Rock and Soil Mechanics

Volume 41 | Issue 7 Article 6

12-22-2020

Triple-shear failure criteria and experimental verification for unsaturated soils

Hao CHEN
1. School of Civil Engineering and Architecture, Nanchang University, Nanchang, Jiangxi 330031, China

Xiao-rong HU
1. School of Civil Engineering and Architecture, Nanchang University, Nanchang, Jiangxi 330031, China, huxiaorong@ncu.edu.cn

Custom Citation

CHEN Hao, HU Xiao-rong, . Triple-shear failure criteria and experimental verification for unsaturated soils[J]. Rock and Soil Mechanics, 2020, 41(7): 2380-2388.

Rock and Soil Mechanics 2020 41(7): 2380–2388 ISSN 1000–7598
https: //doi.org/10.16285/j.rsm.2019.6519 rocksoilmech.researchcommons.org/journal

Triple-shear failure criteria and experimental verification for unsaturated soils

CHEN Hao[1], HU Xiao-rong[1]

1. School of Civil Engineering and Architecture, Nanchang University, Nanchang, Jiangxi 330031, China

Abstract: Based on the triple-shear failure criterion of materials and the mechanical properties of unsaturated soils, the triple-shear failure criteria of the single stress variable and the double stress variable for unsaturated soils are put forward, and their characteristics are analyzed. The results show that the existing failure criteria for unsaturated soils can be interpreted approximately with the new criteria by changing the influence coefficient b of the principal stress. For loci on the π plane, the failure criteria proposed in this paper cover all the convex regions from the inner boundary of the single-shear failure criterion to the outer boundary of the triple-shear failure criterion. Therefore, the new criteria are suitable for unsaturated soils under various complex stress states and can reflect the inequality in uniaxial tension and compression of the unsaturated soils. In addition, several true triaxial test results reported by other researchers are adopted to verify the proposed criteria. For the unsaturated clayey sands, the true triaxial test results were found in good agreement with the values predicted by the triple-shear failure criteria of the single stress variable and the double stress variable when the influence coefficient of the principal stress b=0.6, and the predicted values given by the double stress variable criterion are more consistent than that of the single stress variable criterion. For the unsaturated loess, true triaxial test values are also in good agreement with the values predicted by the triple-shear failure criteria when the influence coefficient of principal stress b=0.2, and the difference is smaller between the two kinds of triple-failure failure criteria than that of the clayey sands.

Keywords: unsaturated soils; triple-shear failure criteria; single stress variable; double stress variable; experimental verifications

1 Introduction

The shear failure theory of unsaturated soils mainly includes the effective stress-based shear failure theory put forward by Bishop[1], the shear failure theory of double stress state variable proposed by Fredlund et al.[2], the theory of swelling pressure affecting the strength of unsaturated soils put forward by Lu et al.[3] and the shear failure theory of generalized hyperbolic suction of unsaturated soils put forward by Shen[4]. However, these failure criteria do not consider the effect of intermediate principal stress on the shear strength of unsaturated soils. The constitutive model of unsaturated soils established on the basis of spatial mobilized plane (SMP) criterion[5] considers the effect of intermediate principal stress on the strength of unsaturated soils, but the predicted results under triaxial extension are obviously not in agreement with the actual situation. Based on the new double-shear failure criterion by Mao-hong Yu, Zhang et al.[6] developed a new double-shear failure criterion for the true triaxial status of unsaturated soils. Although the criterion takes into account the influence of intermediate principal stress, it has the problem of the existence of double failure angles[7]. Through the flexible true triaxial test, the influence of intermediate principal stress on the strength of Shanghai fine sand under different influence coefficients of intermediate principal stress was analyzed by Hu et al.[8], and it was concluded that the strength characteristic of fine sand was closely related to the intermediate principal stress. By conducting the true triaxial drained shear test on Fujian standard sand, Zhang et al.[9] found that the density of sand during shear dilatancy changes with different intermediate principal stress coefficients, thus affecting the shear behavior of sand. Hu et al.[10–11] put forward a triple-shear failure criterion which can fully reflect the geotechnical characteristics on the basis of dodecahedral elements. Based on this criterion, the triple-shear failure criteria for unsaturated soils with single stress variable and double stress variable are proposed in this paper, and their characteristics are analyzed. In addition, the proposed criterion is verified by the true triaxial test data.

2 Triple-shear failure criterion for unsaturated soils

The triple-shear failure criterion[12] is

$$\begin{aligned}&\left[\left(\sigma_1'-\sigma_3'\right)^2+b\left(\sigma_1'-\sigma_2'\right)^2+b\left(\sigma_2'-\sigma_3'\right)^2\right]-\\&\left(1+b\right)\left[\left(\sigma_1'\right)^2-\left(\sigma_3'\right)^2\right]\sin\varphi'=\\&2c'\left(1+b\right)\left(\sigma_1'-\sigma_3'\right)\cos\varphi'\end{aligned} \tag{1}$$

Received: 2 September 2019 Revised: 16 December 2019
This work was supported by the National Natural Science Foundation of China (10862004, 11362016).
First author: CHEN Hao, male, born in 1994, Master candidate, majoring in research on strength of unsaturated soils. E-mail: 879977416@qq.com
Corresponding author: HU Xiao-rong, male, born in 1964, PhD, Professor, research interests: constitutive modelling of geomaterials and its application to geotechnical engineering. E-mail: huxiaorong@ncu.edu.cn

where σ_1' is the maximum effective (net) principal stress; σ_2' is the intermediate effective (net) principal stress; σ_3' is the minimum effective (net) principal stress; c' is the effective cohesion; φ' is the effective angle of internal friction; and b is the influence coefficient of the intermediate principal stress which should be obtained from the shear test of a specific soil, and the expression demonstrating its characteristic needs to be specifically studied.

2.1 Failure criterion for unsaturated soils with single stress variable

Based on Terzaghi's effective stress theory, Bishop proposed the effective stress of unsaturated soils with a single stress variable σ', which can be expressed as follows:

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) = (\sigma - u_a) + \chi s \tag{2}$$

where u_a is the pore air pressure; u_w is the pore water pressure; $\sigma - u_a$ is the effective normal stress; $u_a - u_w = s$ is matrix suction; and χ is the effective stress parameter, the value of which ranges from 0 to 1. It is dry soil when χ is equal to 0 and it is saturated soil when χ is equal to 1.

