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# Fractional creep model and experimental study of saturated saline soil

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**Abstract:** The creep behaviors of saturated saline soil are studied in the consolidation creep test under isothermal conditions in this paper. Based on the element model, the fractional creep model of saturated saline soil was established by introducing Abel dashpot and osmotic suction. The phenomenological correlation between salt content and creep behavior of saturated saline soil is discussed. The effects of osmotic suction on the ratio  $C_a/C_c$  of the secondary compression coefficient to the compression index, initial shear modulus and initial shear strain are analyzed by combining experimental results with fractional model fitting. The fractional order model is fitted by the test results of different stress levels and salt content. The validity of the model is verified by the viscosity coefficient formula. The results show that the ratio  $C_a/C_c$  increases exponentially as the osmotic suction increases. The initial shear modulus decreases as the osmotic suction increases, and the initial shear strain exhibits a linear relationship with the osmotic suction. Comparing with the integer element model, the fractional creep model proposed in this paper is more suitable for predicting the creep behavior of saturated saline soil. The results of experiments and model analysis found that the increase of salt content promotes the creep behavior of saline soil.

**Keywords:** saturated saline soil; triaxial creep test; osmotic suction; theoretical model; creep characteristics

## 1 Introduction

Due to the leaching effect of rainfall, the soil matrix, and the migration of groundwater to the subsurface caused by global warming, the salts in the soil are dissolved to liquid solutions, which breaks the original equilibrium state of the soil. As a result, the soluble salts carried by groundwater and the soil particles undergo ion exchange and ionization re-equilibrium. In this process, due to the chemical-mechanical coupling between the soil and the pore salt solution, a second non-steady and uneven compression occurs in soil, resulting in a series of effects on foundation engineering, underground space structures, and pavement and bridge engineering<sup>[1]</sup>.

To date, there are few research results on the creep characteristics and constitutive theory of saturated saline soils, and the relevant experimental research mainly focuses on the creep and mechanical behaviors of saturated chloride saline soils. Ogata<sup>[2]</sup>, Nixon<sup>[3]</sup>, Wijeweera<sup>[4]</sup>, et al. carried out a series of studies on the effects of salt content on the creep effect and shear strength of frozen sand. The unconfined compression test, triaxial creep test and shear test of constant strain rate under isothermal conditions were tested respectively to conclude that the salt content significantly affect the creep and shear strength of frozen sand. In other word, the creep and creep rate increase with the increase of salt content, while the shear strength

decreases with the increase of salt content. Liu et al.<sup>[5]</sup> studied the compression and permeability characteristics of the soil after the salt solution was saturated with existing experimental data and constructed a theoretical formula capable of predicting the their behaviors. Yan et al.<sup>[6–8]</sup>, Zhang et al.<sup>[9]</sup>, Xu et al.<sup>[10]</sup> studied the compression deformation and strength characteristics of clay saturated with salt solution on the basis of experiments and the results of Barbour et al.<sup>[11]</sup>, and discussed the effect of osmotic suction on mechanical behavior of saturated saline soils after isobaric consolidation. The correlations between osmotic suction and physical parameters such as compression index, rebound modulus and initial consolidation pressure have been established. The current status of the research on creep of saturated saline soils are inadequate both in experiments and theoretical models, as well as in research objects and theoretical research methods.

In the study of the theoretical model for describing soil creep, the element model shows strong applicability due to its simple theoretical derivation and easy access to material parameters. However, almost all element models establish the nonlinear creep relationship based on the exponential function. When departing from the exponential function, those models degenerate into linear creep models (Bingham model, Burgers model, and Nishihara model), and fail to predict soil creep effectively. Therefore, many scholars<sup>[12–17]</sup> have proposed using

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fractional differential theory instead of integer order theory to establish a corresponding creep model. The improved model not only improves the non-linearity and full elasticity/viscosity constraints of the element model, but also strengthens the capability of model to describe the soil creep path dependence.

Based on the fractional-order differential theory, a fractional-order creep model of the soil is established, and the chemical-mechanical coupling of the saturated saline soil is described by osmotic suction. Then, based on the results of consolidation tests, triaxial undrained tests, and fractional-order models, the correlation between the creep characteristics of saturated saline soils and osmotic suction is discussed, and the prediction formulas are given. Finally, based on the creep test results at different stress levels and salt contents, the fractional creep model of saturated saline soil is analyzed and verified, and the formula for calculating the viscosity coefficient of saturated saline soil creep model is developed.

