, the matric suction s, Donnan osmotic pressure Π_D, and generalized osmotic pressure Π all show an increasing trend. In other words, with the decrease of degree of saturation, the difference between pore gas pressure and pore water pressure, the effect of double electric layer and the internal micro physicochemical interactions of soil mass become more significant. These microscopic interactions will affect the values of the real pore water pressure and the mean intergranular stress. Therefore, it is necessary to quantitatively characterize these two macroscopic variables representing physicochemical interactions in soils: Donnan osmotic pressure and surface force potential.
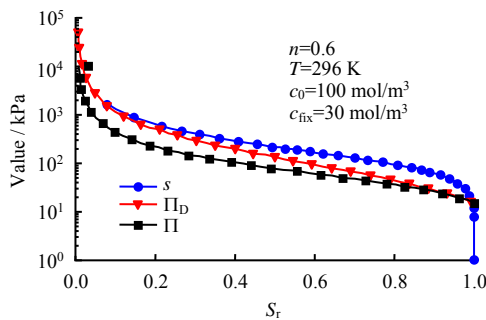

Fig. 4　Variations of s, Π_D, and Π with S_r

The composition of the formula of mean intergranular stress under different conditions is described below in order to better understand the physical meaning of the terms in the formula.

When the soil is unsaturated and the chemical osmosis effect is not considered (the Donnan osmotic pressure characterizing the chemical osmosis action is not required to be calculated), the mean intergranular stress can be expressed as follows:

$$\sigma_1'' = \sigma - p^g + n^1 s + n_0^1 \rho^1 \Omega_0^1 + \int_{n^1}^{n_0^1} s \, dn^1 \qquad (9)$$

When the soil is fully saturated, the pores of the soil are completely filled with liquid, and the volume fraction of liquid phase n^1 is the porosity n. Without considering the terms related to the unsaturated state (i.e. the matric suction s and the integral term related to the volume fraction of liquid phase in the surface force potential), the mean intergranular stress can be expressed as follows:

$$\sigma_2'' = \sigma - p_w^1 + n\left(-\Pi_D + \rho^1 \Omega_0^1\right) \qquad (10)$$

When the soil is fully saturated and the physico-chemical interactions between soil particles are not taken into account, the last term on the right hand side of Eq.(10) is ignored, and the mean intergranular stress can be expressed as follows:

$$\sigma_3'' = \sigma - p_w^l \qquad (11)$$

Equation (11) is the Terzaghi effective stress formula for saturated soils.

3　Determination of the solid liquid interface interaction of surface force potential

The two components of generalized osmotic pressure are calculated from Eq.(7) and Eq.(8), respectively. Eq.(7) can be directly quantified, while Eq.(8) for calculating the surface force potential consists of two parts: the first part is $n_0^1 \rho^1 \Omega_0^1$ and the second part is the integral of matric suction and Donnan osmotic pressure over the volumetric water content. When the relationship between the basic physical soil properties and the soil-water characteristic curve is given, the second part can be directly calculated by integral. The first part $n_0^1 \rho^1 \Omega_0^1$ is the only unknown variable and it characterizes the solid liquid interface interaction of surface force potential. The sum of solid liquid interface interaction and gas liquid interface interaction leads to the total surface force potential at the unsaturated state. Therefore, the physicochemical interactions between soils can be quantitatively described only when the solid liquid interface interaction of surface force potential is determined.

When the soil medium is in thermodynamic equilibrium, it shall satisfy the following mechanical equilibrium conditions[24]:

$$p^1 - p^s - p_m^1 = 0 \qquad (12)$$

where p^s is the thermodynamic pressure of the solid phase. This formula refers to the compatibility conditions at the microscopic interface of the porous medium, i.e. the substance cannot penetrate the interface between different phases (except through the chemical reaction)[27]. Therefore, the pressure difference between the two sides of the interface for various phases of substances does not vary randomly, but is restricted by Eq.(12). This macroscopic pressure difference is a function of the material state variable, $p_m^1 = n^1 \rho^1 (\partial \Omega^1 / \partial n^1)$ where the volume fraction of liquid phase is equal to the porosity n when the soil is saturated. According to the mechanical equilibrium condition of saturated soils, it