Substituting Eq.(2) into Eq.(1) gives:

$$\begin{aligned}&\left[(\sigma_1-\sigma_3)^2+b(\sigma_1-\sigma_2)^2+b(\sigma_2-\sigma_3)^2\right]-\\&(1+b)(\sigma_1-\sigma_3)(\sigma_1+\sigma_3-2u_a+2\chi s)\sin\varphi'=\\&2c'(1+b)(\sigma_1-\sigma_3)\cos\varphi'\end{aligned} \tag{3}$$

where σ_1, σ_2, σ_3 are the maximum principal stress, the intermediate principal stress and the minimum principal stress, respectively.

In order to replace the principal stresses in Eq.(3) by other stress variables, the total stress space shown in Fig.1 is introduced.

The following equations can be obtained from Fig.1,

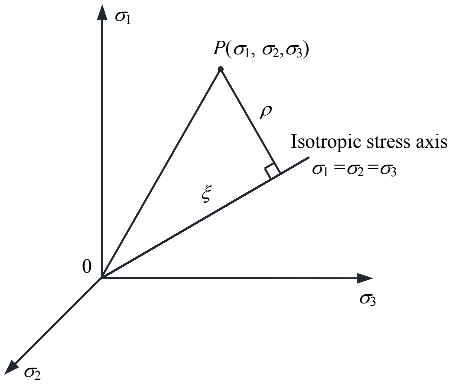


Fig. 1 Geometric space of principal stresses

$$\left.\begin{aligned}\sigma_1-u_a&=\frac{\xi}{\sqrt{3}}+\sqrt{\frac{2}{3}}\rho\cos\theta-u_a=\\&\sigma'_{oct}+\sqrt{\frac{2}{3}}\rho\cos\theta=\\&p'+\frac{2}{3}q\cos\theta\\\sigma_2-u_a&=\frac{\xi}{\sqrt{3}}+\sqrt{\frac{2}{3}}\rho\cos(\frac{2\pi}{3}-\theta)-u_a=\\&\sigma'_{oct}+\sqrt{\frac{2}{3}}\rho\cos(\frac{2\pi}{3}-\theta)=\\&p'+\frac{2}{3}q\cos\left(\frac{2\pi}{3}-\theta\right)\\\sigma_3-u_a&=\frac{\xi}{\sqrt{3}}+\sqrt{\frac{2}{3}}\rho\cos(\frac{2\pi}{3}+\theta)-u_a=\\&\sigma'_{oct}+\sqrt{\frac{2}{3}}\rho\cos(\frac{2\pi}{3}+\theta)=\\&p'+\frac{2}{3}q\cos\left(\frac{2\pi}{3}+\theta\right)\end{aligned}\right\} \tag{4}$$

where ξ is the normal distance from the stress point P to the origin; ρ is the normal distance from the stress point P to the isotropic stress axis; σ'_{oct} is the octahedral effective stress (net normal stress), expressed as $\sigma'_{oct} = \frac{\xi}{\sqrt{3}} - u_a$; p' is the average effective principal stress; q is the generalized shear stress; and θ is the Lode angle.

Substituting Eq.(4)into Eq.(3) yields:

$$\rho=\frac{(1+b)\cos\left(\theta-\frac{\pi}{6}\right)\left[(\sigma'_{oct}+\chi s)\sin\varphi'+c'\cos\varphi'\right]}{\frac{\sqrt{2}}{4}\left[2\cos^2\left(\theta-\frac{\pi}{6}\right)+2b\cos^2\left(\theta+\frac{\pi}{6}\right)+2b\sin^2\theta-(1+b)\frac{2}{\sqrt{3}}\sin\varphi'\cos\left(\theta-\frac{\pi}{6}\right)\cos\left(\theta+\frac{\pi}{3}\right)\right]} \tag{5}$$

$$\rho'=\frac{\partial\rho}{\partial\theta}=\frac{M_1'M_2-M_1M_2'}{M_2^{\ 2}} \tag{6}$$

where

$$\left.\begin{aligned}
&M_1 = 2\sqrt{2}(1+b)\cos\left(\theta-\frac{\pi}{6}\right)\cdot \\
&\qquad\left[(\sigma'_{\text{oct}}+\chi s)\sin\varphi'+c'\cos\varphi'\right] \\
&M_1' = \frac{\partial M_1}{\partial\theta} = -2\sqrt{2}(1+b)\sin\left(\theta-\frac{\pi}{6}\right)\cdot \\
&\qquad\left[(\sigma'_{\text{oct}}+\chi s)\sin\varphi'+c'\cos\varphi'\right] \\
&M_2 = 2\cos^2\left(\theta-\frac{\pi}{6}\right)+2b\cos^2\left(\theta+\frac{\pi}{6}\right)+2b\sin^2\theta- \\
&\qquad(1+b)\frac{2}{\sqrt{3}}\sin\varphi'\cos\left(\theta-\frac{\pi}{6}\right)\cos\left(\theta+\frac{\pi}{3}\right) \\
&M_2' = \frac{\partial M_2}{\partial\theta} = -2\sin\left(2\theta-\frac{\pi}{3}\right)-2b\sin\left(2\theta+\frac{\pi}{3}\right)+ \\
&\qquad 2b\sin 2\theta+(1+b)\frac{2}{\sqrt{3}}\sin\varphi'\sin\left(2\theta+\frac{\pi}{6}\right)
\end{aligned}\right\} \tag{7}$$

