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Analysis of the evolution of excess pore water pressure in soft soil under linear unloading

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Abstract: At present, the research on the consolidation deformation of soft soil under unloading mainly focuses on the instantaneous unloading. In order to clarify the evolution of consolidation deformation of soft soil under linear unloading, through the one-dimensional consolidation theory of Terzaghi and the principle of effective stress, an effective method is proposed to study the general solution of the soft soil foundation under the arbitrary unloading rate and solve the analytical solution of the consolidation equation under linear unloading. The results show that: (1) the negative excess pore pressure generated in soft soil foundation under linear unloading can be divided into three stages: growth period, rapid dissipation period and slow dissipation period; (2) unloading rate affects the growth path and dissipation rate of negative excess pore pressure; and (3) unloading affects the maximum negative excess pore pressure with the unloading amount approximately linearly increasing. Finally, taking engineering examples for comparative analysis, it is found that the negative excess pore pressure and its dissipation rate induced by unloading are the highest at the end of unloading. The theoretical results with evolution trends are in good agreement with field measurements.

Keywords: geotechnical engineering; one-dimensional consolidation; linear unloading; consolidation equation; unloading rebound; negative excess pore water pressure

1 Introduction

With the continuous investment in infrastructure construction in China, the deformation analysis of soft soil foundation after unloading is faced in many projects. The most obvious characteristic of soft soil foundation under unloading is rebound deformation, which is closely related to the change of effective stress in soil. According to the principle of effective stress, the key to solve the rebound deformation of foundation under unloading is to fully understand the evolution of excess pore water pressure of foundation under unloading [1−5]. Many achievements have been made in the study of the deformation of soft soil under the action of unloading. For example, Li et al. [6] found that the swelling uplift of the soil mass at the bottom of the pit was caused by the gradual decrease of effective stress in that soil mass when there was the dissipation of negative pore pressure at the excavation of the pit. Through unloading triaxial test, Chen et al. [7] found that the stress–strain relationship was affected by the stress path and had good normalization characteristics. Through theoretical analysis and experiments, Zhou et al. [8] found that the relationship between pore pressure and principal strain was closely related to unloading path. Through one-dimensional loading compression and unloading rebound tests, Lin et al. [9] showed that the excess pore pressure increased rapidly in the initial stage of unloading, and then the growth rate was approximately constant. Hao et al. [10−11] explained the phenomenon of small, fast and stable rebound deformation of foundation after removing the preloading and improved the calculation method of equivalent transformation from three-dimensional (3D) consolidation to one-dimensional (1D) consolidation in sand well area under unloading. Shi et al.[12−13] obtained through tests that the rebound deformation of the mud under unloading could be divided into three parts, and the water absorption volume induced by unloading at the end of preloading of the sample is equal to the volume change of the sample. Through tests, Zhang et al. [14] obtained the conclusion that the consolidation coefficient is not a fixed constant under loading/unloading.

The above scholars have seldom theoretical analysis and discussion on the topic of the evolution of excess pore water pressure under linear unloading. In this paper, the analytical solution for one-dimensional consolidation under linear unloading is derived based on the Terzaghi's one-dimensional consolidation theory, and the rationality of the analytical solution is verified by comparing the actual monitoring with the theoretical value, so as to provide reference for the prediction of the rebound deformation and the maintenance of the foundation under unloading.

2 1D consolidation equation of soft soil foundation under unloading

2.1 Terzaghi's 1D consolidation theory

Under the action of external load q, seepage consolidation occurs in saturated soil. The effective stress σ' of soil

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skeleton and the pore water pressure u (as shown in Fig. 1 for the two curved surface parts) are function of changing time t and depth z. Assume that at the instant of the load, the external load is entirely carried by the pore water pressure u, then u gradually begins to dissipate over time, combined with the effective stress σ' increasing gradually, the external load q is carried by the soil skeleton, gradually, until the excess pore water pressure u=0.

