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Analysis of energy evolution during the step loading and unloading creep experiments of sandstone

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Abstract: The uniaxial step loading and unloading creep experiments were conducted on the sandstone samples from Shaanxi Province to analyze the law of energy evolution of the samples during the process of deformation and failure. The results show that with the increase of cyclic series, the dissipation energy of each grade exhibits a nonlinear increase and the plastic strain energy of each grade is relatively stable. The dissipation energy exceeding plastic strain energy can be regarded as a precursor energy characteristic of the sample failure. By defining the correlation coefficient and establishing the relationship between energy and deformation, it can be found that with increasing cyclic series, there is a positive correlation between the plastic strain energy and the new adding plastic strain of each grade, and the dissipation energy of each grade is positively correlated with the plastic strain accumulation. According to the calculation of the energy of loading, creep, unloading and recovery stages, the evolution rate of energy in each grade increases with the increase of cyclic series. When the rates of loading and unloading are constant, the change rate of energy at the loading stage is larger than that at the unloading stage, and the change rate of energy at the creep stage is greater than that at the recovery stage under the same grade. Finally, by analyzing the change law of energy at each cyclic loading stage, an energy attenuation coefficient λ_i is proposed, which decreases as a power function with increasing stress level. Thus, an effective method is introduced for predicting failure stress.

Keywords: creep; sandstone; energy evolution; plastic strain energy; dissipation energy

1 Introduction

As a special geological material, rock is not only elastic and plastic, but also shows distinct rheological characteristics. With the increase of the depth of underground engineering, the stress on the underground rock gradually increases, and the deformation and failure phenomenon caused by the rheological properties of rock become more obvious^[1–2]. In addition, mining and other engineering activities will also trigger periodic disturbance to the surrounding rock, which will induce instability of rock. At present, there are still many shortcomings in the mechanism of rock failure under cyclic stress loading and unloading. Therefore more and more scholars have paid attention to the study of rock mechanics from the perspective of energy.

Rock failure is a process with the initiation, extension and transfixion of internal micro cracks, which is accompanied by the conversion of energy with different forms in the rock interior and the exchange of energy with the external^[3]. Thus, it is necessary to consider the time effect of rock rheology and the principle of energy transformation and dissipation under the periodic stress loading and analyze the relationship between

energy evolution and deformation in various stress states, which is of great significance to investigate the mechanism of deformation and failure of rock.

In recent years, many scholars have conducted the research on the mechanism of energy evolution in the process of rock failure. Xie et al.^[4] discussed the law of absorption, release and dissipation of energy in the process of deformation and failure of rock. He pointed out that rock failure and deformation is a comprehensive result of energy dissipation and release and the energy dissipation is one-way and irreversible, while energy release is reversible in both directions. Zhang et al.^[5] conducted conventional tri-axial tests on three kinds of rocks and investigated the nonlinear evolution characteristics of energy in the process of rock deformation. Zhang et al.^[6–7] established a self-inhibited evolution model related to the energy transformation of rocks under loading. His results highlighted that the energy evolution is characterized by bifurcation and chaos. A damage model was proposed by Peng et al.^[8] from the perspective of energy, which vividly reflected the process of damage evolution of rock. Xu et al.^[9] studied the relationship between hysteretic loop area and deformation by conducting cyclic loading and unloading experiments on fine

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sandstones. A series of uniaxial cyclic loading and unloading experiments of sandstone was carried out by Deng et al. [10], in whose work the characteristics of hysteretic loops in the loading and unloading curves were analyzed and the relationship between energy dissipation and residual strain were discussed. He et al. [11] conducted tri-axial tests on sandstone samples under different stress paths, and obtained the influence relationship among elastic strain energy, dissipated energy, initial vertical pressure and confining pressure. Xu et al. [12] performed a series of tri-axial cyclic loading and unloading creep tests, and analyzed the evolution law and deformation characteristics of the creep rate of samples by establishing a corresponding creep model. Bagde et al. [13] conducted dynamic uniaxial cyclic loading experiments on sandstone and obtained the relationship of the energy release of sandstone with the loading frequency and amplitude. Li et al. [14] studied the nonlinear behavior of deformation characteristics of rock under cyclic loading, and established a damage model of fractured rock from the view of energy dissipation. Gong et al. [15] analyzed the laws of energy storage and consumption in the process of loading and unloading of rock samples by performing the Brazil splitting, SCB and point loading tests. Yang et al. [16] investigated the corresponding relationship between acoustic emission characteristics and non-uniform deformation of rock samples in the process of cyclic loading and unloading by conducting uniaxial cyclic loading and unloading experiments on granite. Zhai et al. [17] analyzed the energy evolution and deformation characteristics of rock joints under shear loading-creep-unloading.

Most of studies aforementioned are based on common cyclic loading and unloading experiments to analyze the rock deformation and its laws of energy evolution, but the step loading and unloading tests with creep and creep recovery are rarely reported. In this study, the energy evolution characteristics of sandstone samples at each level are analyzed by conducting step cyclic loading and unloading creep tests and the relationship between the energy of each stage and the deformation of samples are discussed.