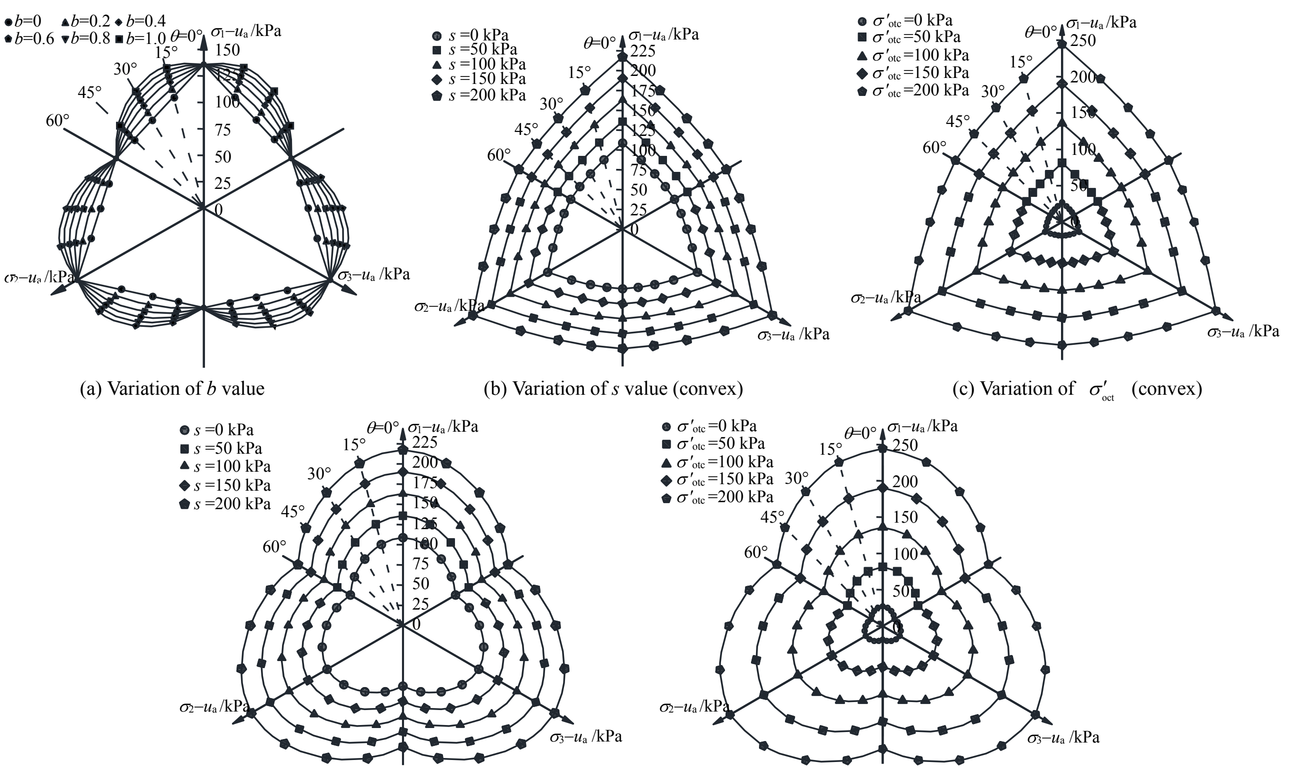
$$\begin{aligned}
q &= A\sin\varphi'(p'+\chi s)+Ac'\cos\varphi' = \\
&\quad A\sin\varphi' p'+A(c'\cos\varphi'+\chi s\sin\varphi')
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
A = {}& 6(1+b)\cos\left(\theta-\frac{\pi}{6}\right)\Big/ 2\sqrt{3}\left[\cos^2\left(\theta-\frac{\pi}{6}\right)+\right. \\
& \left. b\cos^2\left(\theta+\frac{\pi}{6}\right)+b\sin^2\theta\right]-(1+b)\cos\left(2\theta+\frac{\pi}{6}\right)\sin\varphi'
\end{aligned} \tag{9}$$

2.2 Characteristic analysis of the failure criterion for unsaturated soils with a single stress variable

The model characteristic analysis can be divided into the loci analysis and meridian analysis on the *π* plane. The yield locus of the new triple-shear failure criterion (Eq.(3)) on the *π* plane is shown in Fig.2, in which the effective stress parameter χ is 0.5 for the analysis of unsaturated soil failure criterion. Figure 2(a) is the loci analysis diagram on the *π* plane with the variation of the intermediate principal stress influence coefficient *b* when the net normal stress $\sigma'_{\text{oct}} = 100$ kPa and the matrix suction $s = 50$ kPa. Figure 2(b) is the loci analysis diagram on the *π* plane with the variation of the matrix suction *s* when the net normal stress $\sigma'_{\text{oct}} =$ 100 kPa and the intermediate principal stress influence coefficient $b = 0.1$. Figure 2(c) is the loci analysis diagram on the *π* plane with the variation of the net normal stress σ'_{oct} when the matrix suction $s = 50$ kPa and the intermediate principal stress influence coefficient $b = 0.1$. Figure 2(d) is the loci analysis diagram on the *π* plane with the variation of the matrix suction *s* when the net normal stress $\sigma'_{\text{oct}} = 100$ kPa and the intermediate principal stress influence coefficient $b = 0.6$. Figure 2(e) is the loci analysis diagram on the *π* plane with the variation of the net normal stress σ'_{oct} when the matrix suction $s = 50$ kPa and the intermediate principal stress influence coefficient $b = 0.6$.



(a) Variation of *b* value (b) Variation of *s* value (convex) (c) Variation of σ'_{oct} (convex)

(d) Variation of *s* value (non-convex) (e) Variation of σ'_{oct} (non-convex)

Fig. 2 Loci on the *π* plane of the triple-shear failure criterion of the single stress variable for unsaturated soils

It can be seen from Fig.2(a) that the loci intersects at a point of 60° coordinate line, and there is a point at which one of the loci is tangent to the normal line of 60° coordinate line. The influence coefficient of intermediate principal stress of this loci is b_0. When the value of b satisfies $0 \leqslant b \leqslant b_0$, the new triple-shear failure criterion produces the loci of a convex plane, otherwise it leads to the loci of a non-convex plane. The relation is shown as follows:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y'(\theta)}{x'(\theta)}=\frac{\rho'\cos\theta-\rho\sin\theta}{-\rho'\sin\theta-\rho\cos\theta}=\sqrt{3} \tag{10}$$

From Eq. (10), we can obtain that,

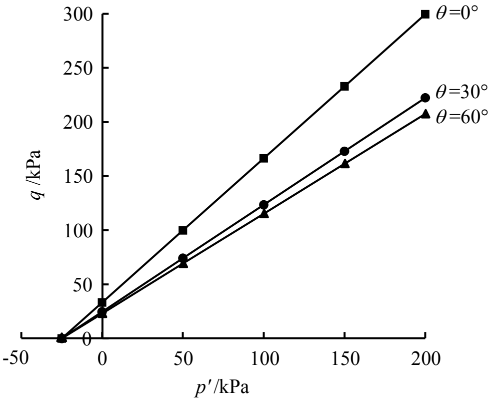
$$b_0=\frac{2\sin\left(2\theta-\frac{\pi}{3}\right)-\frac{2}{\sqrt{3}}\sin\varphi'\sin\left(2\theta+\frac{\pi}{6}\right)-\left[1+\cos\left(2\theta-\frac{\pi}{3}\right)-\frac{1}{\sqrt{3}}\sin\varphi'\cos\left(2\theta+\frac{\pi}{6}\right)\right]\tan\left(\theta-\frac{\pi}{6}\right)}{\left[2-\sin\left(2\theta+\frac{\pi}{6}\right)-\frac{1}{\sqrt{3}}\sin\varphi'\cos\left(2\theta+\frac{\pi}{6}\right)\right]\tan\left(\theta-\frac{\pi}{6}\right)-2\cos\left(2\theta+\frac{\pi}{6}\right)+\frac{2}{\sqrt{3}}\sin\varphi'\sin\left(2\theta+\frac{\pi}{6}\right)} \tag{11}$$

$b_0=0.128$ can be calculated from Eq.(11).