## 2 Experimental program

### 2.1 Sample preparation

The experimental program includes a consolidation test and a triaxial undrained creep test. The data collection is accomplished by instruments such as a medium pressure consolidation meter and a SR-6 triplex triaxial creep meter and a displacement meter. The sizes of the samples are  $\phi 6.18 \text{ cm} \times 2 \text{ cm}$  and  $\phi 6.18 \text{ cm} \times 12.5 \text{ cm}$ . The soil tested was sampled from Qilihe District, Lanzhou. The soil was washed with salt before sample preparation to ensure the accuracy of the test. The maximum dry density of the soil after salt washing was  $1.7 \text{ g/cm}^3$ , the initial moisture content of the sample was 16.88%, and the dry density was  $1.641 \text{ g/cm}^3$ .

### 2.2 Testing procedure

The preparation process of consolidated sample is as follows: desalting, air drying, crushing and sieving (0.5 mm), sample preparation by adding water and air drying. The method of saturation is to put the soil sample into the test tank and add solution to saturate it. The solutions used for saturation are distilled water, sodium chloride solution and sodium sulfate solution, respectively. The mass molar concentrations of sodium chloride and sodium sulfate solutions are 0.6, 1.2, and 1.8 mol/kg, respectively. The loading test is performed after saturation by adding the solution. The solution level of the test tank is kept substantially constant during the test, and the temperature is maintained at  $25 \pm 1^\circ \text{C}$ . The duration of each stage of the consolidation test is 24 h, and the loading order is 50, 100, 200, 300, 400, 800, 1 600 kPa. A total of 10 groups were designed for the measurements.

Triaxial sample is saturated by using a vacuum saturation

device after desalting and air drying. After installation of the saturated sample, the isostatic drainage consolidation is performed first, and vertical stress is applied after the consolidation is completed. The confining pressure and vertical stress are applied by an air compressor and a calibration weight. A balance weight corresponding to the confining pressure is applied to eliminate the back pressure on the axial drive shaft caused by confining pressure. Secondly, in order to ensure that the vertical loading system is in full contact with the specimen, a tray (12.5 kPa) is applied in advance. After the consolidation is stable, the drain valve is closed first, and the design weight is applied at one time to start undrained shear creep. Amongst, the confining pressure is 200, 300, 400 kPa, respectively; the magnitude of the deviating stress is 200, 300, 400 kPa, respectively.

## 3 Results and discussions

### 3.1 Consolidation test

Figure 1 shows the results of consolidation tests of soils saturated with different solutions. According to the test results, the compression indexes of the samples after saturation with distilled water, sodium chloride solution and sodium sulfate solution are all basically equal, and the compression index is  $C_c = 0.087$ . The test results are in good agreement with those of Yan et al.<sup>[7]</sup> Table 1 lists the secondary consolidation coefficients of the soil ( $C_a$ ) after saturation with different solutions, based on the results of the 24 h consolidation test in Figure 1.  $C_a$  is obtained using the formula  $C_a = \Delta e / \lg(t/t_0)$ . It can be found that the sub-consolidation coefficients of the soil saturated with sodium sulfate solution and sodium chloride solution are larger than soil saturated with distilled water. In addition, the secondary consolidation coefficient increases with the increase of the solution concentration. The soil secondary consolidation coefficients are not equal when saturated with sodium sulfate solution and sodium chloride solution of the same concentration.

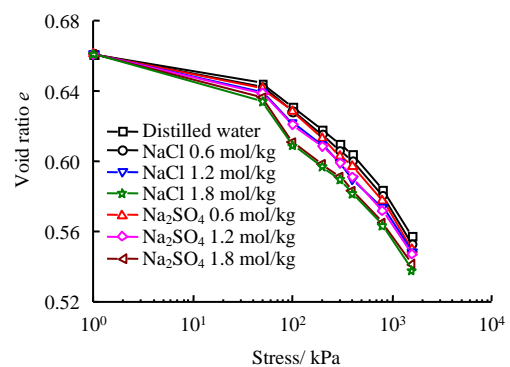


Fig.1 Consolidation test results

**Table 1 Average secondary consolidation coefficient of saturated saline soil**

Pore solution	Mass molar concentration $m$ / (mol/kg)	Secondary consolidation coefficient $C_a$
Distilled water	0.0	0.001 330
	0.6	0.001 375
	1.2	0.001 690
NaCl solution	1.8	0.002 035
	0.6	0.001 464
Na <sub>2</sub> SO <sub>4</sub> solution	1.2	0.001 748
	1.8	0.002 010

