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Analysis of one-dimensional thermal consolidation of saturated soil considering heat conduction of semi-permeable drainage boundary under varying loading

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Abstract: In order to study the influence of the temperature change on the consolidation for saturated soils, this paper presents a semi-analytical solution to the one-dimensional thermal consolidation equations considering the heat conduction of the semi-permeable drainage boundary on basis of the seepage and heat transfer equations. The semi-analytical solutions of excess pore pressure, temperature difference and soil settlement are firstly derived by using the Laplace transform upon the one-dimensional thermal consolidation equations considering the coupled and uncoupled thermo-mechanical models, respectively. Then the analytical solutions for a given time domain are obtained by the Crump's method. Verification is conducted by reducing the proposed solutions into those under the conditions of the single drainage boundary and the assumptions of Terzaghi's consolidation, which shows that the proposed solutions are reliable and in good agreement with the existing solutions from literature. Finally, several examples are analyzed to investigate the effects of semi-permeable boundary parameters, thermal diffusion coefficient, consolidation coefficient and temperature increase on the consolidation process of saturated soils under varying loading. All results demonstrate that the consolidation behavior is significantly affected by the investigated parameters.

Keywords: saturated soils; semi-permeable boundary; thermal consolidation; coupled thermo-mechanical model; Laplace transform

1 Introduction

Since Terzaghi [1] established the one-dimensional consolidation theory of saturated soil, scholars have carried out a large number of theoretical and experimental researches on it and extended it to complex situations such as variable loads, semi-permeable boundary, stratified foundation, and nonlinear rheological constitutive models[2–6]. However, these theories do not take temperature into account in the mechanical properties of soil. Considering the effect of temperature, thermal consolidation involves many aspects of the interaction between heat, water and force [7]. Studying the effect of temperature on the engineering properties of soil is of great practical value in such fields as thermal energy storage, geothermal resource development, nuclear waste disposal and heat supply pipeline design [8]. Therefore, it is necessary to analyse the thermal consolidation characteristics of soil.

Paaswell [9] first proposed the concept of thermal consolidation by considering the effect of temperature in soil consolidation tests. Subsequently, Campanella et al. [10] studied the thermal consolidation of saturated soil for volume and pore water pressure changes induced by temperature changes. Since then, considering the effect of temperature on the consolidation characteristics of geomaterials has attracted great attention. Delage et al. [11] conducted experimental research on rapid heating and consolidation of saturated clay and discussed the coupling effect of heat conduction and consolidation by using Fourier's heat conduction equation and Terzaghi's consolidation equation. Monfared et al. [12] after experiments, summarized some basic laws of the thermal consolidation characteristics of soil caused by temperature. Bai et al. [13] developed an axisymmetric thermal consolidation test device and carried out an indoor thermal consolidation test for hollow cylinder samples. Zhang et al. [14] analysed the influence of temperature on the thermal consolidation characteristics of one- dimensional saturated viscous soils through laboratory experiments. In addition, other scholars conducted many theoretical studies on thermal consolidation. Wu et al.[8] established a mathematical model based on the consolidation theory to analyse the thermal consolidation of one-dimensional saturated soils under variable loads. In the traditional consolidation theory, the boundary condition of soil layer is treated as the two extreme cases of completely permeable or completely impermeable, but in the practical engineering, the boundary condition after the cushion is laid on the top surface should be treated as a semi-permeable boundary[2]. In addition, the external load of tradetional consolidation is constant, but in the actual construction process, the load on the top of the soil will not remain unchanged, so it is necessary to take the timevarying characteristics of the external load into consideration.

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In practical engineering, sand cushion is often laid on saturated soil, which will affect the consolidation characteristics and thermal conductivity of saturated soil. Therefore, in this paper, based on the thermal consolidation theory of one-dimensional linear elastic saturated soil with coupled and uncoupled thermo-mechanical models, the top surface boundary conditions are considered as the semi-permeable boundary conditions with thermal conductivity, and the cyclic sectional loads more similar to the actual engineering are considered. A semi-analytical solution to thermal consolidation of one-dimensional saturated soil under external load and constant heat source is derived by analytical method. Then the semi-analytical solutions are reduced to the results of relevant studies to verify the correctness of the results of this paper. On this basis, the effects of external load, semi-permeable boundary parameters, temperature increment, thermal diffusion coefficient and consolidation coefficient on the thermal consolidation characteristics of one-dimensional saturated soil are analysed by numerical examples.

2 Thermal consolidation calculation model

As for the analysis of the thermal consolidation theory, Wu et al.[8] supplemented the assumptions of Terzaghi's one-dimensional consolidation theory based on the seepage consolidation theory. In this paper, based on the assumptions, the boundary conditions are considered as follows: the top surface of the soil layer is semipermeable, heat-exchangeable, and the bottom surface is impermeable and adiabatic. In addition, the external load is considered as a physical quantity that varies with time.

This paper intends to analyse the thermal consolidation problem of one-dimensional saturated soil under semipermeable boundary condition, and its schematic diagram is shown in Fig. 1. In the figure, H is the thickness of soil layer; k_1 and L_1 represent the permeability coefficient and thickness of the semi-permeable boundary layer, respectively; $Q(t)$ is the external load changing with time; $T_{s,1}$ and $T_{s,2}$ are the instantaneous temperature increment applied at the upper and lower surfaces of the cushion, respectively; k_v is the permeability coefficient of soil layer; γ_w is the unit weight of water; and E_s is the compression modulus, which can be expressed as

$$E_s = \frac{(1-\mu)E}{(1+\mu)(1-2\mu)} \tag{1}$$

where μ is Poisson's ratio; E is the deformation modulus of saturated soil.

According to studies [7–8], a one-dimensional thermal consolidation equation of saturated soil with coupled and uncoupled thermo-mechanical models under arbitrary loads can be obtained.