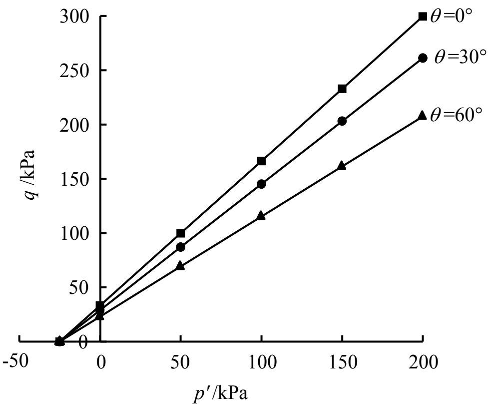
As can be seen from Fig.2, when either two of the net principal stress, matrix suction and the influence coefficients of intermediate principal stress are kept constant, the size of the loci plane increases with the increase of the remaining one. For example, in the loci with a varying influence coefficient of the intermediate principal stress, $b=0$ indicates the simple shear failure theory, and the size of the corresponding loci, i.e. the one based on the Mohr-Coulomb criterion, is the minimum; $b=1$ indicates the triple-shear failure criterion, and the size of the corresponding loci is the largest; b being equal to other arbitrary values leads to the corresponding loci. Therefore, the loci of the triple-shear failure theory for unsaturated soil on π plane covers all the convex regions from the inner boundary representing the simple shear failure theory to the outer boundary representing the triple-shear failure criterion[13]. It also shows that the criterion is suitable for unsaturated soils under various complex stress states.

When the matrix suction $s=50$ kPa and the principal stress influence coefficient b equals 0.1 and 0.6, respectively, the meridian is drawn on the p'-q plane according to Eq.(8), as shown in Fig.3.

It can be seen from Fig.3 that the stress path of the new triple-shear strength of unsaturated soil in the p'-q plane is a straight line that does not pass the origin, and the change of the principal stress influence coefficient b has an effect on the pure shear meridian ($\theta=30°$). When $b_0=0.1$, the pure shear meridian is close to that under conventional triaxial extension status($\theta=60°$); and when $b_0=0.6$, the pure shear meridian is close to that under conventional triaxial compression status ($\theta=0°$).



(a) b=0.1



(b) b=0.6

Fig. 3 Meridians of the triple-shear failure criterion of the single stress variable for unsaturated soils

2.3 Failure criterion for unsaturated soils with double stress variables

Fredlund et al.[14] adopted the double stress variables that consisted of net stress $(\sigma-u_{\mathrm{a}}\delta_{ij})$ and matrix suction $(u_{\mathrm{a}}-u_{\mathrm{w}})\delta_{ij}$ as the characteristic stresses of unsaturated soils, in which the total cohesion of unsaturated soil is formed by combining the matrix suction as the adsorption cohesion c_{s} and the saturated effective cohesion c' [15]. The three net principal stresses are written respectively as

$$\left.\begin{aligned}\sigma_1'&=\sigma_1-u_{\mathrm{a}}\\ \sigma_2'&=\sigma_2-u_{\mathrm{a}}\\ \sigma_3'&=\sigma_3-u_{\mathrm{a}}\end{aligned}\right\} \tag{12}$$

Substituting Eq.(12) into Eq.(1) gives

$$\begin{aligned}&\left[(\sigma_1-\sigma_3)^2+b(\sigma_1-\sigma_2)^2+b(\sigma_2-\sigma_3)^2\right]-\\&(1+b)(\sigma_1-\sigma_3)(\sigma_1+\sigma_3-2u_{\mathrm{a}})\sin\varphi'=\\&2(c'+c_{\mathrm{s}})(1+b)(\sigma_1-\sigma_3)\cos\varphi'\end{aligned} \tag{13}$$

Adopting various fundamental formulations may results in $c_{\mathrm{s}}=(u_{\mathrm{a}}-u_{\mathrm{w}})\theta_{\mathrm{w}}\tan\varphi'$, or $c_{\mathrm{s}}=(u_{\mathrm{a}}-u_{\mathrm{w}})\tan\varphi^{\mathrm{b}}$, or $c_{\mathrm{s}}=(u_{\mathrm{a}}-u_{\mathrm{w}})S^k\tan\varphi'$, etc., where θ_{w} is the volumetric water content; S is the degree of saturation; k is the fitting parameter; and φ^{b} is the suction angle.

Substituting Eq.(4) into Eq.(13) leads to

$$\rho=(1+b)\cos\left(\theta-\frac{\pi}{6}\right)\left[\sigma'_{\mathrm{oct}}\sin\varphi'+(c'+c_{\mathrm{s}})\cos\varphi'\right]\Bigg/ \frac{\sqrt{2}}{4}\left[2\cos^2\left(\theta-\frac{\pi}{6}\right)+2b\cos^2\left(\theta+\frac{\pi}{6}\right)+2b\sin^2\theta-(1+b)\frac{2}{\sqrt{3}}\sin\varphi'\cos\left(\theta-\frac{\pi}{6}\right)\cos\left(\theta+\frac{\pi}{3}\right)\right] \tag{14}$$

$$\rho'=\frac{\partial\rho}{\partial\theta}=\frac{K_1{}'K_2-K_1K_2{}'}{K_2{}^2} \tag{15}$$