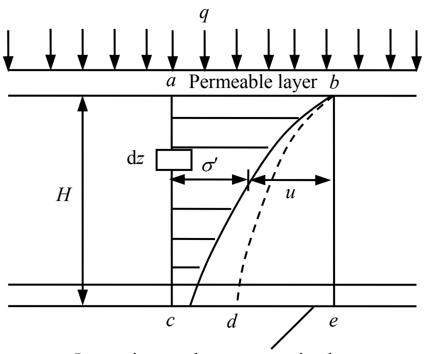

Fig.1 Schematic diagram of the one-dimensional consolidation theory of Terzaghi

Figure 1 shows a saturated soft soil layer with a thickness of H and consolidated under dead-weight. Assume that the surface of soil layer is under a uniformly distributed load q, the additional stress σ_z caused by q is uniformly distributed in depth, which is equal to p, shown as the line be in Fig. 1. The upper part of the saturated soft soil is a permeable sand layer, while the bottom part is a hard impermeable layer. During the consolidation process, the water in the soil can only be discharged from the upper permeable sand layer. The curve bd represents the change curve of effective stress σ' and excess pore water pressure u along depth z in the soil at any time t, as shown in the dotted line in Fig. 1. The effective stress σ' and pore water pressure u are function of depth z and time t, expressed as $\sigma'=\sigma'$ (z, t) and $u=u\,(z, t)$, respectively.

The differential equation of pore water pressure $u=u\,(z, t)$ [15] is

$$C_{\mathrm{v}}\frac{\partial^2 u}{\partial z^2}=\frac{\partial u}{\partial t} \quad (1)$$

2.2 Calculation diagram and related assumption under unloading

In this paper, the basic equation of 1D consolidation of saturated soft soil foundation under any unloading rate is presented. Figure 2 is the calculation diagram.

The load acting in Fig. 2 is assumed to be a uniformly distributed load over a large area. Compared with the hypothesis of Terzaghi's 1D consolidation theory, the other conditions are the same as the hypothesis of Terzaghi's 1D consolidation theory except for the load acting on the soft soil. The basic equation of 1D consolidation of soft soil at any rate of unloading [15] can be obtained as follows:

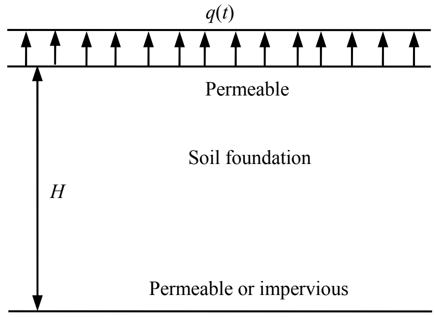

Fig.2 One-dimensional consolidation calculation diagram

$$C_{\mathrm{ve}}\frac{\partial^2 u}{\partial z^2}=\frac{\partial u}{\partial t}-f(t) \quad (2)$$

According to the reference [10], we could know $C_{\mathrm{ve}}=(5-10)C_{\mathrm{v}}$, where C_{v} is the consolidation coefficient of soil under loading, and it is not applicable to unloading. C_{ve} can be expressed as

$$C_{\mathrm{ve}}=\frac{k_{\mathrm{v}}}{\gamma_{\mathrm{w}} m_{\mathrm{v}}}=\frac{k_{\mathrm{v}}\left(1+e_0\right)}{a_{\mathrm{v}}\gamma_{\mathrm{w}}} \quad (3)$$

where C_{ve} is the coefficient of water absorption and consolidation (m^2/s), which is a coefficient of dissipation rate of negative excess pore pressure induced by soil unloading; $f(t)$ is the unloading rate (kPa), $f(t)=\frac{\mathrm{d}q(t)}{\mathrm{d}t}$; u is the pore water pressure (kPa); k_{v} is the unloading permeability coefficient of soil (m/s); m_{v} is the volume rebound coefficient (kPa^{-1}), $m_{\mathrm{v}}=1/E_{\mathrm{n}}$, E_{n} is the rebound modulus; γ_{w} is the pore liquid unit weight (kN/m^{-3}); a_{v} is the rebound coefficient (kPa^{-1}); and e_0 is the initial void ratio of soil.