The coupled thermo-mechanical model is expressed by

$$C_v \frac{\partial^2 \sigma(z,t)}{\partial z^2} = \frac{\partial \sigma(z,t)}{\partial t} - A\frac{\partial T(z,t)}{\partial t} \tag{2}$$

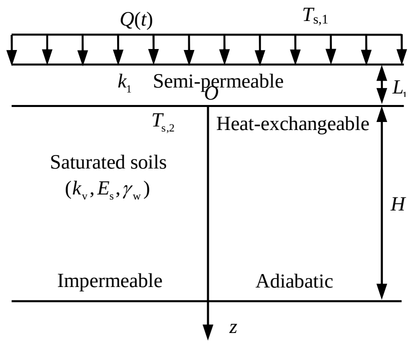

Fig.1 1D thermal consolidation model of saturated soils with semi-permeable boundary

$$\left(C + \frac{T_0\beta^2}{E_s}\right)\frac{\partial T(z,t)}{\partial t} - \frac{T_0\beta}{E_s}\frac{\partial \sigma(z,t)}{\partial z} = K\frac{\partial^2 T(z,t)}{\partial z^2} \tag{3}$$

where σ is the effective stress in the soil; T is the temperature difference between the soil temperature and the initial reference temperature; T_0 is the initial reference temperature; C_v is the consolidation coefficient; $C_v = k_v E_s / \gamma_w$; $A = \beta - \alpha E_s$. $\beta = \alpha E / (1-2\mu)$, is the thermal coefficient; $\alpha = (1-n)\alpha_s + n\alpha_w$, is the thermal expansion coefficient of soil; α_s and α_w are the thermal expansion coefficients of soil particles and water; respectively. n is porosity; K is the thermal conductivity of the soil; C is the specific heat of soil; z is the ordinate; and t is time.

The uncoupled thermo-mechanical model is

$$C_v \frac{\partial^2 \sigma(z,t)}{\partial z^2} = \frac{\partial \sigma(z,t)}{\partial t} - A\frac{\partial T(z,t)}{\partial t} \tag{4}$$

$$C\frac{\partial T(z,t)}{\partial t} = K\frac{\partial^2 T(z,t)}{\partial z^2} \tag{5}$$

For the semi-permeable top surface, heat transfer, undrained bottom surface, adiabatic, temperature increment and external load as shown in Fig.1, the initial condition and boundary condition are as follows:

$$t = 0: \sigma(z,t) = 0,\ T(z,t) = 0 \tag{6}$$

$$z = 0: T(z,t) = T_{s,2},\ \frac{\partial \sigma(z,t)}{\partial z} - \frac{R_1}{H}\left[\sigma(z,t) - Q(t)\right] = 0 \tag{7}$$

$$z = H: \frac{\partial \sigma(z,t)}{\partial z} = 0,\ \frac{\partial T(z,t)}{\partial z} = 0 \tag{8}$$

where $R_1 = Hk_1/(L_1 k_v)$ reflects the drainage rate of semi-permeable boundary layer.

Considering the heat transfer in semi-permeable boundary layer, based on a one-dimensional heat transfer theory [15–17], we have

$$q_z'' = K_1 \frac{T_{s,1} - T_{s,2}}{L_1} \tag{9}$$

where q_z'' is the cushion heat flux and K_1 is the thermal conductivity coefficient of cushion, it shows heat conduction rate in a semi-permeable boundary layer.

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3 Solution of mathematical model

3.1 Solution of coupled thermo-mechanical model

The Laplace transform of Eqs. (2) and (3) is, respectively

$$\frac{\partial^2\tilde{\sigma}}{\partial z^2}=a_1 s\tilde{\sigma}-a_2 s\tilde{T} \tag{10}$$

$$\frac{\partial^2\tilde{T}}{\partial z^2}=-a_3 s\tilde{\sigma}+a_4 s\tilde{T} \tag{11}$$

where $\tilde{\sigma}$ and $\tilde{T}$ are the results of σ and T by Laplace transforms, respectively; $a_1=1/C_{\mathrm{v}}$, $a_2=A/C_{\mathrm{v}}$, $a_3=\beta T_0/(KE_{\mathrm{s}})$, $a_4=(CE_{\mathrm{s}}+\beta^2 T_0)/(KE_{\mathrm{s}})$; and s is the Laplace variable.

From Eq. (10), we could get

$$\tilde{T}=\frac{a_1}{a_2}\tilde{\sigma}-\frac{1}{a_2 s}\frac{\partial^2\tilde{\sigma}}{\partial z^2} \tag{12}$$

Substituting Eq. (12) into Eq. (11), we could get

$$\frac{1}{a_2 s}\frac{\partial^4\tilde{\sigma}}{\partial z^4}-\frac{a_1+a_4}{a_2}\frac{\partial^2\tilde{\sigma}}{\partial z^2}+\left(\frac{a_1 a_4}{a_2}-a_3\right)s\tilde{\sigma}=0 \tag{13}$$

Solving Eq. (13), we could get

$$\tilde{\sigma}=C_1\mathrm{e}^{\xi z}+C_2\mathrm{e}^{-\xi z}+D_1\mathrm{e}^{\eta z}+D_2\mathrm{e}^{-\eta z} \tag{14}$$

where C_1, C_2, D_1 and D_2 are undetermined coefficients.