2 Analysis of energy evolution mechanism

The process of crack extension and damage accumulation in rock is accompanied by the variation of energy. Some scholars [4–8] analyzed cyclic loading and unloading experimental curves from the point of energy circulation. The consensus is that the area under the loading curve represents the absorbed total strain energy by samples and the area under the unloading curve represents recoverable elastic strain energy of samples. The area enclosed by a complete loading and unloading curves represents the energy dissipated by the sample at that stage. The total strain energy is equal to the sum of the elastic strain energy and the dissipated energy. With the further study of cyclic loading and unloading tests, more and more

attentions have been paid to the hysteresis loop in the test curves [18–22], whose area is considered to represent the energy dissipation during the test.

It is a common approach for energy calculation to analyze the experimental stress–strain curve by applying the integral method to image area. As shown in Fig.1, according to the laws of thermodynamics and energy conservation, it can be known that for the first-order cycle:

$$W_1 = W_{r1} + W_{u1} \tag{1}$$

where W_1 is the total strain energy under the first step cyclic, which corresponds to the area of polygon $OABO_3$ under the loading-creep curve; W_{r1} is the recoverable strain energy under the first step cycle, which corresponds to the area of polygon O_1DCBO_3 under the unloading-recover curve; W_{u1} is irrecoverable strain energy under the first step cycle, which corresponds to the difference between above two polygons.

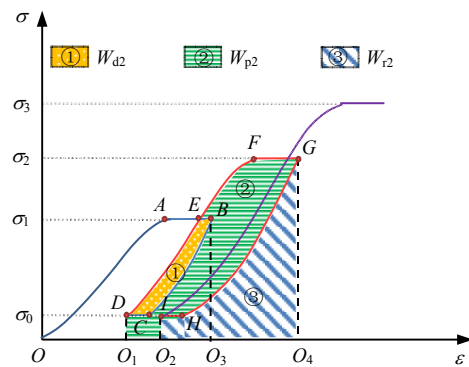


Fig.1 Energy evolution process of step loading and unloading

After the first step cycle, the hysteresis loop appears in the test curve and the analysis of energy evolution is conducted on the dissipated energy characterized by the hysteresis loop. The hysteresis loop represents the consumed energy induced by the re-compression of the opening cracks caused by the previous step cyclic unloading [19]. At the same time, the irrecoverable strain energy can be subdivided into plastic strain energy and dissipated energy:

$$W_2 = W_{r2} + W_{p2} + W_{d2} \tag{2}$$

where W_2 is the total strain energy under the second cyclic, which corresponds to the area enclosed by polygon O_1DFGO_4 in Fig.1; W_{r2} is the recoverable strain energy under the second step cycle, which corresponds to the area of polygon ③; W_{d2} is the dissipated energy under the second step cycle, which corresponds to the area of polygon ①; W_{p2} is the plastic strain energy under the second step cycle, which is obtained by the total strain energy minus the sum of recoverable strain energy and dissipated energy and thus corresponds to the area of

polygon ② in Fig. 1.

3 Experimental samples and programs

3.1 Preparations of samples and experimental device

The sandstones in this study are collected from Ningtiaota coal mine in Shaanxi province. The samples were cored from original rock and grinded into a cylinder with 50 mm in diameter and 100 mm in height. The samples were divided into two groups. Group A was subjected to uniaxial compression experiments, and group B was subjected to uniaxial step loading and unloading creep experiments. Basic parameters of samples and compressive strength of group A samples are listed in table 1.

Table 1 Physical and mechanical parameters of samples

| Specimen No. | Diameter /mm | Height /mm | Density /($\text{g} \cdot \text{cm}^{-3}$) | Compressive strength /MPa |
|--------------|--------------|------------|--|---------------------------|
| A1 | 49.00 | 99.20 | 2.35 | 44 |
| A2 | 48.70 | 100.00 | 2.42 | 44 |
| A3 | 48.38 | 99.90 | 2.40 | 37 |
| B1 | 48.56 | 98.65 | 2.35 | — |
| B2 | 48.43 | 98.85 | 2.35 | — |
| B3 | 48.82 | 98.67 | 2.34 | — |

The experimental device adopts the five-connected rheological testing system of the State Key Laboratory for Geomechanics and Deep Underground Engineering in China University of Mining and Technology (Beijing), as shown in Fig.2.



Fig.2 Five-connected rheological testing system

3.2 Experimental program of step loading and unloading

The mean compressive strength σ_c of sample was obtained by conducting uniaxial compression tests and each step cyclic stress was also determined by referring to this strength. The experimental loading and unloading stress path and each step stress are shown in Fig. 3. The experimental loading and unloading rates were set as 0.25 MPa/s for all cases in order to facilitate the results comparison and analysis. For each step, cyclic loading should keep constant for 24 h, then unloaded to 2 MPa and keep constant for 10 h. Following this cyclic experiments until the sample was broken, each cyclic creep stage is maintained as steady creep stage before failure.