The initial boundary condition of the 1D consolidation basic Eq. (2) is

$$\left.\begin{array}{l}
\text{when } t=0, 0\leqslant z\leqslant H,\ u|_{t=0}=u_0(z)\\
\text{when } 0<t<\infty,\ u|_{z=0}=0(\text{top permeable})\\
\text{when } 0<t<\infty,\ \left.\frac{\partial u}{\mathrm{d}z}\right|_{z=H}=0(\text{bottom impermeable})\\
\qquad u|_{z=H}=0(\text{bottom permeable})
\end{array}\right\} \quad (4)$$

2.3 General solution for 1D consolidation under arbitrary unloading rate

According to Fig. 2, the case of top permeable with bottom impermeable is analysed as an example. Combining the Fourier series method and separation variable method and using the boundary condition (4), the 1D consolidation of equation (2) can be solved under arbitrary unloading rate as follows:

$$u(z,t)=\sum_{n=1}^{\infty}M_n(t)\sin\frac{N}{H}z \quad (5)$$

$$f(t)=\sum_{n=1}^{\infty}X_n(t)\sin\frac{N}{H}z \quad (6)$$

The Eq. (6) is further derived as

$$X_n(t)=\frac{2}{H}\int_0^H f(t)\sin\frac{N}{H}z\mathrm{d}z=\frac{N}{H}f(t) \quad (7)$$

$$N=\frac{2n-1}{2}\pi \quad (n=1,2,3,\cdots) \quad (8)$$

Substituting Eqs.(5) and (6) into Eq.(2), we have

$$M_n'(t)+C_{\mathrm{ve}}\frac{N^2}{H^2}M_n(t)-X_n(t)=0 \quad (9)$$

Given the a as

$$a=C_{\mathrm{ve}}\frac{N^2}{H^2} \quad (10)$$

The general solution for Eq. (9) is

$$M_n(t)=\mathrm{e}^{-at}\left(\frac{2}{N}\int_0^t f(\tau)\mathrm{e}^{at}\mathrm{d}t+C\right) \quad (11)$$

Substituting Eq.(11) into Eq.(5) and based on the first part of Eq.(4), we have

$$C=\frac{2}{H}\int_0^H u_0(z)\sin\frac{N}{H}z\mathrm{d}z \quad (12)$$

Then, the solution can be found as

$$M_n(t)=\mathrm{e}^{-at}\left[\frac{2}{N}\int_0^t f(t)\mathrm{e}^{at}\mathrm{d}t+\frac{2}{H}\int_0^H u_0(z)\sin\frac{N}{H}z\mathrm{d}z\right] \quad (13)$$

Substituting Eq.(13) into Eq.(5) leads to the general solution for 1D consolidation under arbitrary unloading rate for Eq. (2):

$$u(z,t)=\sum_{n=1}^{\infty}\mathrm{e}^{-at}\sin\frac{N}{H}z\left[\frac{2}{N}\int_0^t f(t)\mathrm{e}^{at}\mathrm{d}t+\frac{2}{H}\int_0^H u_0(z)\sin\frac{N}{H}z\mathrm{d}z\right] \quad (14)$$

3 Analytical solution for 1D consolidation of soft soil foundation under linear unloading

Load $q(t)$ is discharged linearly, and the change of load with time is shown in Fig. 3.

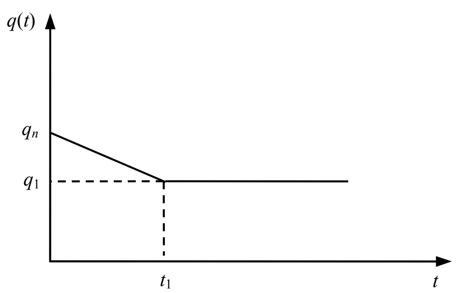

Fig.3 Curve of load versus time

$$q(t)=\begin{cases}\dfrac{q_1-q_n}{t_1}t+q_n & 0\leqslant t\leqslant t_1\\ q_1 & t_1\leqslant t\end{cases} \quad (15)$$

where q_n is the initial load during history, q_1 is the load after unloading, t_1 is the end time of unloading.

The stress condition of each stage is expressed as

$$f(t)=\begin{cases}\dfrac{q_1-q_n}{t_1} & 0\leqslant t\leqslant t_1\\ 0 & t_1\leqslant t\end{cases} \quad (16)$$

The continuous condition of pore water pressure for each stage is

$$u_1(t=t_1)=u_2(t=t_1) \quad (17)$$

Given the unloading rate of each stage as follows:

$$\left.\begin{array}{ll} f_1(t)=\dfrac{q_1-q_n}{t_1}=k & 0\leqslant t\leqslant t_1\\ f_2(t)=0 & t_1\leqslant t\end{array}\right\} \quad (18)$$

where k is the unloading rate.