where

$$\left.\begin{aligned}
&K_1=2\sqrt{2}(1+b)\cos\left(\theta-\frac{\pi}{6}\right)\left[\sigma'_{\mathrm{oct}}\sin\varphi'+(c'+c_{\mathrm{s}})\cos\varphi'\right]\\
&K_1'=\frac{\partial K_1}{\partial\theta}=-2\sqrt{2}(1+b)\sin\left(\theta-\frac{\pi}{6}\right)\cdot\\
&\qquad\left[\sigma'_{\mathrm{oct}}\sin\varphi'+(c'+c_{\mathrm{s}})\cos\varphi'\right]\\
&K_2=2\cos^2\left(\theta-\frac{\pi}{6}\right)+2b\cos^2\left(\theta+\frac{\pi}{6}\right)+2b\sin^2\theta-\\
&\qquad(1+b)\frac{2}{\sqrt{3}}\sin\varphi'\cos\left(\theta-\frac{\pi}{6}\right)\cos\left(\theta+\frac{\pi}{3}\right)\\
&K_2{}'=\frac{\partial K_2}{\partial\theta}=-2\sin\left(2\theta-\frac{\pi}{3}\right)-2b\sin\left(2\theta+\frac{\pi}{3}\right)+\\
&\qquad 2b\sin2\theta+(1+b)\frac{2}{\sqrt{3}}\sin\varphi'\sin\left(2\theta+\frac{\pi}{6}\right)
\end{aligned}\right\} \tag{16}$$

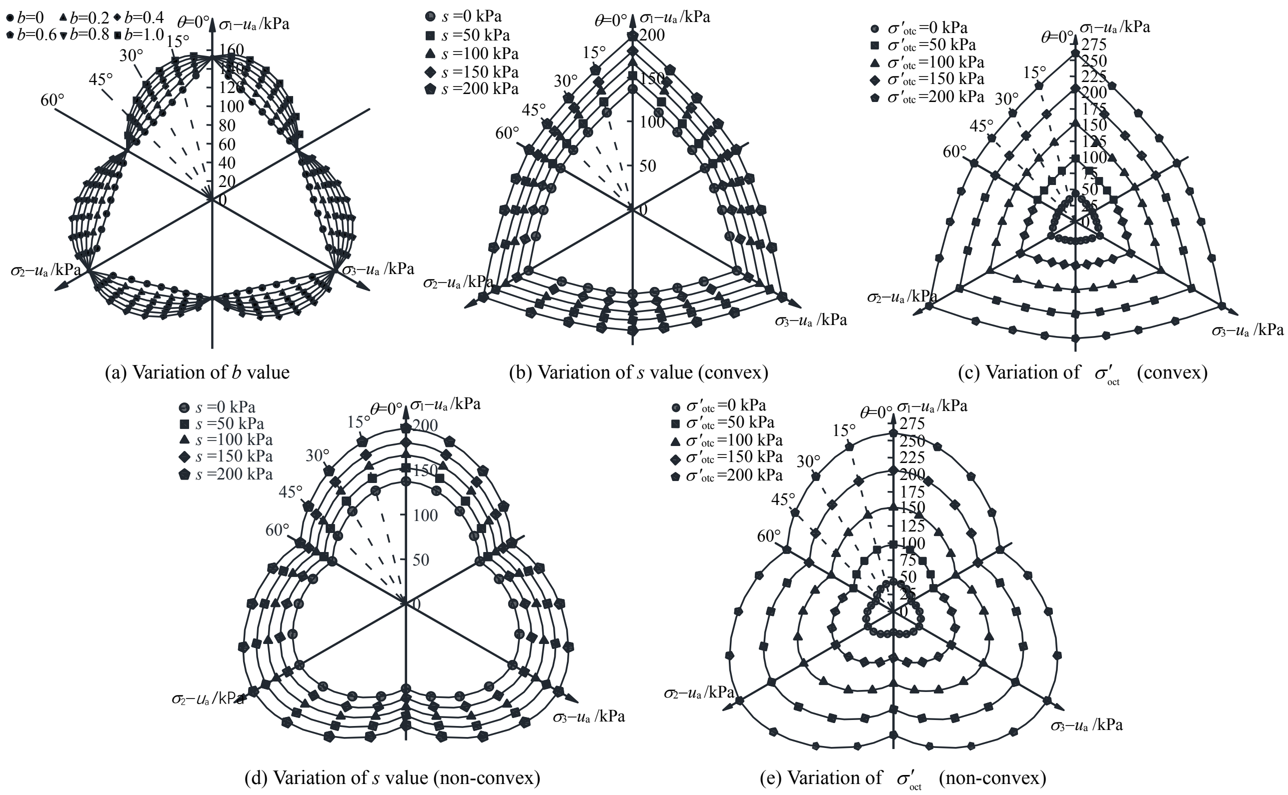
$$q=A\sin\varphi' p'+A(c'+c_s)\cos\varphi' \tag{17}$$

where

$$A=6(1+b)\cos\left(\theta-\frac{\pi}{6}\right)\Bigg/2\sqrt{3}\left[\cos^2\left(\theta-\frac{\pi}{6}\right)+b\cos^2\left(\theta+\frac{\pi}{6}\right)+b\sin^2\theta\right]-(1+b)\cos\left(2\theta+\frac{\pi}{6}\right)\sin\varphi' \tag{18}$$

2.4 Failure criterion for unsaturated soils with double stress variable

Similar to the characteristics of the unsaturated soil failure criterion with the single stress variable, the yield locus of the new triple-shear failure criterion based on Eq.(13) on the π plane is shown in Fig.4. Figure 4(a) is the loci diagram on the π plane corresponding to the change of the influence coefficient of principal stress b when the net normal stress $\sigma'_{\mathrm{oct}}=100$ kPa and matrix suction $s=50$ kPa. Figure 4(b) is the loci diagram on the π plane corresponding to the change of matrix suction s when the net normal stress $\sigma'_{\mathrm{oct}}=100$ kPa and the influence coefficient of intermediate principal stress $b=0.1$. Figure 4(c) is the loci diagram on the π plane corresponding to the change of net normal stress σ'_{oct} when the matrix suction $s=50$ kPa and the intermediate principal stress influence coefficient $b=0.1$. Figure 4(d) is the loci diagram on the π plane corresponding to the



(a) Variation of b value (b) Variation of s value (convex) (c) Variation of σ'_{oct} (convex)

(d) Variation of s value (non-convex) (e) Variation of σ'_{oct} (non-convex)

Fig. 4 Loci on the π plane of the triple-shear failure criterion of the double stress variable for unsaturated soils

change of matrix suction s when the net normal stress influence coefficient $b=0.6$. Figure 4(e) is the loci diagram on the π plane corresponding to the change of net normal stress σ'_{oct} when the matrix suction s=50 kPa and the medium principal stress influence coefficient $b=0.6$. The influence coefficient of principal stress b_0 at the boundary loci is equal to 0.128 as calculated by Eq.(11).