Substituting Eq. (13) into Eq. (11), we could get

$$\tilde{T}=C_1 b_1\frac{\mathrm{e}^{\xi z}}{a_2}+C_2 b_1\frac{\mathrm{e}^{-\xi z}}{a_2}+D_1 b_2\frac{\mathrm{e}^{\eta z}}{a_2}+D_2 b_2\frac{\mathrm{e}^{-\eta z}}{a_2} \tag{15}$$

where

$$\left.\begin{aligned}&\xi=\sqrt{\frac{\left(a_1+a_4-\sqrt{a^2+4a_2a_3-2a_1a_4+a_4{}^2}\right)s}{2}}\\&\eta=\sqrt{\frac{\left(a_1+a_4+\sqrt{a^2+4a_2a_3-2a_1a_4+a_4{}^2}\right)s}{2}}\\&b_1=a_1-\xi^2/s,\ b_2=a_1-\eta^2/s\end{aligned}\right\} \tag{16}$$

Taking the first derivative of Eq. (14) and (15), respectively leads to

$$\frac{\partial\tilde{u}}{\partial z}=\xi C_1\mathrm{e}^{z\xi}-\xi C_2\mathrm{e}^{-z\xi}+\eta D_1\mathrm{e}^{z\eta}-\eta D_2\mathrm{e}^{-z\eta} \tag{17}$$

$$\frac{\partial\tilde{T}}{\partial z}=\xi b_1 C_1\frac{\mathrm{e}^{z\xi}}{a_2}-\xi b_1 C_2\frac{\mathrm{e}^{-z\xi}}{a_2}+\eta b_2 D_1\frac{\mathrm{e}^{z\eta}}{a_2}-\eta b_2 D_2\frac{\mathrm{e}^{-z\eta}}{a_2} \tag{18}$$

Laplace transforms of the boundary conditions Eqs. (7) and (8) are expressed as

$$\frac{\partial\tilde{\sigma}(0,s)}{\partial z}-r\left[\tilde{\sigma}(0,s)-Q(s)\right]=0,\ \tilde{T}(0,s)=\varsigma \tag{19}$$

$$\frac{\partial\tilde{\sigma}(H,s)}{\partial z}=0,\ \frac{\partial\tilde{T}(H,s)}{\partial z}=0 \tag{20}$$

where $r=R_1/H$, $\varsigma=T_{\mathrm{s},2}/s$.

Substituting Eqs. (14) and (15) into Eqs. (19) and (20), we could get

$$\tilde{\sigma}(z,s)=\frac{1}{\gamma_1}\left\{\chi_1\cosh[\xi(H-z)]-\chi_2\cosh[\eta(H-z)]\right\} \tag{21}$$

$$\begin{aligned}\tilde{T}(z,s)=\frac{1}{a_2\gamma_1}\{&b_1\chi_1\cosh[\xi(H-z)]-\\&b_2\chi_2\cosh[\eta(H-z)]\}\end{aligned} \tag{22}$$

where,

$$\begin{aligned}\gamma_1=&b_1\eta\cosh(H\xi)\sinh(H\eta)+\cosh(H\eta)\cdot\\&\left[(b_1-b_2)r\cosh(H\xi)-b_2\xi\sinh(H\xi)\right]\end{aligned} \tag{23}$$

$$\chi_1=(a_2\varsigma-b_2Q)r\cosh(H\eta)+a_2\varsigma\eta\sinh(H\eta) \tag{24}$$

$$\chi_2=(a_2\varsigma-b_1Q)r\cosh(H\xi)+a_2\varsigma\xi\sinh(H\xi) \tag{25}$$

The settlement $w(t)$ of foundation at a certain time t can be calculated using the following formula:

$$w(t)=\int_0^H\varepsilon_{\mathrm{v}}\mathrm{d}z \tag{26}$$

where ε_{v} is the strain of soil layer.

According to the generalized thermoelastic Hooke's law of thermo-elasticity theory, there is

$$w(t)=\int_0^H\frac{\sigma-\beta T}{E_{\mathrm{s}}}\mathrm{d}z \tag{27}$$

After the Laplace transform, we could get

$$\tilde{w}(s)=\frac{1}{E_{\mathrm{s}}}\int_0^H\tilde{\sigma}(z,s)-\beta\tilde{T}(z,s)\mathrm{d}z \tag{28}$$

Substituting Eqs. (21) and (22) into Eq. (28), we could get

$$\tilde{w}=\frac{\left[\chi_1(a_2-\beta b_1)\eta\sinh(H\xi)+\chi_2(-a_2+\beta b_2)\xi\sinh(H\eta)\right]}{a_2E_{\mathrm{s}}\gamma_1\xi\eta} \tag{29}$$

3.2 Solution of uncoupled thermo-mechanical model

For the uncoupled thermo-mechanical model, the thermal diffusion coefficient is defined as $C_{\mathrm{t}}=K/C$. Laplace transforms of Eqs. (4) and (5) are expressed as

$$\frac{\partial^2\tilde{\sigma}}{\partial z^2}=a_1 s\tilde{\sigma}-a_2 s\tilde{T} \tag{30}$$

$$\frac{\partial^2\tilde{T}}{\partial z^2}=-a_3 s\tilde{\sigma}+a_4 s\tilde{T} \tag{31}$$

where $a_1=1/C_{\mathrm{v}}$, $a_2=A/C_{\mathrm{v}}$, $a_3=0$, and $a_4=C/K$.

Solving Eq. (31) ,we could get

$$\tilde{T}(z,s)=C_1\mathrm{e}^{m_1 z}+C_2\mathrm{e}^{-m_1 z} \tag{32}$$

where $m_1=\sqrt{a_4 s}$.

Substituting Eq. (32) into Eq. (30), we could solve

$$\tilde{\sigma}(z,s)=D_1\mathrm{e}^{m_2 z}+D_2\mathrm{e}^{-m_2 z}+\frac{a_2 s\left(C_1\mathrm{e}^{-m_1 z}+C_2\mathrm{e}^{m_1 z}\right)}{m_1{}^2-m_2{}^2} \tag{33}$$

where $m_2 = \sqrt{a_1 s}$.