Given the initial pore water pressure of $u_1(z)=q_0$, substituting the equation (18) into (14), and considering the equations (4) and (17), the solution for 1D consolidation of soft soil foundation under linear unloading is derived as follows:

$$u(z,t)=\begin{cases}\sum\limits_{n=1}^{\infty}\dfrac{2\mathrm{e}^{-at}}{N}\left[\dfrac{k}{a}\left(\mathrm{e}^{at}-1\right)+q_0\right]\sin\left(\dfrac{N}{H}z\right) & 0\leqslant t\leqslant t_1\\ \sum\limits_{n=1}^{\infty}\dfrac{2\mathrm{e}^{-at}}{N}\left[\dfrac{k}{a}\left(\mathrm{e}^{at_1}-1\right)+q_0\right]\sin\left(\dfrac{N}{H}z\right) & t_1\leqslant t\end{cases} \quad (19)$$

where H is the maximum drainage length, which is the thickness of the soil layer for one-way drainage and half of the thickness for two-way drainage.

According to the principle of effective stress and the relationship between the rebound modulus and the effective stress, the rebound deformation is solved as

$$S(t)=\begin{cases}-\dfrac{H}{E_n}\left\{kt+\sum\limits_{n=1}^{\infty}\dfrac{2q_0}{N^2}-\sum\limits_{n=1}^{\infty}\dfrac{2\mathrm{e}^{-at}}{N^2}\left[\dfrac{k}{a}\left(\mathrm{e}^{at}-1\right)+q_0\right]\right\} & 0\leqslant t\leqslant t_1\\ -\dfrac{H}{E_n}\left\{q_1-q_n+\sum\limits_{n=1}^{\infty}\dfrac{2q_0}{N^2}-\sum\limits_{n=1}^{\infty}\dfrac{2\mathrm{e}^{-at}}{N^2}\left[\dfrac{k}{a}\left(\mathrm{e}^{at_1}-1\right)+q_0\right]\right\} & t_1\leqslant t\end{cases} \quad (20)$$

From Fig. (19) of pore pressure under the linear unloading, it can be seen that the excess pore water pressure under unloading is caused by unloading rate, unloading capacity, together with the coefficient of water absorption-consolidation and the initial pore water pressure. The negative excess pore water pressure in this paper during the unloading process is the difference between the excess pore water pressure and that at the initial unloading at a such time. Equation (19) is coded in Matlab, and relevant parameters are assigned. Let q_0 = 0; C_{ve} = 6.85×10^{-5} m^2/s; H =20 m; the rest of the parameters are variables, and the results are shown in Figs. 4 and 5.

It can be seen from Fig. 4 that when the unloading rate k < 2 kPa/d, the maximum negative excess pore water pressure is independent of the unloading amount. When unloading amount is constant, the effect of unloading rate k on the maximum negative excess pore water pressure has critical values k_1 and k_2. When k < k_1, the generated maximum negative excess pore water pressure is very small even to zero, this is due to the dissipation of negative excess pore water pressure produced by unloading during the process of unloading. When $k_1 < k < k_2$, the maximum negative excess pore water pressure caused by unloading increases as unloading rate increases. When $k_2 < k$, the increase of unloading rate has no effect on the maximum negative excess pore water pressure, this is because unloading is so fast that the unloading is completed before the negative excess pore water pressure is dissipated, namely when $k_2 < k$, is equivalent to instantaneous unloading. When the unloading rate is constant, the maximum negative excess pore water pressure caused by unloading increases approximately linearly with the increase of unloading amount. When other conditions are constant, the greater the unloading amount, the greater the critical rate k_2.

It can be seen from Fig. 5 that unloading rate affects the dissipation rate of negative excess pore water pressure, which increases with the increase of unloading rate. The unloading rate affects the growth path of negative excess pore water pressure while the unloading amount affects the maximum negative excess pore water pressure. The change of negative excess pore water pressure with time under unloading can be divided into three stages: increasing period, rapid dissipation period and slow dissipation period.