When the matrix suction s=50 kPa and the intermediate principal stress influence coefficient is equals 0.1 and 0.6, respectively, the meridian is plotted on the $p'-q$ plane according to the Eq.(17), as shown in Fig.5.

It can be seen from Fig.5 that the meridian characteristics of the failure criterion for unsaturated soils with the double stress variable is basically similar to those for unsaturated soils with the single stress variable, however, according to the specific numerical analysis, the longitudinal and transverse intersections are increased on the $p'-q$ plane.

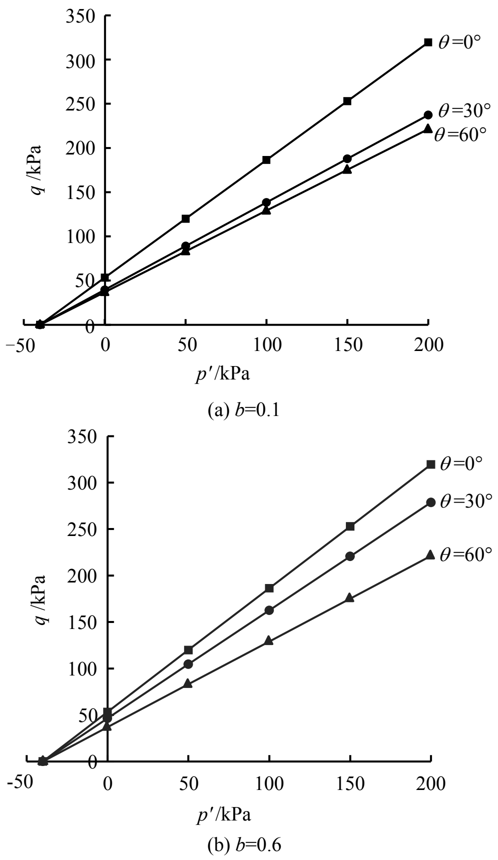


Fig. 5 Meridians of the triple-shear failure criterion of the double stress variable for unsaturated soils

3 Verification of the failure criterion by true triaxial test results of unsaturated soils

The theoretical verification includes two types: verification by laboratory tests and verification by results in the literature. In order to ensure the verification is objective and convincing, this paper uses the results in the literature in the verification.

3.1 Verification by using flexible true triaxial test results of unsaturated clay sand (SP-SC)

Hoyos et al.[16] carried out flexible true triaxial tests using compacted clay sand (SP-SC). In their tests, dry weight γ_d=15.344 kN/m^3, effective cohesion $c'=0$ kPa, effective internal friction angle $\varphi'=33°$, and different matrix suctions (50, 100 and 200 kPa) corresponded to different cohesions (26.0, 34.7 and 52.5 kPa) were employed. The test procedures included 8 fully drained static compression tests (HC), 6 fully drained conventional triaxial tests (CTC), 6 fully drained triaxial compression tests (TC), 3 fully drained triaxial tensile tests (TE) and 3 fully drained direct shear tests (SS). The predictions of the triple-shear failure criterion for unsaturated soil proposed in this paper is compared with the experimental results, as shown in Figs.6–8. As the reference[16] did not give the value of the effective stress parameter χ, this value with the single stress variable was calculated based on the experimental data as approximately 0.5.

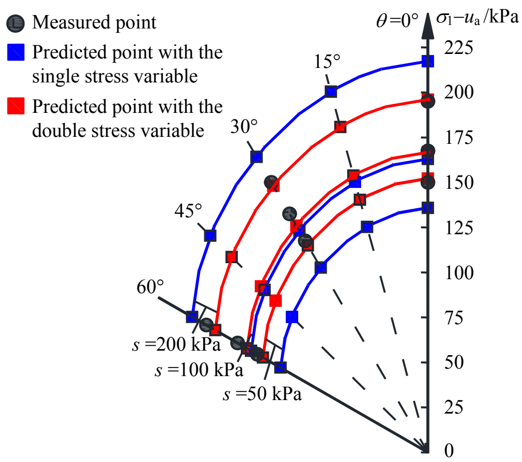


Fig. 6 Comparisons between the new triple-shear failure criteria and flexible true triaxial shear test results

It can be seen from Fig. 6 that when the average net stress $\sigma'_{oct}=100$ kPa and the influence coefficient of intermediate principal stress $b=0.6$, the following conclusions can be drawn: (1) When the triple-shear failure criterion with the double stress variable is adopted, the test data corresponding to different matrix suctions and different Lode angles (0°, 30°and 60°) are in good agreement with the predicted values by the criterion; when the matrix suction reaches 200 kPa, the predicted value of the triple-shear failure criterion under the single stress variable is larger, while it is smaller when the matrix suctions are 50 kPa and 100 kPa. (2) With the increase of the matrix suction s, the loci based on the unsaturated soil strength theory will expand.

It can also be seen from Fig.6 that the predicted value of the triple-shear failure criterion for unsaturated soil with the double stress variable is in good agreement with the test data from Hoyos et al.[16], thus verifying its applicability. The difference between the predicted value using the triple-shear failure criterion for unsaturated soils with the single stress variable and the experimental results is relatively larger, but the same evolution behavior is observed for both, which demonstrates the rationality of the criterion to a certain extent.

When the matrix suction $s = 50$ kPa and the Lode angle θ=0° are employed, the predicted values using the triple shear failure criterion with the double and single stress variable on the $p' - q$ plane are compared with the experimental data, respectively, as shown in Fig.7.

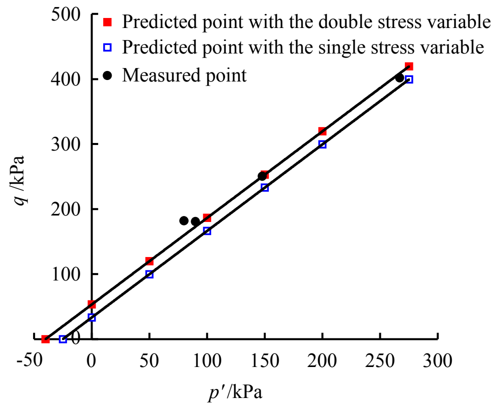


Fig. 7 Meridian validations with the true triaxial shear test results

As can be seen from Fig.7, the experimental data are basically consistent with the predicted values using the triple-shear failure criterion under double stress variable condition, which verifies its correctness. Although it is also different from the predicted value of the triple-shear failure criterion with the single stress variable, the overall trend is still the same.