**3.2 Creep under triaxial undrained shear**

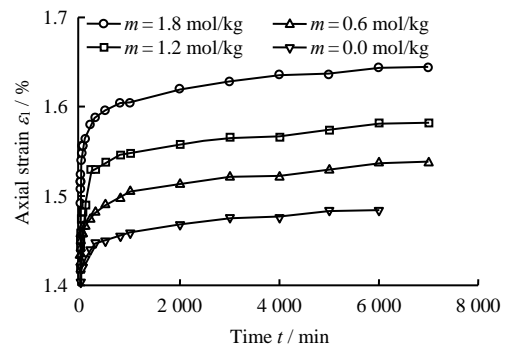
Results of triaxial shear creep tests of saline soils saturated by sulphate under different deviatoric stresses are shown in Fig. 2. It can be seen from the figure that the creep of saturated sulphate saline soil obviously correlates with the salt content. That is to say, under the same stress level, the creep behavior of sulfated soil is clearly different from that of non-saline soil, and the difference between the two is related to the change of salt content.

The creep behaviors under deviating stress of 200, 300, and 400 kPa conditions are compared respectively. It is found that the initial creep amount, rate, and stabilization time under different bias stresses will vary with the changes in salt content. When the deviator stress is 200 kPa and the mass molar concentration  $m$  is 0.6, 1.2, and 1.8 mol/kg, the strain differences  $\Delta\varepsilon$  ( $\Delta\varepsilon = \varepsilon_{t=1\,000} - \varepsilon_{t=end}$ ) are 0.032%, 0.034%, and 0.04%, respectively. At  $m = 1.8$  mol/kg, the strain differences  $\Delta\varepsilon$  are 0.04%, 0.058%, and 0.064% when the deviator stresses are 200, 400, and 800 kPa, respectively. However, the creep deformation under different conditions are generally attenuated. As the strain rate gradually approaches to zero, the strain tends to keep a constant value.

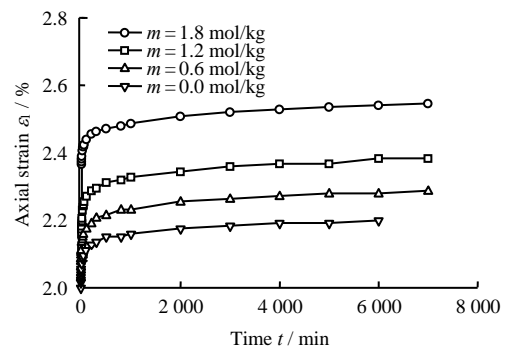
**4 Fractional creep model**

**4.1 Description of water chemistry**

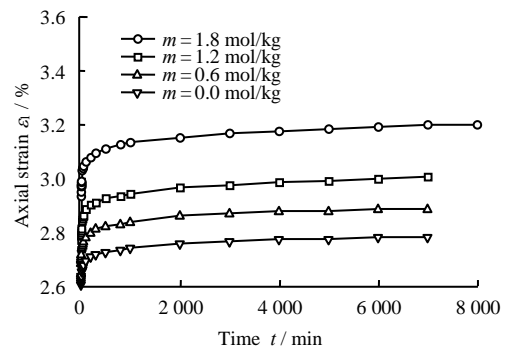
Under loading condition, the original balance of saturated saline soil is broken. As a result, the pore salt solution will undergo ionization equilibrium and ionic interactions with the soil particles, resulting in different stress fields due to seepage and changing the stress state of the soil. Therefore, the effect of water chemistry on soil-water system needs to be considered for establishing the constitutive model of saline soil, so as to reflect the effect of pore salt solution on soil characteristics. According to the existing research results<sup>[8,9,11]</sup>, it is found that the chemical-mechanical coupling effect of saturated saline soils can be described by osmotic suction. The osmotic suction can be calculated using Van't Hoff equation<sup>[18]</sup>:



(a)  $\sigma_1 - \sigma_3 = 200$  kPa,  $\sigma_3 = 200$  kPa



(b)  $\sigma_1 - \sigma_3 = 300$  kPa,  $\sigma_3 = 300$  kPa



(c)  $\sigma_1 - \sigma_3 = 400$  kPa,  $\sigma_3 = 400$  kPa

**Fig.2 Triaxial shear creep test of saturated saline soil**

$$\pi = \omega RTc\phi \tag{1}$$

where  $\omega$  is the total number of anions and cations in the salt solution;  $R$  is the molar gas constant,  $R = 8.315$  kg/(mol · K);  $T$  is the thermodynamic temperature;  $c$  is the quantity solubility of the salt solution substance (mol/L);  $\phi$  is the permittivity, calculated by the equation proposed by Pitzer et al.<sup>[19–20]</sup>:

$$\phi = 1 + |z_M z_X| f_\phi + 2m \frac{\omega_M \omega_X}{\omega} B_\phi^{MX} + 2m^2 \frac{(\omega_M \omega_X)^{3/2}}{\omega} C_\phi^{MX} \tag{2}$$

In the formula,  $z_M, z_X$  are the charge numbers of cations  $M$  and anions  $X$  constituting the solute;  $\omega_M$  is the total number of ions of the cation;  $\omega_X$  is the total number of ions of the anion;  $\omega = \omega_M + \omega_X$ ;  $f_\phi$  is the Debye-Hückel term;  $B_\phi^{MX}$  and  $C_\phi^{MX}$  are the coefficient of ion interaction.

Fredlund et al.<sup>[21]</sup> proposed an independent stress state variable method for describing the effective stress expression of unsaturated soils, which reflected the contribution of suction to effective stress. Therefore, this paper uses the concept of independent stress state variable method to reflect the effective stress  $\sigma^*$  of saturated saline soil through the sum of single effective stress and effective osmotic stress, which is

$$\sigma^* = \sigma_a^* + \sigma_\pi^* \quad (3)$$

where  $\sigma_a^*$  is the Terzaghi's effective stress, i.e.,  $\sigma_a^* = \sigma - \mu_w$ ;  $\sigma$  and  $\mu_w$  are the total stress and pore water pressure, respectively;  $\sigma_\pi^*$  is the effective osmotic stress generated by the interaction of particles in saturated saline soil,  $\sigma_\pi^* = \chi\pi$  and  $\chi$  is the model parameter.

### 4.2 Fractional calculus theory

From the Abel integral theory, when  $I \in [0, +\infty)$ ,  $r \in R^+$  and  $t > 0$ , the Riemann-Liouville (RL) fractional-order integral equation at order  $r \in (0, 1)$  and  $t \in (t_0, t)$  is

$${}_t I_t^r f(t) = \frac{1}{\Gamma(r)} \int_{t_0}^t (t - \xi)^{r-1} f(\xi) d\xi \quad (4)$$

where  $\Gamma(\cdot)$  is the gamma function:

$$\Gamma(r) = \int_0^\infty t^{r-1} e^{-t} dt \quad (5)$$

The fractional derivative of RL is the result of the inverse operation of its fractional integral. Therefore, the time-fractional derivative expression is

$${}_t D_t^r f(t) = D^1 {}_t I_t^{1-r} f(t) \quad (6)$$

The Abel damper is a typical method to transform the integer-order element model into a fractional-order model. Its constitutive relationship is<sup>[17, 22]</sup>

$$\sigma(t) = \eta D^r \varepsilon(t) \quad (0 \leq r \leq 1) \quad (7)$$

where  $\eta$  is the viscosity coefficient.

### 4.3 Visco-elastoplastic model

The use of the Abel damper to replace the Newtonian viscous element in the model, can not only eliminate the limitations of Newtonian damping's complete elasticity and extreme viscosity, but also can establish a nonlinear creep model after getting rid of the exponential function dependence. Figure 3 shows a fractional-order creep model constructed using Newtonian springs, Abel dampers, and plastic elements.

The total strain, based on the model in Figure 3, can be decomposed as

$$\varepsilon = \varepsilon_e + \varepsilon_v + \varepsilon_{vp} \quad (8)$$

In the formula, the subscripts "e", "v" and "vp" respectively

represent elastic deformation, viscous deformation and viscoplastic deformation.