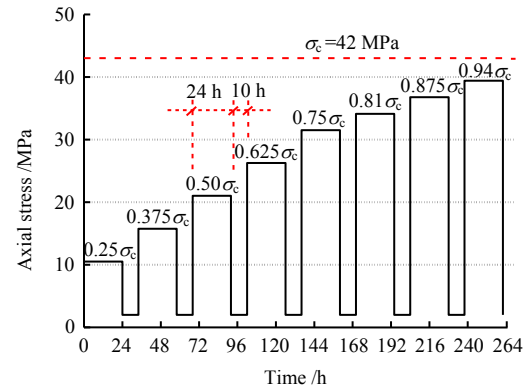
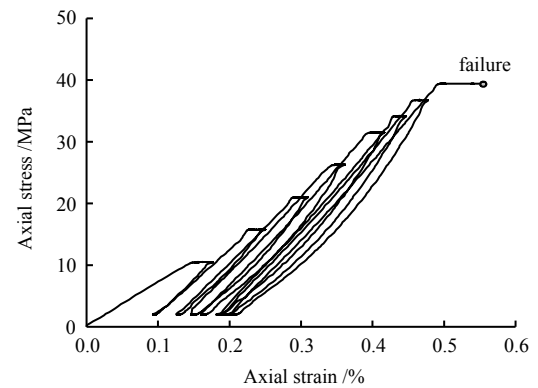
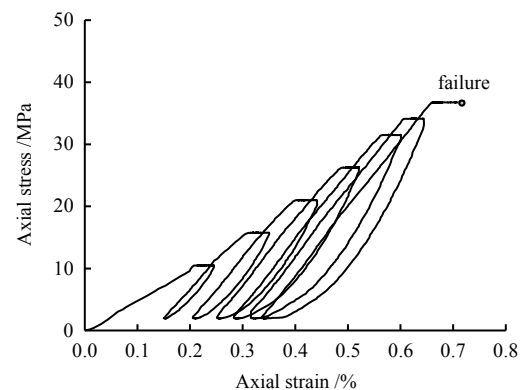


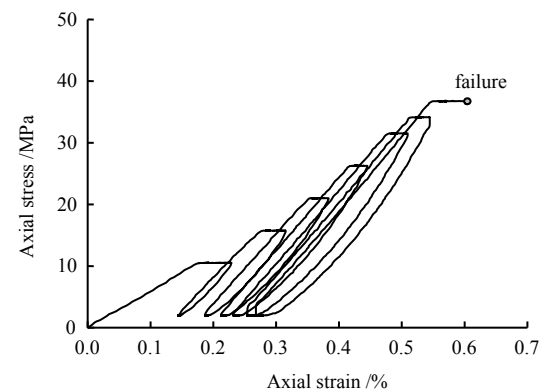
Fig.3 Loading and unloading path of the creep test



(a) Specimen B1



(b) Specimen B2



(c) Specimen B3

Fig.4 Stress-strain curves

The experimental curves of step loading and unloading

creep tests are described in Fig. 4. It can be seen that the sample B1 is broken at the 8th cyclic creep stage whereas samples B2 and B3 are destroyed at the 7th cyclic creep stage. After the step loading and unloading creep, the failure mean stress of samples is less than the mean peak stress in the uniaxial compression test, which indicates that the step cyclic loading and unloading creep processes result in severer deterioration of the samples, leading to the decrease of rock strength.

4 Analysis of experimental results

The calculated results of cyclic energy of sandstone samples under each step are listed in table 2. The evolution

laws of cyclic energy under each step can be obtained by analyzing the experimental data in table 2.

4.1 The evolution laws of recoverable and unrecoverable strain energy

The total strain energy in the cyclic process is composed of recoverable strain energy and unrecoverable strain energy. It can be seen in table 2 and Fig. 5, the total strain energy, recoverable strain energy and unrecoverable strain energy all increase with the increase of cyclic steps (stress). Among them, the recoverable strain energy increases linearly and remarkably while the unrecoverable strain energy increases nonlinearly and marginally.

Table 2 Calculation results of energy at each step

| Specimen No. | Number of cycles | Stress lever /MPa | $W_i/(kJ \cdot m^{-3})$ | $W_{ri}/(kJ \cdot m^{-3})$ | $W_{ui}/(kJ \cdot m^{-3})$ | $W_{di}/(kJ \cdot m^{-3})$ | $W_{pi}/(kJ \cdot m^{-3})$ |
|--------------|------------------|-------------------|-------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
| | | | 10.736 | 5.025 | 5.711 | — | — |
| | 2 | 15.75 | 14.765 | 10.225 | 4.540 | 0.025 | 4.515 |
| | 3 | 21.00 | 22.445 | 16.050 | 6.395 | 0.827 | 5.568 |
| | 4 | 26.25 | 32.678 | 24.805 | 7.873 | 3.733 | 4.140 |
| | 5 | 31.50 | 44.558 | 32.988 | 11.571 | 3.793 | 7.778 |
| | 6 | 34.15 | 48.842 | 38.442 | 10.400 | 4.997 | 5.403 |
| | 7 | 36.75 | 57.368 | 44.481 | 12.887 | 7.710 | 5.177 |
| B2 | 1 | 10.50 | 14.098 | 4.892 | 9.206 | — | — |
| | 2 | 15.75 | 21.216 | 10.921 | 10.295 | 1.208 | 9.087 |
| | 3 | 21.00 | 30.355 | 17.540 | 12.815 | 2.572 | 10.243 |
| | 4 | 26.25 | 41.493 | 25.945 | 15.549 | 5.920 | 9.629 |
| | 5 | 31.50 | 58.301 | 35.251 | 23.049 | 10.337 | 12.712 |
| | 6 | 34.15 | 63.969 | 40.188 | 23.781 | 15.824 | 7.957 |
| B3 | 1 | 10.50 | 14.726 | 4.548 | 10.177 | — | — |
| | 2 | 15.75 | 18.187 | 9.417 | 8.770 | 1.018 | 7.752 |
| | 3 | 21.00 | 24.667 | 16.022 | 8.645 | 1.889 | 6.755 |
| | 4 | 26.25 | 34.954 | 24.338 | 10.616 | 3.575 | 7.041 |
| | 5 | 31.50 | 48.924 | 34.002 | 14.922 | 5.235 | 9.687 |
| | 6 | 34.15 | 56.305 | 37.988 | 18.317 | 10.287 | 8.030 |