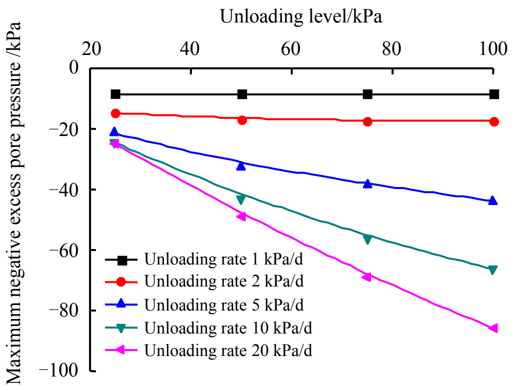

Fig.4 Maximum negative excess pore pressure induced by different unloading levels and rates

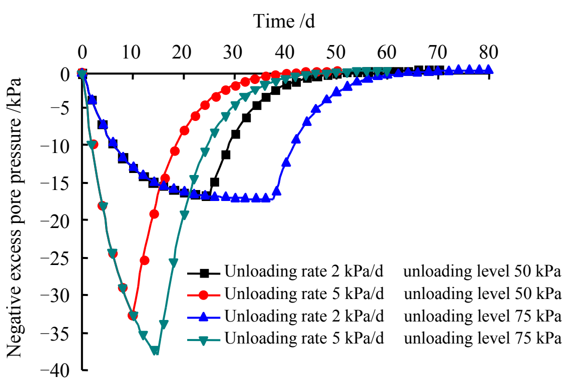

Fig.5 Negative excess pore pressure induced by different unloading levels and rates with time

4 Case analysis

The runway foundation of a civil airport project [11] is a typical soft clay foundation in China, and the soft clay layer is relatively uniform in the airport. For the soft clay foundation of the airport runway, the construction company intends to use the bagged sand well pre-consolidation scheme for reinforcement. The depth of the sand well is 20 m, the diameter of the sand well is 0.14 m, the spacing is 1.4 m, and the quincunx layout is adopted. The preloaded uniform load of the runway foundation is 82.65 kPa, and then the runway foundation is preloaded under this load for 260 days, and finally unloaded to 54.87 kPa. The specific unloading curve is shown in Fig. 6, and the measured and calculated points are selected at the center of the airport runway.

In practice, however, due to the water absorbing consolidation of the foundation after unloading is three-dimensional, through the equivalent calculation method of Hao et al.[11], the three-dimensional consolidation of sand well zone can be equivalently converted into a one-dimensional consolidation for calculation. After calculations, the equivalent one-dimensional vertical coefficient of consolidation of sand well zone is $C_v = 7.34\times10^{-6}$ m^2/s. Through the indoor unloading tests of Lin et al. [9], C_{ve}/C_v=9.3, namely the water absorption coefficient of consolidation when unloading is C_{ve}= 6.85×10^{-5} m^2/s. In this case, the calculated depth is 20 m, and the drainage conditions are double-sided.

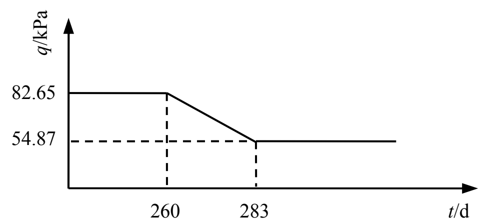

Fig.6 Unloading schedule of airport foundation

Figure 7 shows the three-dimensional plot of negative excess pore water pressure changing with time and depth in the foundation of civil airport project[11] according to Eq. (19) during unloading and within a period after unloading. It can be seen that in the process of unloading, the negative excess pore water pressure gradually increases with the start of unloading until it reaches its maximum value at the end of unloading. As the unloading rate is constant, and the negative excess pore water pressure increases more and more slowly, indicating that the dissipation rate of negative excess pore water pressure increases gradually in the unloading process. The dissipation rate of negative excess pore water pressure gradually decreases on completion of unloading, which indicates that the dissipation rate of negative excess pore water pressure is the highest right after unloading. After unloading, the negative excess pore water pressure dissipates rapidly and has basically dissipated in a very short time. However, complete dissipation takes a long time, which corresponds to the primary and secondary rebound. Since the dissipation of excess pore water pressure is caused by water absorption of foundation, which indicates that most of the rebound deformation of soil after unloading will be completed soon, and that Eq. (19) can predict the duration of water absorption rebound of foundation after unloading.