can be obtained that $p^s = p_w^l$ (Schematic diagrams of solid phase pressure of saturated soils and equilibrium solution are given in Fig.5).
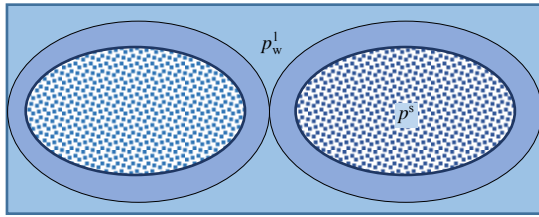

Fig. 5 Schematic of stress of saturated soil

When the soil is saturated, substituting the above conditions into Eq.(12) gives

$$p^l - p_w^l - n\rho^l \frac{\partial \Omega^l}{\partial n} = 0 \qquad (13)$$

Substituting $p^l = p_w^l + \Pi_D - \rho^l \Omega_0^l$, which is the relationship between real pore water pressure and measured pore water pressure when the soil is saturated (the expression of Eq.(5) when the soil is saturated), into Eq.(13) yields

$$n\rho^l \frac{\partial \Omega_0^l}{\partial n} + \rho^l \Omega_0^l = \Pi_D \qquad (14)$$

That is

$$\frac{\partial(n\rho^l \Omega_0^l)}{\partial n} = \Pi_D \qquad (15)$$

Integrating Eq.(15) gives

$$n\rho^l \Omega_0^l = \int_0^n \Pi_D dn \qquad (16)$$

Equation (16) is the expression of the solid liquid interface interaction of surface force potential, and is the physical variable characterizing the physicochemical interactions between solid phases and liquid phases of the soil. According to Eq.(16), the second part of the difference between the real pore water pressure in saturated soils and the measured pore water pressure is

$$\rho^l \Omega_0^l = \frac{\int_0^n \Pi_D dn}{n} \qquad (17)$$

The above expression is similar to the formula of adsorption stress derived by Gor et al.[28]. However, the expression obtained in this study determines values at the macro level. It is a function of porosity and basic physical properties, and the parameters required for calculation are simple. The expression proposed by Gor et al. needs the integral over the water film thickness h, which is a measurement at the micro level and is not convenient for quantitative calculation. Parametric analysis of $\rho^l \Omega_0^l$ is carried out and the analysis results are shown in Fig.6 and Fig.7.
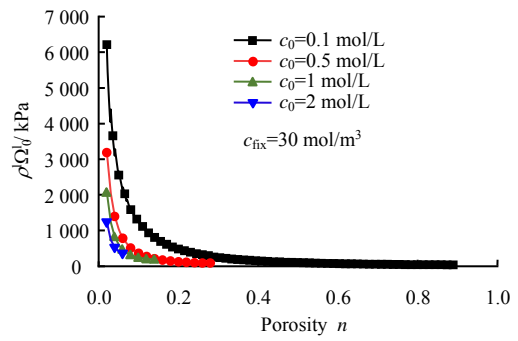

Fig. 6 Relationship between $\rho^l \Omega_0^l$ and porosity at different pore solution concentrations as the fixed charge concentrations is constant
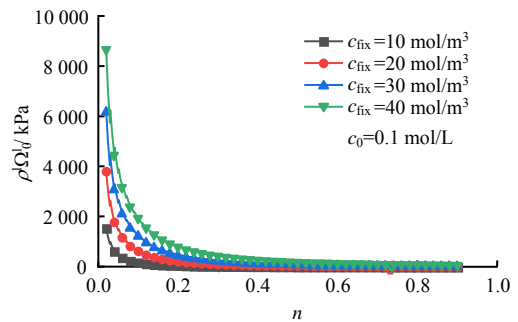

Fig. 7 Relationship between $\rho^l \Omega_0^l$ and porosity at different fixed charge concentrations as the concentration of the pore solution is constant

It can be seen from Figs.6 and 7 that $\rho^l \Omega_0^l$ decreases gradually with the increase of porosity. That is, when the soil is loose, the physicochemical inter- actions between soil particles are weak. For dense porous media, the adsorption action increases with the decrease of porosity. With identical porosity, when the fixed charge density c_{fix} is larger, $\rho^l \Omega_0^l$ is lager; when the concentration of equilibrium solution c_0 is smaller, $\rho^l \Omega_0^l$ is larger as well.