Note: W_i is the total strain energy under the i step cycle; W_{ri} is the recoverable strain energy under the i step cycle; W_{ui} is the unrecoverable strain energy under the i step cycle; W_{di} is the dissipated energy under the i step cycle; W_{pi} is the plastic strain energy under the i step cycle.

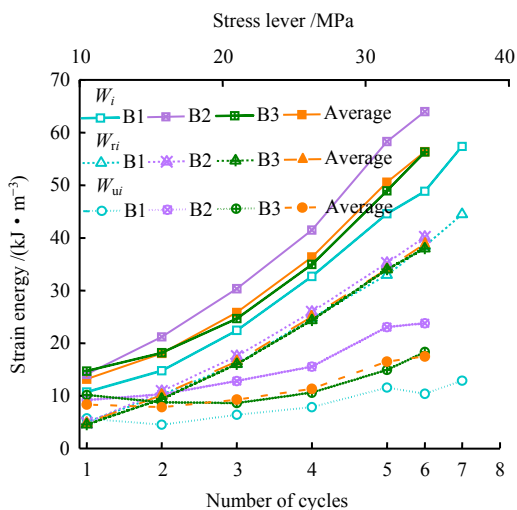


Fig.5 Curves of recoverable strain energy and unrecoverable strain energy

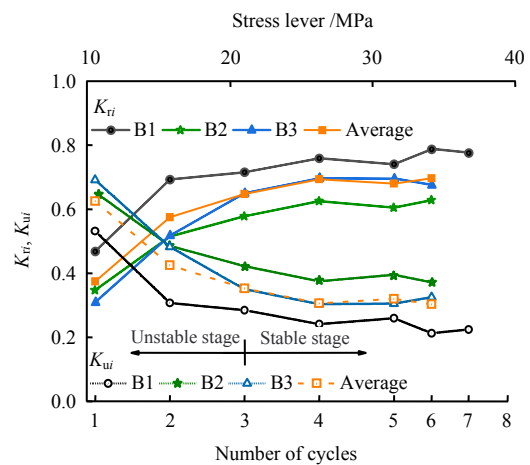


Fig.6 Relationship between K_{ri} and K_{ui}

The recoverable strain energy coefficient K_{ri} is defined as

the ratio of the recoverable strain energy under the i step cycle to total strain energy. The irrecoverable strain energy coefficient K_{ui} is defined as the ratio of the irrecoverable strain energy under the i step cycle to total strain energy. The variations between both are depicted in the Fig. 6. With the increase of cyclic steps, the recoverable strain energy coefficient K_{ri} increases whereas the irrecoverable strain energy coefficient K_{ui} decreases gradually. Both K_{ri} and K_{ui} vary considerably before $0.5\sigma_c$, followed by slight fluctuations after $0.5\sigma_c$. The total strain energy of each step cyclic tends to transform into recoverable strain energy restored in the samples.

4.2 The evolution laws of plastic strain energy and dissipated energy

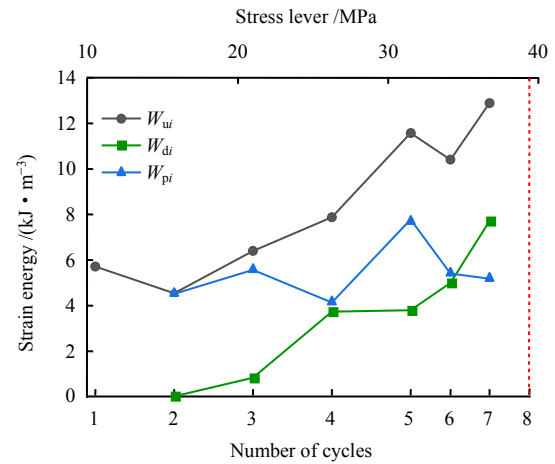
In order to accurately predict the possibility of coal seam outburst, Xiao et al. [19] separated the energy represented by the hysteresis loop from the irrecoverable strain energy when he calculated the elastic energy index, and claimed that the energy was consumed by the compression of cracks. Xin et al. [20] assumed that the energy represented by the hysteresis loop was consumed by particle rotation and cracks friction.. Although the points mentioned above have discussed the dissipated energy, the relationship between plastic strain energy, dissipated energy and plastic strain has not been explained. In this study, the dissipated energy (hysteretic loop) is separated from the irrecoverable strain energy and the variation laws of plastic strain energy and dissipated energy under each level are obtained. The relationship between the dissipated energy and the plastic strain energy is explored via analyzing the corresponding relationship between the transformation of energy and the change of strain.