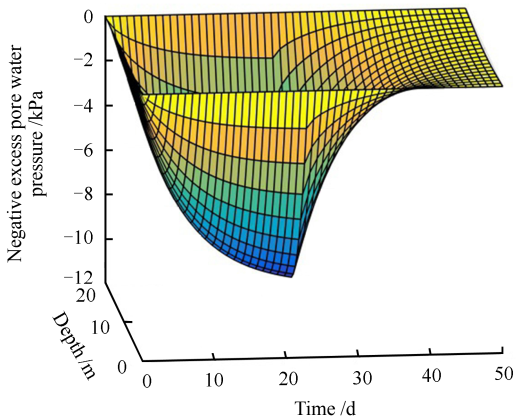

Fig.7 Evolution of negative excess pore water pressure with time and depth

Figure 8 shows the comparison between theoretical and experimental results of negative excess pore water pressure at different times. It can be seen that theoretical values are generally larger than measured values, but they both follow the same path and are in good agreement. Thus, the Eq. (19) can be used to predict the negative excess pore water pressure of the foundation at a certain time and depth under unloading.

Figure 9 shows the comparison between theoretical and experimental results of the amount of rebound at different times. From this figure, the rebound deformation of the foundation starts at the beginning of unloading. The velocity of rebound increases first and then decreases. At 283 days, that is the end of the unloading, the rebound velocity reaches its maximum, which corresponds to the dissipation process of negative excess pore water pressure. It explains that the rebound amount and velocity of foundation are closely related to the negative excess pore water pressure and its dissipated rate under unloading. By comparing the theoretical result with the experimental one, it can be seen that the measured and the calculated results of the rebound amount of foundation are in good agreement and follow the same path, indicating that Eq. (20) can be used to predict the rebound amount of foundation at a certain time under unloading.

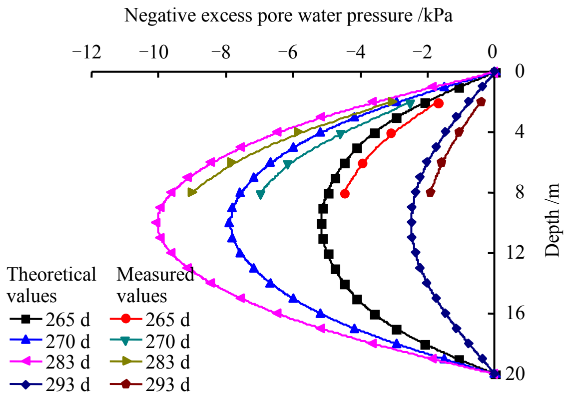

Fig.8 Comparison of theoretical and measured values of negative excess pore water pressure at different times

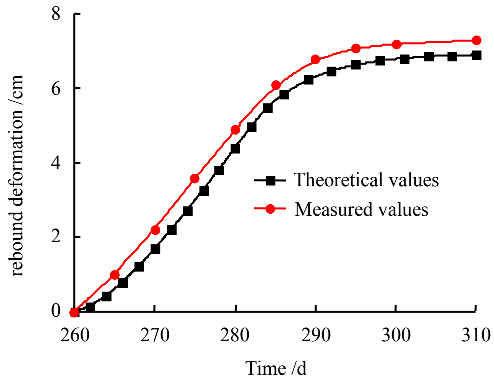

Fig.9 Comparison of theoretical and measured results of swelling under unloading

5 Conclusion

In this paper, through Terzaghi's one-dimensional consolidation theory, a general solution for the one-dimensional consolidation equation under arbitrary unloading rate is derived, and the analytical solution for the consolidation equation under linear unloading is derived.

When the unloading rate is constant, the maximum negative excess pore water pressure induced by unloading is approximately linearly increased with the unloading volume. When the unloading volume is constant, the influence of unloading rate k on the maximum negative excess pore water pressure has critical values k_1 and k_2. The unloading rate influences the growth path and dissipation rate of negative excess pore water pressure while the unloading volume influences the maximum negative excess pore water pressure.

Under unloading, the change of negative excess pore water pressure generated in soft soil foundation with time can be divided into three stages: growth period, rapid dissipation period and slow dissipation period. The rebound amount and the rebound velocity are closely related to the negative excess pore water pressure and its dissipation rate.

By comparing and analyzing the excess pore water pressure caused by unloading in soft soil foundation with a real engineering case, it is found that the theoretical results and their variation are in good agreement with the actual measurements, which indicates that the derived analytical solution for pore water pressure under unloading has practically engineering significance.

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