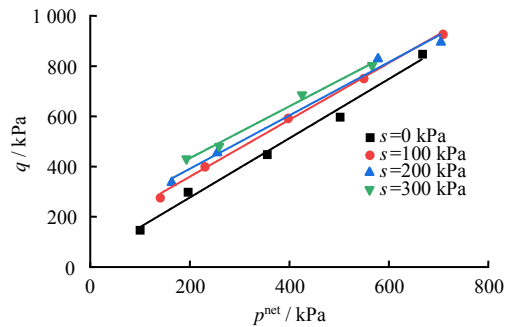## 4 Verification of shear strength in the critical state

When the soil reaches its critical state by shearing, both stress and volume remain unchanged, but the shear strain will continue to develop. At this point, the interaction between soil particles is reduced to the minimum due to the effect of soil fabric. The inter- actions between soil particles are mainly the friction between soil particles caused by external forces and the physicochemical interactions in the soil. Therefore, if there exists the "mean intergranular stress", i.e. effective stress, which can uniformly describe the effects of external force and the physicochemical interactions between soil particles, then there should be a unique effective stress value (effective stress principle) corresponding to a certain shear strength. Based on this theory, the existence and uniqueness of

the mean intergranular stress are verified by the data of the test on the critical state of three different soils in this section.
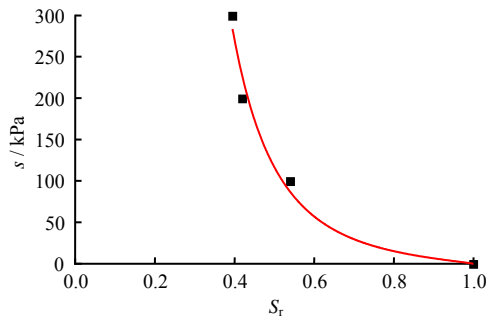
4.1 Verification by the tests on the low-plastic silt

Estabragh et al.[29] conducted the triaxial shear tests on silty soils with a low plasticity under fully saturated conditions and under matric suctions of 100, 200, 300 kPa, respectively. The results of the tests at the critical state of soil samples are shown in Fig.8(a). VG model was applied to fit the soil-water characteristic curve (SWCC), and the fitting results were given in Fig.8(b).

The mean intergranular stress of this soil was calculated, and the relationship between mean intergranular stress and shear strength was obtained, as shown in Fig.9. When the mean intergranular stress is greater than 300 kPa, the relationship between the mean intergranular stress and the shear strength is represented by the zonal distribution within a small range, but the trend of this relationship basically follows a straight line.

(a) Relation between net stress and shear strength

(b) SWCC fitted by VG model

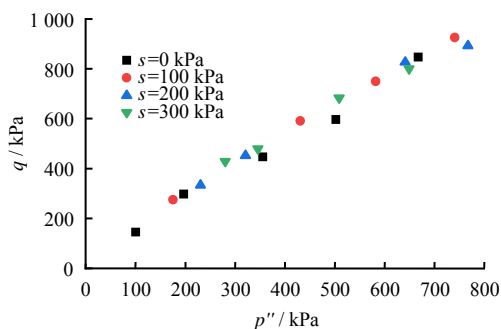**Fig. 8 Experimental data of critical state of silt with low plasticity**