Substituting Eqs. (32) and (33) into Eqs. (19) and (20), we could get

$$\tilde{\sigma}(z,s) = \frac{1}{\gamma_2}\left\{\chi_3 \cosh\left[m_1(H-z)\right] + \chi_4 \cosh\left[m_2(H-z)\right]\right\} \tag{34}$$

$$\tilde{T}(z,s) = \varsigma \cosh[m_1(H-z)]/\cosh(Hm_1) \tag{35}$$

where

$$\chi_3 = a_2\varsigma s\left[-m_2\cosh(Hm_2) + r\cosh(Hm_2)\right] \tag{36}$$

$$\chi_4 = \left\{\left[\left(m_1^2 - m_2^2\right)Qr - a_2\varsigma rs\right]\cosh(Hm_1) - a_2\varsigma m_1 s\sinh(Hm_2)\right\} \tag{37}$$

$$\gamma_2 = \cosh(Hm_1)\left(m_1^2 - m_2^2\right)\cdot\left[m_2\sinh(Hm_2) + r\cosh(Hm_2)\right] \tag{38}$$

By substituting Eqs. (34) and (35) into Eq. (28), the analytical solution of the thermal consolidation settlement of one-dimensional saturated soil with uncoupled thermo-mechanical model in the Laplace transform domain is obtained:

$$\tilde{w} = \frac{\lambda - \beta\gamma_2 m_2\varsigma\tanh\left(Hm_1\right)}{E_s\gamma_2 m_1 m_2} \tag{39}$$

where

$$\lambda = m_2\chi_3\sinh\left(Hm_1\right) + m_1\chi_4\sinh(Hm_2) \tag{40}$$

3.3 Inverse numerical transformation

In this paper, the analytical solution of the thermal consolidation problem of one-dimensional saturated soils with coupled and uncouple thermo-mechanical models in the Laplace transform domain is obtained by analytical method. By inverse Laplace transform, the solution in the physical space domain can be obtained. As for numerical inversion of Laplace change, Dubner and Abate[18], Stehfest [19] and Crump[20] have proposed their algorithms, respectively. According to the research of He et al.[21], the convergence speed of Crump method is faster than that of Dubner and Abate method. Moreover, compared with Stehfest method, Crump method is more accurate and reliable, and it can overcome the defects of oscillation and dispersion when the curve of solution has a steep change.

The inverse Laplace transform of this paper adopts the Crump method [18]. The inverse Laplace transform of Eqs. (21), (22), (29), (34), (35), and (39) are expressed as

$$\sigma(z,t) = \frac{e^{st}}{T}\left\{\frac{1}{2}\tilde{\sigma}(z,s) + \sum_{k=1}^{\infty}\{\mathrm{Re}[\tilde{\sigma}(z,s+\frac{k\pi i}{T})]\cdot \cos\frac{k\pi t}{T} - \mathrm{Im}[\tilde{\sigma}(z,s+\frac{k\pi i}{T})]\sin\frac{k\pi t}{T}\}\right\} \tag{41}$$

$$T(z,t) = \frac{e^{st}}{T}\left\{\frac{1}{2}\tilde{T}(z,s) + \sum_{k=1}^{\infty}\{\mathrm{Re}[\tilde{T}(z,s+\frac{k\pi i}{T})]\cdot \cos\frac{k\pi t}{T} - \mathrm{Im}[\tilde{T}(z,s+\frac{k\pi i}{T})]\sin\frac{k\pi t}{T}\}\right\} \tag{42}$$

$$w(t) = \frac{e^{st}}{T}\left\{\frac{1}{2}\tilde{w}(s) + \sum_{k=1}^{\infty}\{\mathrm{Re}[\tilde{w}(s+\frac{k\pi i}{T})]\cdot \cos\frac{k\pi t}{T} - \mathrm{Im}[\tilde{w}(s+\frac{k\pi i}{T})]\sin\frac{k\pi t}{T}\}\right\} \tag{43}$$

where $i = \sqrt{-1}$.

Accordingly, the corresponding calculation program is compiled, and the specific operation and parameter selection are described in the study [4].

4 Verification and examples

4.1 Verification

In order to verify the correctness of the method and numerical calculation program in this paper, the semi-analytical solution is reduced to those for the thermal consolidations of one-dimensional saturated soil under permeable boundary condition and under cyclic loading and semi-permeable boundary condition. The solutions are compared with the results from the literatures. Wu[15] obtained the analytic solution of thermal consolidation of one-dimensional saturated soils by theoretical derivation. The parameters adopted in this paper are consistent with those in Reference [15]. In the verification calculation, the semi-permeable boundary parameters are from Reference [2], as listed in Table 1.

The relevant parameters in the formula can be calculated by using the parameter values in Table 1, as shown in Table 2.

Table 1 Parameters of saturated soil and semi-permeable cushion

Permeability coefficient of cushion k_1 /(m · a^{-1})	Permeability coefficient of soil layer k_v /(m · a^{-1})	Coefficient of linear expansion of soil α_s /(10^{-4}℃$^{-1}$)	Water line expansion coefficient α_w /(10^{-4}℃$^{-1}$)	Cushion thickness L_1 /m	Heat flux density of cushion q''_z /(W·m^{-2})	Thermal conductivity of soil layer K /(W·m^{-1}·℃$^{-1}$)	Thermal conductivity of cushion K_1 /(W·m^{-1}·℃$^{-1}$)	Temperature increase of upper interface of cushion $T_{s,1}$ /℃	External load q_0 /MPa	Poisson's ratio μ	Unit weight of soil γ_w /(kN·m^{-3})	Thickness of soil layer H /m	Deformation modulus E /MPa	Initial temperature T_0 /℃
0.04	0.01	2	4	0.5	0.006	0.02	0.015	15	0.2	0.3	10	10	10	10

Table 2 Parameters of calculation model

Volume specific heat C / (10^6 J · m^{-3} · ℃$^{-1}$)	Compression modulus E_s / MPa	Coefficient of consolidation C_v / (10^{-7} m^2 · s^{-1})	Thermal diffusion coefficient C_t / (10^{-9} m^2 · s^{-1})	Semi-permeable coefficient R	Thermal stress coefficient β / (10^{-3} MPa · ℃$^{-1}$)	Coefficient A / (10^{-3} MPa · ℃$^{-1}$)
3.552	13.46	4.27	5.6	4	5	1.230 8

Figure 2 shows the influence of different initial reference temperatures T_0 on the thermal consolidation settlement of one-dimensional saturated soils with the coupled thermo-mechanical model. As can be seen from the figure, this indicates that the initial temperature has little influence on the thermal consolidation of one-dimensional saturated soils with the coupled thermo-mechanical model. Therefore, when the influence of initial temperature on thermal consolidation is not considered (assume $T_0 = 0$ ℃), the coupled thermo-mechanical term can be ignored, namely, the coupled thermo-mechanical model is reduced to the uncoupled thermo-mechanical model.