In Fig. 7, it can be found that in the early stage of cyclic experiments, the plastic strain energy under each step cycle is larger than the dissipated energy. With the increase of cyclic step (stress), the dissipated energy under each step increases successively. Although the plastic strain energy under each step has certain fluctuations, it is relatively stable overall. The dissipated energy eventually exceeds the plastic strain energy at a certain stage of the cycle before failure occurs and becomes the dominant energy consumption in the irrecoverable strain energy. The dissipated energy exceeding the plastic strain energy during the cycle can be referred to as the precursor energy characteristic of sample fracture.

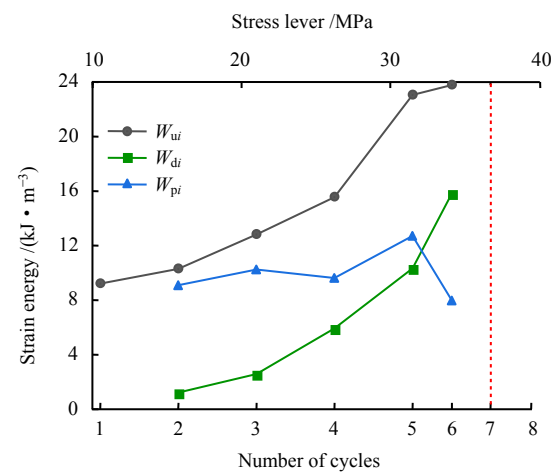
4.3 Relationship between the plastic strain energy, the dissipated energy and the plastic strain

The plastic strain energy coefficient K_{pi} is defined as the ratio of plastic strain energy under the i step cycle to irrecoverable strain energy and can be expressed as

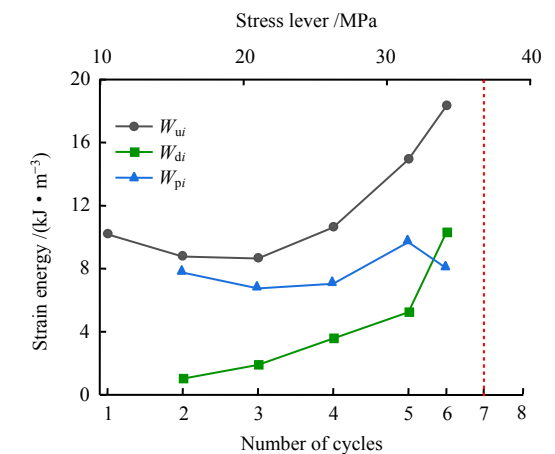
$$K_{pi} = \frac{W_{pi}}{W_{ui}} \tag{3}$$



(a) Specimen B1



(b) Specimen B2



(c) Specimen B3

Fig.7 Relationship between plastic strain energy, dissipated energy and cyclic series

where W_{pi} is the plastic strain energy under the i step cycle and W_{ui} is the irrecoverable strain energy under the i step cyclic.

The plastic strain increment coefficient $K_{\Delta ei}$ is defined as the ratio of the additional plastic strain under the i step cycle to

the total plastic strain of samples. It can be calculated as

$$K_{\Delta \varepsilon_i} = \frac{\Delta \varepsilon_i}{\varepsilon_i} \quad (4)$$

where the $\Delta \varepsilon_i$ is the additional plastic strain under the i step cycle and the ε_i is the total plastic strain of samples after experiments.

The dissipated energy coefficient K_{di} is defined as the ratio of the dissipated energy under the i step cycle to the irrecoverable strain energy. And its formula is shown as follows:

$$K_{di} = \frac{W_{di}}{W_{ui}} \quad (5)$$

The plastic strain accumulation coefficient K_{ei} is defined as the ratio of the residual strain after the i step cycle to the total plastic strain of samples. It can be written as

$$K_{ei} = \frac{\varepsilon_i}{\varepsilon_i} \quad (6)$$

where ε_i is the residual strain after the i step cycle.

As shown in Fig. 8, with the increase of cyclic steps, the plastic strain energy coefficient and the plastic strain increment coefficient both exhibit decrease trends to some degree, and there is a good consistency between them. As can be seen from Fig. 9, with the increase of cyclic steps, the dissipated energy coefficient and the plastic strain accumulation coefficient both experience increase to a certain degree, and they are positively correlated to each other. The above behaviour indicates that the new plastic strain is generated continuously with the increase of the upper limit of cyclic stress. Although the increment of plastic strain at each step tends to decrease, the total plastic strain accumulation continues to increase, which corresponds to the continuous increase of dissipated energy at each step. The dissipated energy represented by the hysteresis loop is closely related to the accumulation of plastic strain.