Fig. 9 Relationship between mean intergranular stress–shear strength

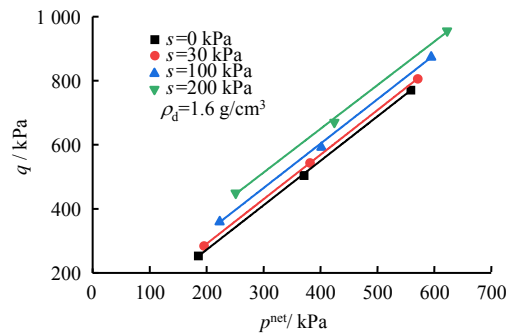### 4.2 Verification by the tests on the critical state of unsaturated silty sand from the region of Beijing

Cai et al.[30] conducted the triaxial shear tests on the silty sand from the region of Beijing under fully saturated conditions and under matric suctions of 100, 200, 300 kPa, respectively. The test results on the critical state of the silty sand with an initial dry density of 1.6 g/cm^3 and of 1.96 g/cm^3 are shown in Fig.10(a) and Fig.11(a), respectively. VG model was applied to fit the soil–water characteristic curve (SWCC), and the fitting results are shown in Figs.10(b) and 11(b), respectively.

4.2.1 Verification of the mean intergranular stress with a dry density of 1.6 g/cm^3

For the silty sand with a dry density of 1.6 g/cm^3, the mean intergranular stress was calculated, and the relationship between the mean intergranular stress and the shear strength was obtained, as shown in Fig.12. It can be seen that the relationship between mean intergranular stress and shear strength follows a straight line, that is, there is a unique mean intergranular stress corresponding to a certain shear strength.

4.2.2 Verification of the mean intergranular stress with a dry density of 1.96 g/cm^3

For the silty sand with a dry density of 1.96 g/cm^3, the mean intergranular stress was calculated, and the relationship between mean intergranular stress and the shear strength was obtained, as shown in Fig.13. It can be seen that the relationship between mean intergranular stress and shear strength basically follows a straight line.

(a) Relation between net stress and shear strength
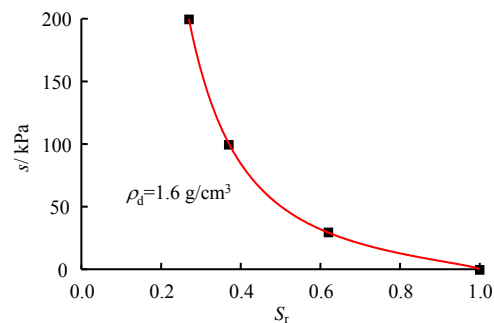
(b) SWCC fitted by VG model

Fig. 10 Experimental data of unsaturated silty sand in the critical state
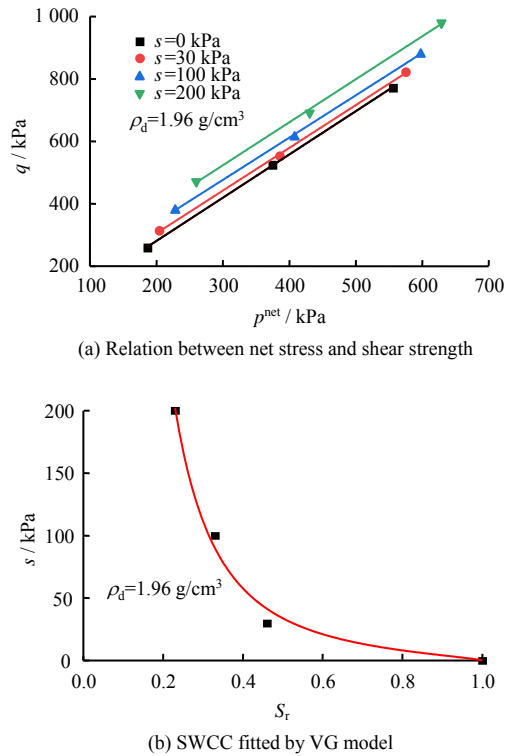
(a) Relation between net stress and shear strength