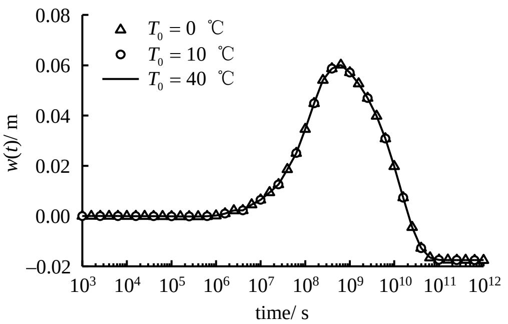


Fig.2 Effect of T_0 on the settlement according to the coupled thermal-mechanical model

For the analytic solutions of thermal consolidation of one-dimensional saturated soils with the coupled thermo-mechanical model, namely, Eqs. (21), (22) and (29), when $T_0 = 0$ ℃ is assumed, they can be degraded to the analytic solutions of thermal consolidation of one-dimensional saturated soils ignoring thermo-mechanical coupling, and their expressions are the same as those of Eqs. (34), (35) and (39). In order to verify the correctness of the analytical method presented in this paper, Eqs. (34), (35) and (39) are reduced to the solutions for the thermal consolidation of one-dimensional saturated soils at the permeable boundary and the cyclic loading consolidation of one-dimensional saturated soils at the semi-permeable boundary.

4.1.1 Thermal consolidation of one-dimensional saturated soil with constant load permeable boundary

If there is no semi-permeable boundary layer and the external load remains unchanged, then there is no heat conduction, that is, $T_{s,2} = T_{s,1}$. For Eq.(39), $R_1 \to \infty$, and $Q(t) = q$, the semi-permeable boundary discussed in this paper can be reduced to a permeable boundary. Then, the thermal consolidation settlement of one-dimensional saturated soils with the uncoupled thermo-mechanical model has an analytic solution in the Laplace transform domain as follows:

$$w = \frac{\lambda_1 \tanh[Hm_1] + \lambda_2 \tanh[Hm_2]}{E_s m_1 \sqrt{a_1}\left(m_1^2 - m_2^2\right) m_2} \tag{44}$$

where

$$\left.\begin{aligned} \lambda_1 &= m_2\left[-\sqrt{a_1}\beta\varsigma\left(m_1^2 - m_2^2\right) + a_2 m_2 \varsigma\sqrt{s}\right] \\ \lambda_2 &= m_1\left[\sqrt{a_1} q\left(m_1^2 - m_2^2\right) - a_2 m_2 \varsigma\sqrt{s}\right] \end{aligned}\right\} \tag{45}$$

The calculation results in this paper are compared with those in Reference [15]. The results are shown in Fig.3. It can be seen that the settlement first increases and then decreases with the development of time, and finally reaches a stable settlement value. The results in this paper are highly consistent with those in Reference [15]. Therefore, the solution in this paper can be used to calculate the thermal consolidation problem of one-dimensional saturated soils with different permeable boundary conditions by changing R_1.

4.1.2 One-dimensional consolidation under cyclic loading of saturated soil at the semi-permeable boundary

When the increment of local surface temperature $T_s = 0$℃, and $H = 5$ m, $z = 0.5$ m, $R = 400$, $Q(t)$ adopts the triangular cyclic load with a period of 40 d and a peak load $q_{max} = 100$ kPa, Eq. (39) can be reduced to the solution for the one-dimensional consolidation settlement under the cyclic load and the condition of semi-permeable boundary in the Laplace domain:

$$w = \frac{Qr \sinh(Hm_2)}{E_s m_2 r \cosh(Hm_2) + E_s m_2^2 \sinh(Hm_2)} \tag{46}$$

The calculation results are shown in Fig. 4. As can be seen from the figure, the semi-analytical solution inversion results derived by the method in this paper are basically consistent with the results in Reference [16].

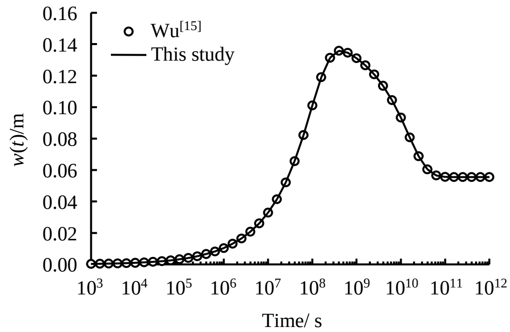


Fig.3 Soil settlement degeneration of one-dimensional thermal consolidation for semi-permeable saturated soil layer

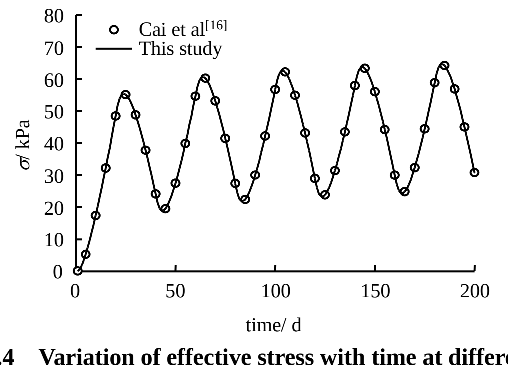


Fig.4 Variation of effective stress with time at different types of load $Q(t)$ for 1D thermal consolidation of saturated soils

4.2 Calculation examples

According to the analysis of the Section 4.1, a one-dimensional thermal consolidation of saturated soil problem doesn't need to consider the effect of initial reference temperature. Therefore, in this section, the solution to one-dimensional thermal consolidation of saturated soil with the coupled thermo-mechanical model is adopted to investigate the effects of semi-permeable boundary heat conduction coefficient K_1, semipermeable boundary parameter R_1, thermal diffusion coefficient C_t and the coefficient of consolidation C_v on the thermal consolidation settlement, for analyzing the thermal consolidation properties of saturated soils. The calculated parameters are consistent with those in Section 4.1.