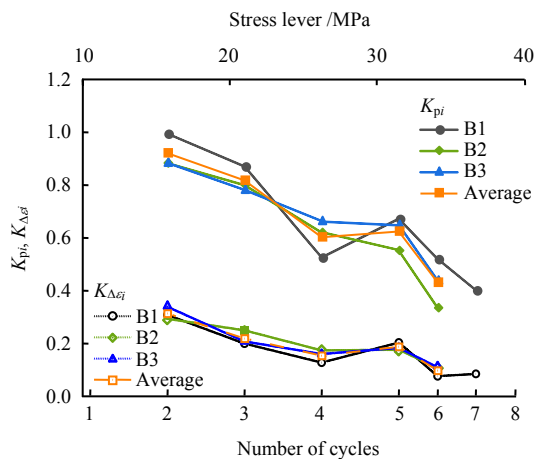


Fig.8 Relationship between plastic strain energy coefficient and plastic strain increment coefficient

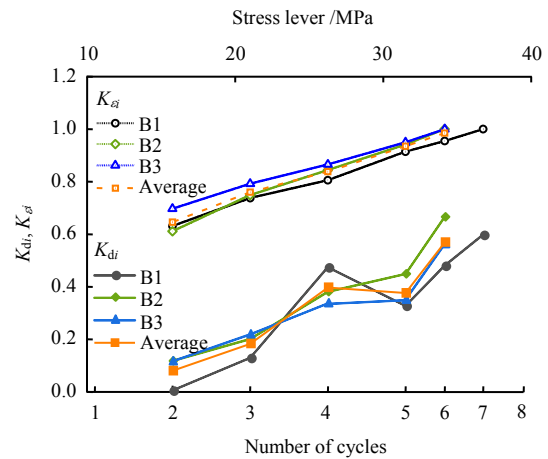


Fig.9 Relationship between dissipated energy coefficient and plastic strain accumulation coefficient

According to the phenomena mentioned above, it can be inferred that the plastic strain energy formed by the stress difference between different step cycles causes the plastic deformation of the sample. It is the main energy causing the irreversible destruction of the internal skeleton, dislocation and displacement of the crack surface and the pore compaction in the sample. The recoverable elastic strain energy stored in the sample during loading will make structure deform without destruction. This part of energy will release along with the elastic recovery of unbroken skeleton during unloading. This process is reversible. The dissipated energy represented by the hysteresis loop is associated with the accumulation of plastic deformation. The greater the plastic deformation accumulated in the sample, the greater the energy consumption.

4.4 The laws of energy evolution at different testing stages

The loading and creep stages are the processes of energy accumulation while the unloading and recovery stages are the processes of energy release. As shown in Fig.10, by calculating and analyzing the energy variations at each stage of loading, creep, unloading and recovery, the law of energy evolution with the increase of cyclic steps can be obtained.

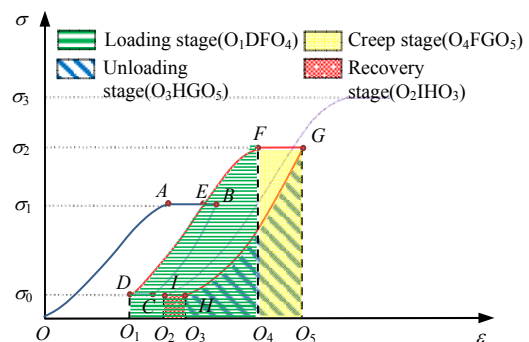


Fig.10 Energy evolution at different stages

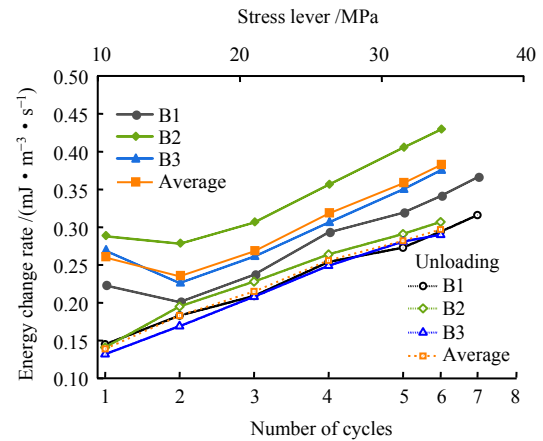
As shown in table 3, in the creep stage, with the increase of cyclic step (stress), the proportion of accumulated energy in the total accumulated energy decreases continuously. For instance, sample B1 decreases from 29.3% to 10.7%; sample B2 decreases from 30.6% to 12.7%; and sample B3 decreases from 37.9% to 13.4%. The released energy in the recovery stage exhibits a downward trend with the increase of cyclic step, although it only accounts for a relative small proportion of the total released energy. Sample B1 decreases from 1.8% to 0.5%. Sample B2 and B3 decrease from 1.8% to 0.9% and from 1.3% to 0.8%, respectively.

Table 3 Energy evolution at each stage (unit: $\text{kJ} \cdot \text{m}^{-3}$)