(b) SWCC fitted by VG model

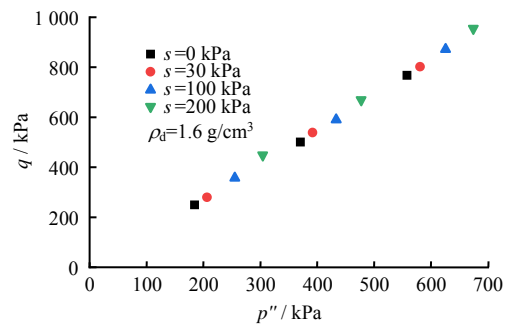**Fig. 11   Experimental data of unsaturated silty sand in the critical state**

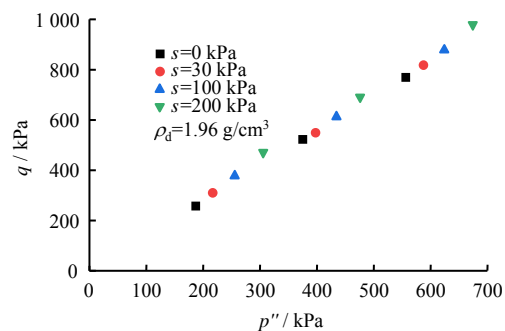**Fig. 12   Relationship between mean intergranular stress–shear strength**

Fig. 13   Relationship between mean intergranular stress–shear strength

4.3 Verification by the tests on the remolded residual clayey soils in Turkey

Kayadelen et al.[31] conducted the triaxial shear tests on soil samples of remolded residual clays from Turkey under the fully saturated condition and under matric suctions of 100, 200, 300 kPa. The relationship between net stress and shear strength at the critical

state obtained by the tests is shown in Fig.14(a). VG model was applied to fit the soil–water characteristic curve (SWCC), and the fitting results were given in Fig.14(b).
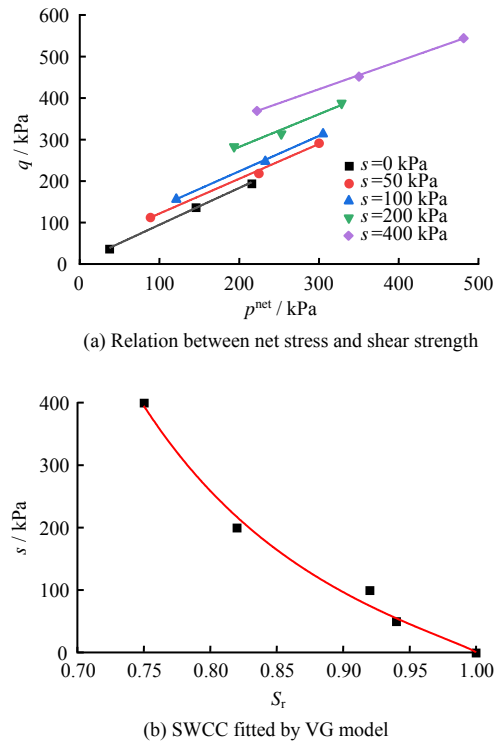

(a) Relation between net stress and shear strength

(b) SWCC fitted by VG model

Fig. 14   Experimental data of undisturbed residual clayey soils from Turkey in the critical state

Based on the test results, the intergranular stress was calculated, resulting in the obtained relationship between mean intergranular stress and shear strength shown in Fig.15. It can be seen that although the calculated data points near the mean intergranular stress of 300 kPa are slightly higher and the calculated data points near the mean intergranular stress of 700 kPa are slightly lower, the relationship between mean intergranular stress and shear strength generally follows a straight line.
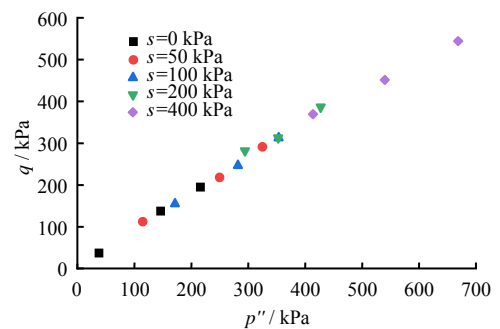

Fig. 15   Relationship between mean intergranular stress–shear strength

This section verifies the uniqueness of the mean intergranular stress through the relationship between mean intergranular stress and shear strength of the

three soils with different properties at the critical state. In the following section, the physicochemical interactions in unsaturated soils are simulated and analysed by using the mean intergranular stress, and the effecttiveness of the mean intergranular stress when physicochemical interactions exist in soils is verified.