4.2.1 Influence of different load types

In this example, three groups of typical loads are selected, i.e., triangle, rectangle and trapezoidal cyclic loads, respectively. $Q(t)$ is shown in Fig. 5.

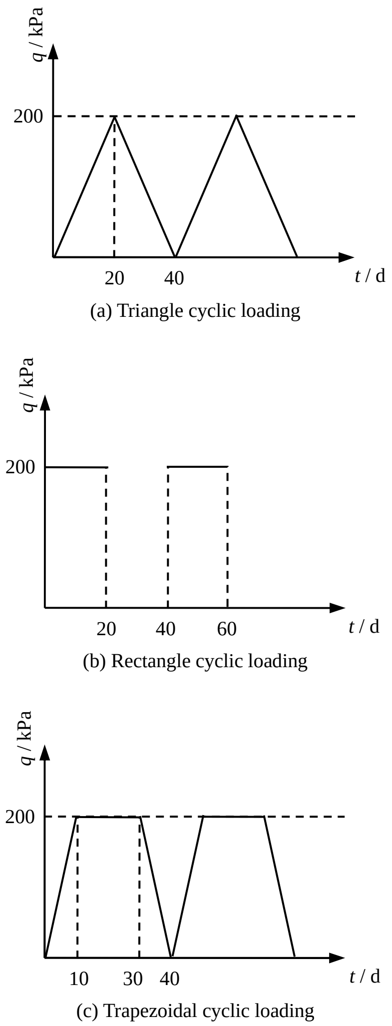


Fig.5 Three typical cycle loadings

By Laplace transform, the cyclic loads of triangle $Q_a(t)$, rectangle $Q_b(s)$, and trapezoidal $Q_c(s)$ are expressed as follows:

$$Q_a(t)=\frac{q_{max}}{t_{20}s^2}\tanh\left(\frac{t_{20}s}{2}\right) \tag{47}$$

$$Q_b(s)=\frac{q_{max}}{2s}\left[1+\tanh\left(\frac{t_{20}s}{2}\right)\right] \tag{48}$$

$$Q_c(s)=\frac{2q_{max}}{t_{10}\left\{\coth(t_{10}s/2)+\coth[(t_{10}+t_{20})s/2]\right\}s^2} \tag{49}$$

where q_{max} is the maximum peak value of cyclic load and t_i is the time required to pass through i days.

By substituting Eqs. (47)–(49) into Eqs. (21), (22), (29), (34), (35) and (39), the expressions of effective stress, temperature increase and settlement of the above three loads in the Laplace domain can be obtained.

It can be seen from Fig. 6(a) that the final settlement of thermal consolidation is affected by the combined action of two factors respectively. First of all, the application of external load will obviously cause soil subsidence. Secondly, with the rise of temperature, the soil will undergo thermal expansion, which will offset part of the settlement. When the external load is small, the soil will eventually expand. The process of thermal consolidation of soil can be roughly divided into two stages: in the first stage, the external load rapidly causes a certain amount of soil settlement; in the second stage, as the compression effect of external load weakens and the temperature in the soil rises, the soil begins to expand to a certain extent and finally reaches a stable settlement.

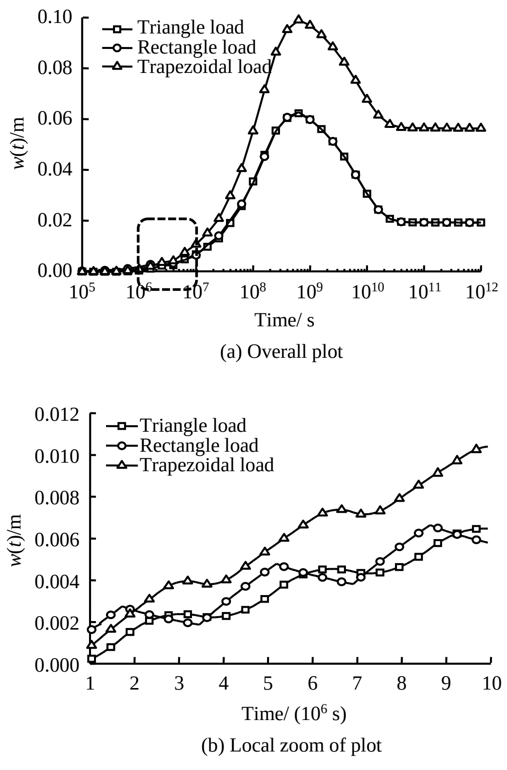


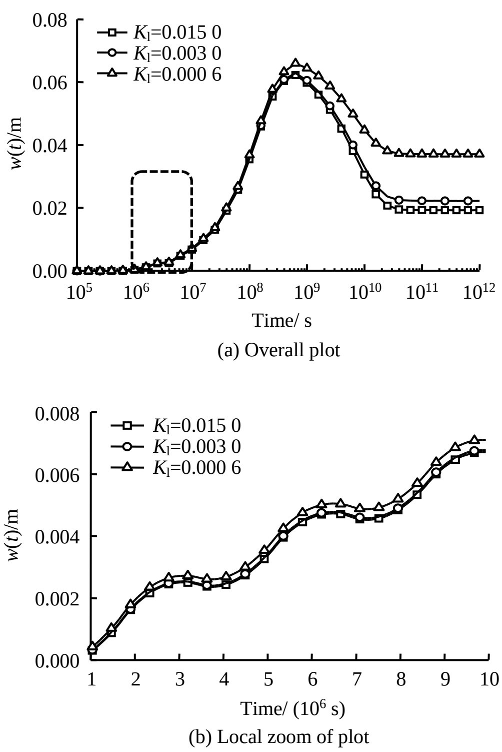
Fig.6 Variation of soil settlement with time at different load types $Q(t)$ for 1D thermal consolidation of saturated soils

Figure 6(b) illustrates that the load type has a significant influence on the settlement process of soil, and the settlement of saturated soil under cyclic loading has a significant periodic characteristic. From the macroscopic development of soil settlement, the settlement process of soil has a close relationship with the average value of external load $Q(t)$. The average values of triangular and rectangular loads are equal, so the two sets of settlement curves basically coincide. The average external load of trapezoidal load is slightly higher than the former two, so its settlement is greater.