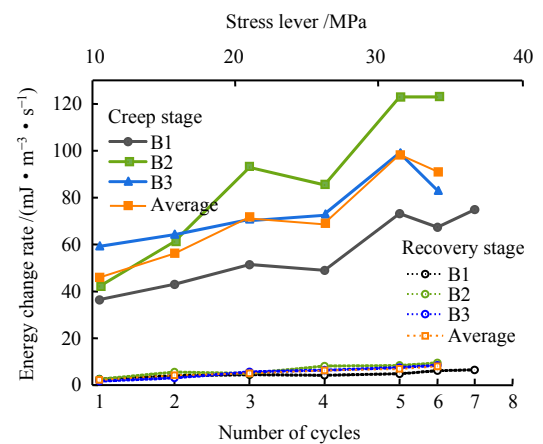
| Sample No. | Cycle step | Loading stage | Creep stage | Unloading stage | Recovery stage |
|------------|------------|---------------|-------------|-----------------|----------------|
| B1 | 1 | 7.588 | 3.148 | 4.934 | 0.090 |
| | 2 | 11.047 | 3.718 | 10.083 | 0.143 |
| | 3 | 18.002 | 4.442 | 15.890 | 0.160 |
| | 4 | 28.449 | 4.229 | 24.655 | 0.150 |
| | 5 | 38.238 | 6.321 | 32.815 | 0.173 |
| | 6 | 44.335 | 4.507 | 38.220 | 0.222 |
| | 7 | 51.233 | 6.136 | 44.248 | 0.233 |
| B2 | 1 | 9.782 | 4.316 | 4.802 | 0.090 |
| | 2 | 15.283 | 5.934 | 10.722 | 0.199 |
| | 3 | 23.231 | 7.124 | 17.360 | 0.180 |
| | 4 | 34.509 | 6.985 | 25.652 | 0.292 |
| | 5 | 48.651 | 9.649 | 34.951 | 0.300 |
| | 6 | 55.827 | 8.142 | 39.847 | 0.341 |
| B3 | 1 | 9.136 | 5.590 | 4.488 | 0.060 |
| | 2 | 12.404 | 5.784 | 9.308 | 0.109 |
| | 3 | 19.816 | 4.851 | 15.820 | 0.202 |
| | 4 | 29.641 | 5.313 | 24.109 | 0.229 |
| | 5 | 41.998 | 6.927 | 33.733 | 0.270 |
| | 6 | 48.788 | 7.518 | 37.686 | 0.303 |

Figure 11(a) shows that the energy absorption rate of loading stage and the energy release rate of unloading stage both increase with the increase of cyclic step when the loading and unloading rates are keeping constant and identical. For a given complete cycle, the energy absorption rate of loading stage overweighs the energy release rate of unloading stage.

In Fig.11(b), the increasing trends are also witnessed for the energy change rate of creep and recovery stages with the increase of cyclic step. The energy change rate of creep stage is one order of magnitude higher than that of recovery stage under the same cyclic step.



(a) Stages of loading and unloading



(b) Stages of creep and recovery

Fig.11 Energy change rate at different stages

5 Prediction of creep failure stress based on energy method

The increase of cyclic step in the step loading and unloading experiments indicates that the upper limit of stress that the sample can withstand is also increasing. In other words, the loading process of the later cycle will build upon the stress state of the previous cycle.

It can be seen from Table 4, the sample consumes the most energy when it reaches the historical maximum stress for the first time and the dissipated energy decreases when the secondary cycle is reloaded to the stress state. The dissipated energy keeps relative stable when the subsequent cycle is reloaded to this stress state. In Fig. 12, when the stress approaches the same state in different cyclic steps, the areas of the loading curves are in the order ① > ② ≈ ③ ≈ ④.

Based on the above discussion, the recoverable strain energy is reversible in both directions and the dissipated energy is related to the accumulation of plastic strain. Neglecting the influence of fatigue effect, the increase of the stress limit will aggravate the damage of the samples and lead to the

transformation of energy to plastic strain energy. Thus, when the cyclic loading process of the later stages does not exceed the historical maximum stress level, the input energy can be

assumed as the converted elastic strain energy and the dissipated energy, and the plastic strain energy can be only produced when it exceeds the historical maximum stress.

Table 4 Energy dissipation of different cyclic series reaching the same stress state

| Sample No. | Stress level /MPa | Energy dissipation of different cyclic series reaching the same stress state /($\text{kJ} \cdot \text{m}^{-3}$) | | | | | | | Energy attenuation coefficient λ_i |
|------------|-------------------|---|--------|--------|--------|--------|--------|--------|--|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| B1 | 15.75 | 11.047 | 10.450 | 10.236 | 10.347 | 10.272 | 10.160 | 10.394 | 1.071 5 |
| | 21.00 | — | 18.862 | 17.696 | 17.873 | 17.671 | 17.821 | 17.681 | 1.062 7 |
| | 26.25 | — | — | 28.449 | 26.982 | 26.809 | 26.957 | 27.114 | 1.055 0 |
| | 31.50 | — | — | — | 38.238 | 37.069 | 37.674 | 37.606 | 1.021 0 |
| | 34.15 | — | — | — | — | 44.335 | 43.948 | 44.319 | 1.004 6 |
| | 36.75 | — | — | — | — | — | 51.233 | 51.219 | 1.000 3 |
| B2 | 15.75 | 15.283 | 11.945 | 10.882 | 10.876 | 10.796 | 10.839 | — | 1.380 8 |
| | 21.00 | — | 23.231 | 20.680 | 20.546 | 19.412 | 19.930 | — | 1.153 4 |
| | 26.25 | — | — | 34.507 | 32.446 | 31.742 | 31.964 | — | 1.076 6 |
| | 31.50 | — | — | — | 48.650 | 45.546 | 46.192 | — | 1.060 6 |
| B3 | 15.75 | 12.404 | 10.715 | 10.419 | 10.429 | 10.008 | 10.232 | — | 1.197 1 |
| | 21.00 | — | 19.816 | 18.445 | 18.283 | 18.391 | 18.574 | — | 1.075 6 |
| | 26.25 | — | — | 29.641 | 28.456 | 28.486 | 28.152 | — | 1.045 0 |
| | 31.50 | — | — | — | 41.997 | 40.678 | 40.723 | — | 1.031 8 |
| | 34.15 | — | — | — | — | 48.786 | 47.980 | — | 1.016 8 |