5 Application of mean intergranular stress in unsaturated clays

Due to the negative charge on the surface of clay particles, the interior of soil mass is greatly affected by the physicochemical interactions, and the mechanical behavior of soils are more significantly affected by the water content and the concentration of pore salt solution. In this section, the mean intergranular stress established above is used to calculate the chemically mechanical performance of clays under the action of unsaturated condition and pore salt solution concentration. The calculation results were compared with some relevant test results to verify the ability of the mean intergranular stress in describing the chemically mechanical characteristics of unsaturated clays.

The constitutive model adopted in this section is the Modified Cam-Clay Model, and the effective stress of saturated soils was replaced by the mean intergranular stress of unsaturated soils. In order to consider the influence of the physicochemical interactions on the soil yielding, the hardening function is[4]

$$p_c = p_{c0}^* \exp(\frac{v\varepsilon_v^p}{\lambda - \kappa})\exp(-\beta\Pi_0)\exp(\beta p_s) \qquad (18)$$

where p_c is the pre-consolidation pressure; v is the specific volume; ε_v^p is plastic volumetric strain; Π_0 is the generalized osmotic pressure when the soil is saturated; p_{c0}^* is the initial pre-consolidation pressure of saturated soils when both ε_v^p and Π_0 are equal to 0; λ is the slope of the loading line of $v - \ln p''$; κ is the slope of the unloading line of $v - \ln p''$; $p_s = n^1(s - \Pi)$ represents the effect of unsaturated condition and physicochemical interactions on soil hardening; and β is the model parameter, which can be obtained from the relationship between the pre-consolidation pressure and the mean intergranular stress.

5.1 Calculation and verification of chemical and mechanical processes under constant matric suction

Complex loading paths involving mechanical and chemical loading were simulated when the matric suction remained constant (500 kPa). Results of the chemical–water–force coupled tests on the compacted clay under the condition of controlled matric suction by Mokni et al.[32] was used for verification.

The specific stress paths are as follows: (1) the matric suction was maintained at 500 kPa, and the vertical net stress was loaded from 20 kPa to 100 kPa; (2) the vertical net stress was maintained at 100 kPa and the concentration of pore salt solution was increased to 6.53 mol/ L; (3) the vertical net stress was loaded from 100 kPa to 1 000 kPa and then unloaded to 100 kPa.

The model parameters are listed in Table 1, and the comparison between calculated results and experimental results is shown in Fig.16. c_{fix} is the fixed charge density in the soil, i.e., the mole number of negative charge per unit volume of soils, which can be calculated by the cationic exchange capacity (mmol/100g) given as $c_{fix} = 10\text{CEC} \times \rho_d$. ρ_d is the dry density of the soil (g/cm³). The cation exchange capacity and the dry density of soils have been given in the literature, so the fixed charge density can be calculated.

Table 1 Parameters of calculation model

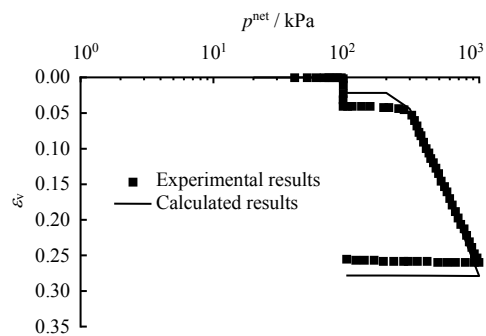| c_{fix} / (mol · m⁻³)	λ	κ	M	P_{c0}^* / kPa	β	n	α
360	0.3	0.001 7	0.856	200	0.034	1.03	7.3

Fig. 16 Numerical simulation of complex loading path under constant matric suction

As seen in Fig.16, the calculated values during the initial loading (20–100 kPa) process with the matric suction unchanged were basically consistent with the test results. When the net stress was maintained at 100 kPa and the matric suction was maintained at 500 kPa, the mean intergranular stress in this study can simulate the compressive deformation of soils caused by the increase of chemical concentration. In the following loading–unload process, the calculation results can generally demonstrate the variation trend of the void ratio of soils.