In order to analyze the influence of other factors on thermal consolidation of cyclically loaded saturated soils more effectively, triangular cyclic loads are adopted as external loads in the following sections.

4.2.2 Influence of thermal conductivity coefficient of semi-permeable boundary

To investigate the effects of thermal conductivity of the semi-permeable boundary on the consolidation process, considering the heat conduction property of sand cushion material, the thermal conductivity coefficient K_1 of semi-permeable boundary is respectively taken as 0.0150, 0.0030 and 0.0006, where 0.0150 for semipermeable boundary material heat transfer coefficient. In order to clearly compare the influence of K_1 on the thermal consolidation of one-dimensional saturated soil, K_1 values of 0.0030 and 0.0006 are taken as parameters, respectively. Figure 7(a) shows the influence of thermal conductivity coefficient of different semipermeable boundaries on thermal consolidation of one-dimensional saturated soil. The greater the thermal conductivity coefficient K_1 of the semi-permeable boundary is, the smaller the final settlement of thermal consolidation of one-dimensional saturated soil will be. It can be seen from Eq. (9) that higher the thermal conductivity coefficient K_1 of the semi-permeable boundary, greater the temperature increment $T_{s,2}$, and greater the soil uplift caused by temperature change in consolidation.



(a) Overall plot

(b) Local zoom of plot

Fig.7 Variations of soil settlement with time at different values of thermal conductivity K_1 of semi-permeable boundary for 1D thermal consolidation of saturated soils

Figure 7(b) shows that no matter how K_1 changes, the change of K_1 in the consolidation process only affects $T_{s,2}$, and does not affect the consolidation speed of soil. Therefore, the effect of periodic load on consolidation of different K_1 is roughly the same, while the stronger thermal expansion effect of $T_{s,2}$ is, the smaller the final settlement will be.

4.2.3 Influence of semi-permeable boundary parameters

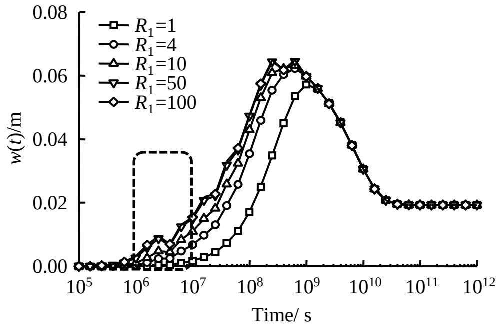
In order to investigate the influence of semipermeable boundary parameters on the thermal consolidation characteristics of one-dimensional saturated soil, in this section, according to the common relative thickness and relative permeability coefficient between the sand cushion and the soil under investigation, R_1 is calculated as 1, 4, 10, 50 and 100, respectively. The calculation results of thermal consolidation settlement under different semi-permeable boundary parameters are shown in Fig. 8(a). During the development of thermal consolidation of saturated soil, the settlement of saturated soil first increases and then decreases, and finally tends to a stable settlement value. Larger the semipermeable boundary parameter R_1 is, higher the permeability of the top surface boundary is, and easier the pore water is discharged, so faster the settlement develops. At the same time, smaller the semipermeable boundary parameter is, slower the consolidation development will be, and smaller the maximum settlement achieved in the consolidation development of saturated soil will be due to the expansion of soil in the later stage of thermal consolidation development. Some of the numerical points in Fig. 8(a) are discontinuous, and some of the curves are messy. In fact, the data points cannot reflect the periodic characteristics of the external cyclic load due to the wide range of calculation time. Figure 8(b) can be seen in a specific local enlarged view.

Figure 8(b) shows that R_1 has a significant influence on the settlement cycle amplitude. Larger R_1 is, better the water permeability is and larger the settlement amount is under the same period, so the soil settlement decreases with time. The periodic characteristics of soil change with external load are more obvious.

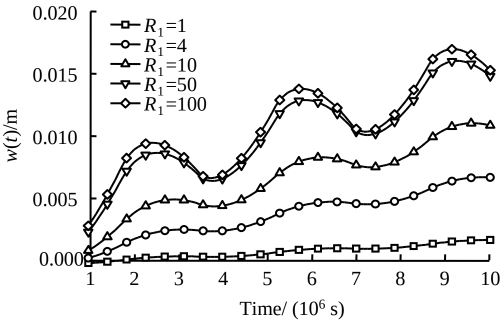
In addition, when R_1 increases to 50, the further increase of R_1 has no significant effect on the consolidation rate. The study of the consolidation of saturated soil at the semi-permeable boundary from Cai et al.[2] suggested that the boundary condition can be considered as the permeable boundary when the boundary parameter is greater than 40.

4.2.4 Influence of thermal diffusion coefficient

In order to examine the influence of thermal diffusion coefficient on the thermal consolidation of one-dimensional saturated soil, The curve of the thermal consolidation settlement of one-dimensional saturated soil with different thermal diffusion coefficients over time is analyzed (Fig.9(a)). It is shown that a larger the thermal diffusion coefficient C_t leads to a more obvious thermal expansion in the soil layer is, and a smaller maximum settlement in consolidation. Since changes of $C_{\rm t}$ do not change $T_{\rm s,2}$, the final settlement amount of soil with different $C_{\rm t}$ is approximately the same.