The triple-shear failure criterion for unsaturated soils proposed in this paper is compared with the Tresca criterion, Drucker-Prager criterion, Mohr-Coulomb criterion, Matsuoka-Nakai criterion and Lade-Duncan criterion listed in the references[17–18], as shown in Fig.8.

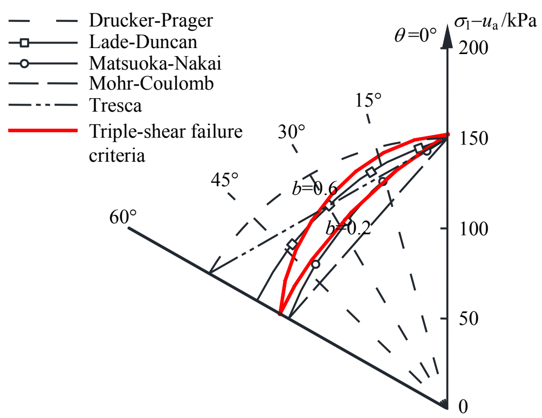


Fig. 8 Comparisons between the triple-shear failure criteria and the other failure criteria

It is concluded from the diagram that: (1) The loci of the new triple-shear failure criterion for unsaturated soils is between the loci of Mohr-Coulomb failure criterion and the loci of outer circle Drucker-Prager failure criterion. According to the verification in the above literature, the predicted values by the outer circle Drucker-Prager failure criterion is overestimated, while that by the Mohr-Coulomb failure criterion is underestimated, indicating that neither of them can well model the strength characteristics of unsaturated soils, which is also shown in the literature[19–20]. (2) When the influence coefficient of intermediate principal stress $b = 0.6$ is adopted, the corresponding loci of the triple-shear failure criterion for unsaturated soils is close to the loci of the Lade-Duncan failure criterion, and when the influence coefficient of intermediate principal stress $b = 0.2$, the corresponding loci of the triple-shear failure criterion is close to that of the Matsuoka-Nakai failure criterion, which indicates that other criteria can be approximately represented by the triple-shear failure criterion.

3.2 Verification of the failure criterion by mixed rigid-flexible true triaxial tests of unsaturated loess

Wang[21] carried out mixed rigid-flexible true triaxial tests on unsaturated loess. In the tests, the effective saturated cohesion $c' = 5.3$ kPa and the effective internal friction angle of the soil $\varphi' = 27.9°$ were measured. When the matrix suction s was 100 kPa, the corresponding cohesion was determined as 48.3 kPa. Meanwhile, through the shear failure test of unsaturated soils, the corresponding effective stress parameter was derived as 0.91 [21].

Figure 9 shows the comparison between the predicted values by the triple-shear failure criterion with both double stress variable and single stress variable and the experimental data when the matrix suction of s=100 kPa, the influence coefficient of intermediate principal stress of $b = 0.2$, the net principal stress of 100, 200 and 300 kPa, respectively, and the Lode angles of 0°, 30° and 60°, respectively, were adopted.

It can be seen from the diagram that the test data are in good agreement with the predicted values, validating the triple-shear failure criterion for unsaturated soils. In addition, with the increase of net principal stress, the loci of triple-shear failure criterion of unsaturated soils expands, and the predicted values by the triple-shear failure criterion with both the double stress variable and the single stress variable basically coincide under the same condition.

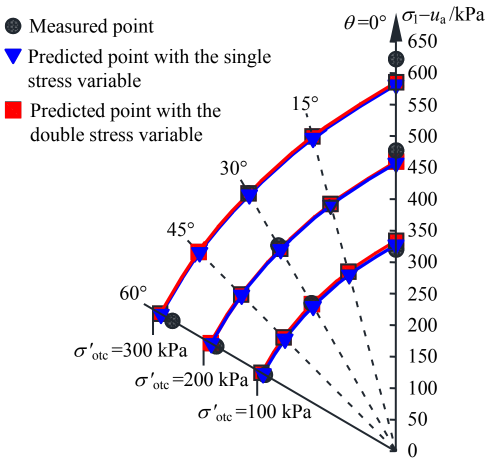
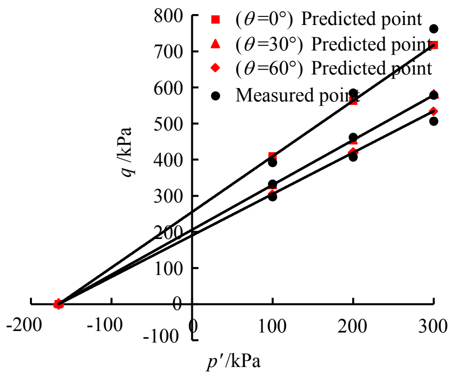
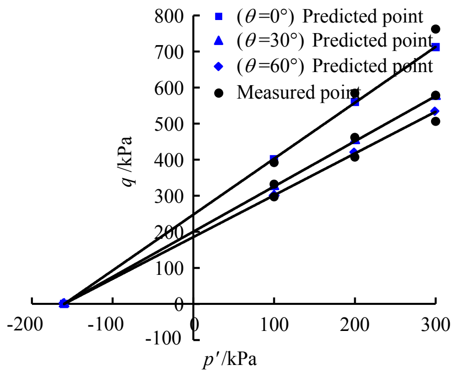


Fig. 9 Comparisons between the new triple-shear failure criteria and rigid-flexible true triaxial shear test results

When the matrix suction s=100 kPa and the influence coefficient of intermediate principal stress $b=0.2$ are employed, the predicted values by the triple-shear failure criterion with both the double stress variable and the single stress variable are compared with the experimental data on the $p'-q$ plane, and the results are shown in Fig.10.



(a) With the double stress variable



(b) With the single stress variable

Fig. 10 Meridian validation with the true triaxial shear test results

It can be seen from the diagram that the meridians of the triple-shear failure criterion with both the double stress variable and the single stress variable on the $p'-q$ plane are basically in agreement with the experimental data. Therefore, the robustness of the triple-shear failure criterion for unsaturated soils can be verified.