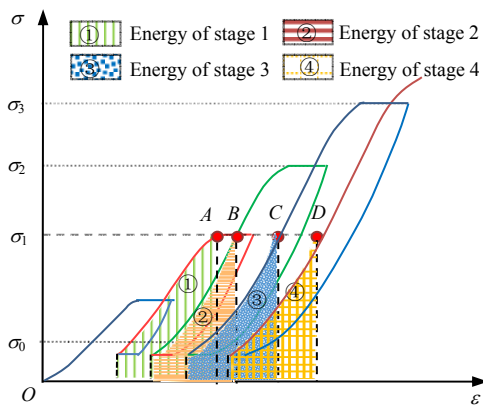


Fig.12 Energy dissipation characteristics during loading

In this paper, the energy attenuation coefficient λ_i is proposed. It is defined as the ratio of the dissipated energy when the sample (excluding the first step cycle) reaches the maximum stress under the i -th step cycle to the average energy consumption of subsequent step of cycle reaching the same stress level again. The values of λ_i corresponding to each step cyclic stress are shown in the Table 4. With the increase of cyclic step (stress), the λ_i decreases gradually and approaches 1. As shown in the Fig. 13, a power function ($Y = aX^b$) is used to fit the data and the fitting parameters are listed in Table 5.

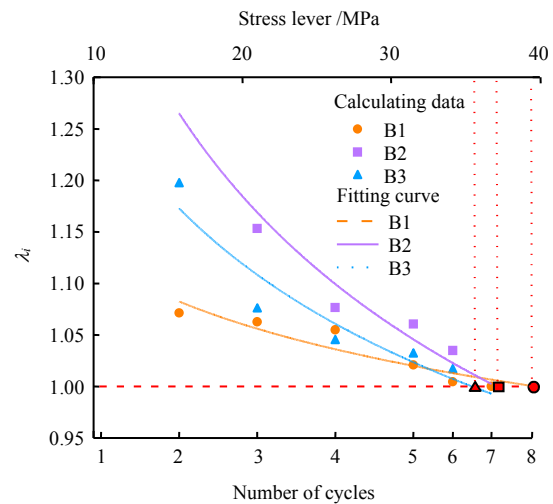


Fig.13 Relationship between λ_i and σ

Table 5 Fitting parameters

| Sample | a | b | R^2 |
|--------|---------|----------|---------|
| B1 | 1.370 8 | -0.085 7 | 0.893 0 |
| B2 | 2.696 7 | -0.274 6 | 0.974 0 |
| B3 | 2.016 8 | -0.196 6 | 0.917 0 |

Substituting the fitting parameters and setting $Y=1$, the X of three groups of samples were calculated as 39.66, 37.06 and 35.45, respectively, which were close to the failure stresses of

B1 (39.4 MPa), B2 and B3 (36.75 MPa). Therefore, this method can be used to predict the failure stress of samples under cyclic loading and unloading. It is worth noting that λ_i is a dynamic parameter that varies with the cyclic process. The more cycles (the greater the value of i), the more accurate the final prediction will be.

6 Summaries

In this paper, a series of uniaxial step loading and unloading creep experiments were conducted on sandstone samples and the laws of energy evolution in the process of rock deformation and failure were analyzed.

(1) The total strain energy is divided into the recoverable strain energy and the irreversible strain energy, and both increase with the increase of cyclic steps. Specifically, the recoverable strain energy increases linearly and dramatically while the irreversible strain energy has a nonlinear growth with a moderate trend. The proportion of recoverable strain energy and unrecoverable strain energy in total strain energy vary greatly before $0.5\sigma_c$ and keep relatively stable afterwards.

(2) The irrecoverable strain energy can be subdivided into the plastic strain energy and the dissipated energy. The dissipated energy under each step increases nonlinearly with the increase of cyclic steps, while the plastic strain energy under each step is relatively stable. The cyclic plastic strain energy under each step is larger than the dissipated energy under each step in the early stage of the test. Before failure of sample, the dissipated energy exceeds the plastic strain energy, which can be considered as a precursor energy characteristic of sample failure.

(3) By defining and comparing the related energy coefficient and strain coefficient, it can be found that with the increase of cyclic step, the plastic strain energy under each step is positively correlated with the increment of plastic strain under each step. Similarly, the dissipated energy under each step is positively correlated with the plastic strain accumulation.

(4) The energy absorption rate of loading stage and the energy release rate of unloading stage both increase with the increase of cyclic step, and the internal structure of the samples became easier to absorb or release energy. The energy absorption rate of creep stage and the energy release rate of recovery stage also increase with the increase of cyclic step, but the energy change rate of creep stage is one order of magnitude higher than that of the recovery stage.

(5) Based on the laws of dissipated energy in the cyclic loading stage, the energy attenuation coefficient λ_i is proposed. Through analyzing its change rule, a novel method is developed to predict the failure stress in the step cyclic loading and unloading experiments

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