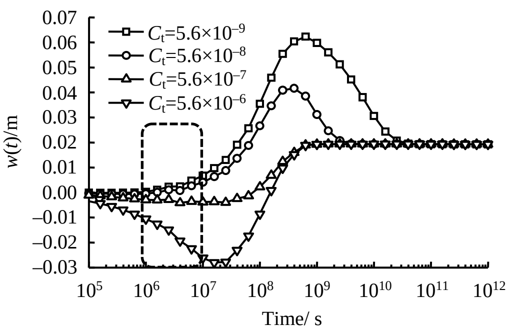


(a) Overall plot

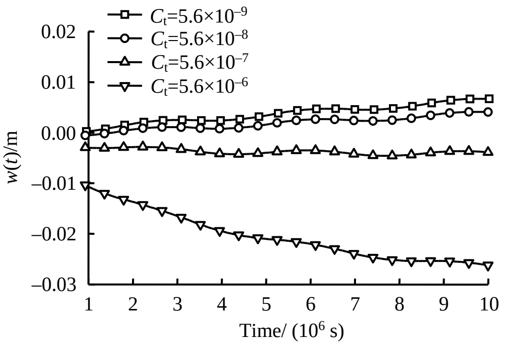


(b) Local zoom of plot

Fig.8 Variations of soil settlement with time at different values of semi-permeable boundary coefficient R_1 for 1D thermal consolidation of saturated soils



(a) Overall plot



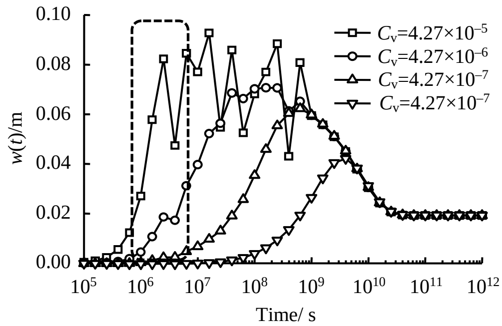
(b) Local zoom of plot

Fig.9 Variations of soil settlement with time at different values of thermal diffusivity $C_{\rm t}$ for 1D thermal consolidation of saturated soils

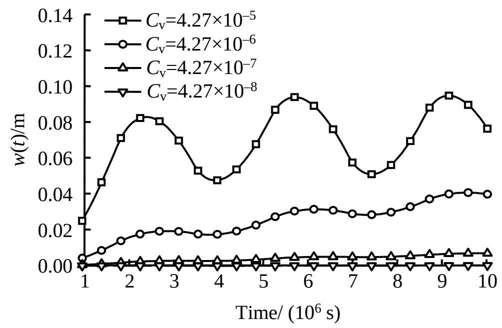
The curves still have certain periodic characteristics and the amplitudes between different curves are consistent under the action of external load (Fig. 9(b)). Since soil thermal diffusion coefficient $C_{\rm t}$ has little influence on soil drainage velocity and consolidation velocity, the periodic characteristics of soil thermal diffusion coefficient $C_{\rm t}$ on soil settlement are consistent.

4.2.5 Influence of consolidation coefficient

Figure 10(a) gives the variation of thermal consolidation settlement of one-dimensional saturated soils. It shows that thermal consolidation of saturated soil develops faster and the maximum settlement is larger with increasing the increase of consolidation coefficient $C_{\rm v}$. Faster dissipation of pore water pressure results in a greater consolidation coefficient. Therefore, the consolidation rate of saturated soil in the thermal consolidation process will be faster, and the effective stress in saturated soil will also increase faster. As a result, a larger maximum settlement amount is achieved in the thermal consolidation process of saturated soil. And $C_{\rm v}$ does not affect $Q(t)$ and $T_{\rm s,2}$, so it has no influence on the final settlement amount of soil.



(a) Overall plot



(b) Local zoom of plot

Fig.10 Variations of soil settlement with time at different values of consolidation coefficient $C_{\rm v}$ for 1D thermal consolidation of saturated soils

Some of the curves in Fig. 10(a) are as messy as those in Fig. 8(a), for reasons shown in Section 4.2.3. It can be clearly seen from Fig. 10(b) that $C_{\rm v}$ has a great influence on the settlement cycle amplitude. Larger $C_{\rm v}$ is, faster the soil consolidation will be, and larger the settlement amount in the same period will be. Therefore, larger the settlement amplitude with time is, more obvious the periodic characteristics will be.

5 Conclusion

(1) This article introduced semi-analytical solutions of one-dimensional thermal consolidation of saturated soils with coupled and uncoupled thermo-mechanical models under semi-permeable boundary with thermal conduction and varying loading, the solution has good generality and can be reduced respectively by changing the parameters of the boundary for the homogeneous boundary problem of one-dimensional thermal consolidation of saturated soil under half analytical solutions and Terzaghi's one-dimensional consolidation of saturated soil solution of the problem.

(2) Semi-permeable boundary parameter R_1 and consolidation coefficient C_v mainly affect the settlement development rate of thermal consolidation of one-dimensional saturated soil. A larger R_1 or C_v makes the soil settle faster. The thermal diffusion coefficient C_t mainly affects the thermal expansion velocity of one-dimensional saturated soil during consolidation. The larger the C_t is, the earlier the thermal expansion of soil will be. The thermal conductivity of semipermeable boundary K_1 mainly affects the final settlement of one-dimensional saturated soil. A greater K_1 results in a smaller the final settlement.

(3) The external load $Q(t)$ has an effect on the thermal consolidation of one-dimensional saturated soil. Firstly, different external loads will significantly affect the settlement, and the settlement under cyclic load will also have corresponding periodic characteristics. Secondly, the average external load will affect the final settlement of soil. The greater the average pressure is, the larger the amount of soil subsidence will be.

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