4 Conclusion

Based on the triple-shear failure criterion, the triple-shear failure criteria for unsaturated soils with both the double stress variable and the single stress variable are proposed. The criterion takes the dodecahedron as the geometric model and considers the joint action of the three principal shear plane stress pairs. A simple expression is adopted which avoids the problem of the existence of the double failure angles in the unified double shear strength criterion.

The loci of the triple-shear failure criteria for unsaturated soils covers all the convex regions from the simple shear failure theory of the inner boundary to the triple-shear failure criterion of the outer boundary, which shows that the criterion is suitable for unsaturated soils under various complex stress states and can model the characteristics of uniaxial tension and compression of unsaturated soils.

The true triaxial test results of unsaturated clay sand is in good agreement with the predicted values by the triple-shear failure criterion when the influence coefficient of medium principal stress $b=0.6$ is adopted. The true triaxial test results of unsaturated loess is in good agreement with the predicted value by the triple-shear failure criterion when the influence coefficient of intermediate principal stress $b=0.2$ is used. The applicability of the triple-shear failure criterion for unsaturated soils is validated. In addition, the predicted values of unsaturated clay sand with the double stress variable are in a better agreement with the experimental values than those with the single stress variable, while difference in the predicted values with both the double stress variable and the single variable are small for unsaturated loess. Moreover, the approach with the single stress variable can directly use σ' to analyze the deformation and strength of unsaturated soils, which is simple and convenient. The approach with the double stress variable uses two independent variables, which can comprehensively characterize the mechanical features of unsaturated soil, although it is relatively more complex in calculating the deformation of unsaturated soils.

The predicted values by triple-shear failure criterion for unsaturated clay sand with the double stress variable is compared with those by other strength criteria. Drucker-Prager failure criterion and Mohr-Coulomb failure criterion cannot well reflect the strength characteristics of unsaturated soils. However, when the triple-shear failure criterion adopts 0.2 and 0.6 for the influence coefficient of intermediate principal stress, the predicted values are very close to those by the Lade-Duncan criterion and Matsuoka-Nakai criterion respectively, and can well reflect the strength characteristics of unsaturated soils. Therefore, the triple-shear failure criterion can approximately represent other failure criteria.

References

[1] BISHOP A W. The principle of effective stress[J]. Teknisk Ukeblad, 1959, 106(39): 113−143.

[2] FREDLUND D G, MORGENSTEM N R , WIDGER R A. The shear strength of unsaturated soils[J]. Canadian Geotechnical Journal, 1978, 15(3): 313−321.

[3] LU Zhao-jun, ZHANG Hui-ming, CHEN Jian-hua, et al. Shear strength and swelling pressure of unsaturated soil[J]. Chinese Journal of Geotechnical Engineering, 1992, 14(3): 1−8.

[4] SHEN Zhu-jiang. Present study issues in unsaturated soils[C]//Proceedings of Regional Geotechnical Problems Symposium. Nanjing: Atomic Energy Press, 1996.1−9.

[5] MATSUOKA H, SUN D A, KOGANE A, et al. Stress-strain behavior of unsaturated soil in true triaxial tests[J]. Canadian Geotechnical Journal, 2002,39(3): 608−619.

[6] ZHANG Chang-guang, FAN Wen, ZHAO Jun-hai, et al. A new twin-shear true triaxial strength criterion of unsaturated soils and its validations[J]. Journal of Tongji University (Natural Science), 2015, 43(9): 1326−1331.

[7] HU Xiao-rong, WEI Xue-ying, YU Mao-hong. Characteristics of rock strenger and failure under triaxial compression analyzed by twin shear theory[J]. Chinese Journal of Rock Mechanics and Engineering, 2003, 22(7): 1093−1098.

[8] HU Ping, HUANG Mao-song, MA Shao-kun, et al. True triaxial tests and strength characteristics of silty sand[J]. Rock and Soil Mechanics, 2011, 32(2): 465–470.

[9] ZHANG Min, XU Cheng-shun, DU Xiu-li, et al. True triaxial experimental research on shear behaviors of sand under different intermediate principal stresses and different stress paths[J]. Journal of Hydraulic Engineering, 2015, 46(9): 1072–1079.

[10] HU Xiao-rong, YU Mao-hong. New research on failure criterion for geomaterial[J]. Chinese Journal of Rock Mechanics and Engineering, 2004, 23(18): 3037–3043.

[11] HU Xiao-rong, YU Mao-hong. Research on triple-shear yield criterion for materials[J]. Engineering Mechanics, 2006, 23(4): 6–11.

[12] HU Xiao-rong, DONG Xiao-long, HU Bo-yang, et al. The triple-shear elasto-plastic constitutive model for saturated over-consolidated clay[J]. Chinese Journal of Solid Mechanics, 2014, 38(6): 579–590.

[13] ZHANG Chang-guang, FAN Wen, ZHAO Jun-hai. Unified strength theory of unsaturated soils and verification with true triaxial test[J]. Chinese Journal of Rock Mechanics and Engineering, 2015, 34(8): 1702−1710.

[14] FREDLUND D G, RAHARDJO H, FREDLUND M D. Unsaturated soil mechanics in engineering practice[M]. New York: John Wiley and Sons lnc., 2012.

[15] ZHANG Chang-guang, ZHAO Jun-hai, ZHU Qian. Classification and summary of shear strength formulae for unsaturated soils[J]. Journal of Architecture and Civil Engineering, 2012, 29(2): 74–82.

[16] HOYOS L R, PEREZRUIZ D D, PUPPALA A J, et al. Modeling unsaturated soil response under suction-controlled true triaxial stress paths[J]. International Journal of Geomechanics, 2012, 12(3): 292–308.

[17] LI Guang-xin. Advanced soil mechanics[M]. Beijing: Tsinghua University Press, 2004.

[18] CHEN Ming-xiang. Elasto mechanics[M]. Beijing: Science Press, 2007.

[19] YU Mao-hong. Twin-shear theory and its application[M]. Beijing: Science Press, 1998.

[20] YU M H. Unified strength theory and its applications[M]. [S. l.]: Springer-Verlag, 2004.

[21] WANG Jiao. Research on soil water characteristics and yielding, strength and deformation properties of unsaturated loess by the true triaxial test[D]. Xi’an: Xi’an University of Technology, 2018.