The relationship between concentration and volumetric deformation during chemical loading is shown in Fig.17. In the calculation of chemical loading, the calculation method was adopted such that the chemical concentration was loaded by steps until the specified concentration was finally reached.
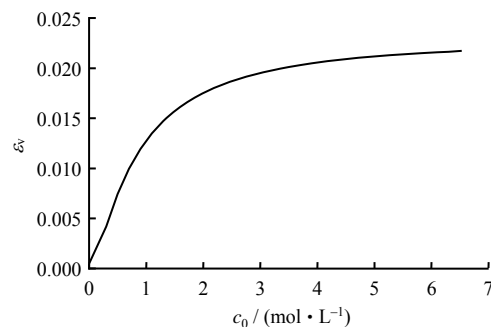

Fig. 17 Concentration-volumetric deformation relationship during chemical loading

As can be seen from Fig.17, as the chemical concentration increases step by step, the volumetric deformation also gradually increases. When the chemical concentration is relatively low, the change of chemical concentration has a significant effect on the volumetric deformation. When the chemical concentration is relatively high, the influence of the change of chemical composition on the volumetric deformation gradually decreases. This is consistent with the variation trend of the mean intergranular stress with the concentration.

5.2 Calculation and verification of consolidation and unloading process under constant matric suction and pore salt concentration

The mechanical loading and unloading paths were simulated under the condition of constant matric suction ($s = 500$ kPa) and different concentrations of pore salt solution (the concentration of pore salt solution was 0, 4, and 5.4 mol/L, respectively). Results of the chemical–water–force coupled tests on the compacted clay under the conditions of controlled matric suction and controlled concentration of pore salt solution by Mokni et al.[32] was used for verification. The adopted model parameters are listed in Table 1, and the comparison between calculated results and experimental results is shown in Fig.18.
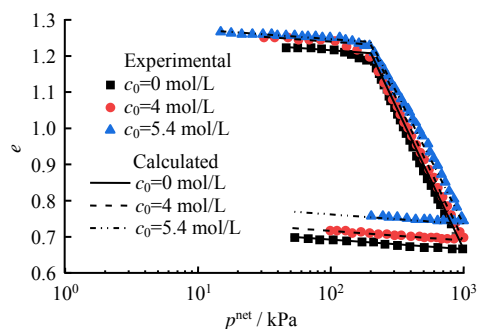

Fig. 18　Vertical net stress–volume ratio relationship under constant matric suction and pore salt solution concentration

In Fig.18, under the condition of constant matric suction and pore salt solution concentration, the comparison between the calculated results of mechanical consolidation and reloading and the experimental data shows that the mean intergranular stress in this study can reflect the mechanical response of soils to different concentrations of pore salt solution under unsaturated conditions.

6　Conclusions

(1) The parametric analysis of each part of the mean intergranular stress formula considering the physicochemical interactions between soil particles was carried out, and the variation law of mean intergranular stress was obtained.

(2) The calculation method of the solid liquid interface interaction of surface force potential was derived, and the quantitative calculation of the mean intergranular stress considering the physicochemical

interactions between soil particles was achieved.

(3) The mean intergranular stress at the critical state was calculated. The relationship between average intergranular stress and shear strength at the critical state follows a straight line, verifying the uniqueness of mean intergranular stress.

(4) The theoretical calculation results were compared with the results of chemically mechanical loading tests and consolidation reloading tests of unsaturated clays. The validity and applicability of the mean intergranular stress considering the physicochemical interactions between soil particles were verified. The formula of mean intergranular stress can provide an important theoretical basis for geotechnical engineering related to environment and energy.

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