The elastic deformation is obtained according to Hooke's law:

$$\varepsilon_e = \frac{\sigma^*}{E} \quad (9)$$

In the formula,  $E$  is the elastic modulus of the spring in Fig.3.

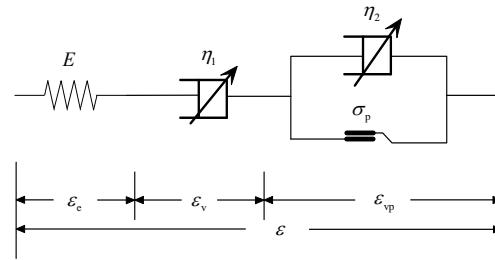


Fig.3 Fractional creep model

Based on the constitutive relationship (7), the viscous deformation is of the Abel damper, using the theoretical relationship under constant stress ( $\sigma = \text{const}$ ), and then Laplace integral transformation is obtained:

$$\varepsilon_v(t) = \frac{\sigma^*}{\eta_1} \frac{t^r}{\Gamma(r+1)} \quad (10)$$

where  $\eta_1$  is the viscosity coefficient.

When the viscoplastic body is under constant stress loading in Figure 3, the stress components satisfy the series-parallel relationship:

$$\sigma^* = \sigma_v^* + \sigma_p^* \quad (11)$$

Based on the fractional-order theory, according to the relationship between the magnitude of stress and the yield limit  $\sigma_s^*$ , the following discussions are carried out:

$$\left. \begin{aligned} \sigma_p^* < \sigma_s^*, \quad \varepsilon = \varepsilon_e + \varepsilon_v \\ \sigma_p^* \geq \sigma_s^*, \quad \varepsilon = \varepsilon_e + \varepsilon_v + \varepsilon_{vp} \end{aligned} \right\} \quad (12)$$

According to the serial relationship between Abel damping and plastic elements, and under the initial conditions  $\varepsilon(0) = 0$ , the analytical solution of the viscoplastic strain equation using Laplace integral transformation is obtained as

$$\varepsilon_{vp} = \frac{\sigma^* - \sigma_s^*}{\eta_2} \frac{t^r}{\Gamma(1+r)} \quad (13)$$

where  $\eta_2$  is the viscosity coefficient of the viscoplastic body element.

The nonlinear creep equation of saturated saline soil obtained by combining Eqs. (9), (10), (12), and (13) is as follows:

$$\varepsilon = \frac{\sigma^*}{E} + \frac{\sigma^*}{\eta_1} \frac{t^r}{\Gamma(1+r)}, \quad \sigma_p^* < \sigma_s^* \quad (14)$$

$$\varepsilon = \frac{\sigma^*}{E} + \left( \frac{\sigma^*}{\eta_1} + \frac{\sigma^* - \sigma_s^*}{\eta_2} \right) \frac{t^r}{\Gamma(1+r)}, \quad \sigma_p^* \geq \sigma_s^* \quad (15)$$

According to the correlation between soil mechanics theory and mechanical parameters, Eqs. (14) and (15) can be transformed into creep Eqs. under three-dimensional stress.

If  $\sigma^* < \sigma_s^*$ ,  $f < 0$ , then

$$\varepsilon_{ij} = \left[ \frac{1}{2G_0} + \frac{1}{2\beta_1} \frac{t^r}{\Gamma(1+r)} \right] s_{ij} \quad (16)$$

If  $\sigma^* \geq \sigma_s^*$ ,  $f \geq 0$ , the associated flow rule can be adopted:

$$\varepsilon_{ij} = \left[ \frac{1}{2G_0} + \frac{1}{2\beta_1} \frac{t^r}{\Gamma(1+r)} \right] s_{ij} + \frac{1}{2\beta_2} \frac{t^r}{\Gamma(1+r)} \left\langle \frac{p^*}{p_0^*} \right\rangle \frac{\partial f}{\partial s_{ij}} \quad (17)$$

where  $\varepsilon_{ij}$  is the component of strain tensor;  $G_0$  is the Hooke's shear modulus;  $\beta_1$  and  $\beta_2$  are the Maxwell model and the viscosity coefficient under complex stress state after considering fractional-order theory;  $p^*$  and  $p_0^*$  are the average principal stresses under the dynamic loading condition and the static reference state, respectively;  $s_{ij}$  is the component of deviator stress,  $s_{ij} = \sigma_{ij}^* - p^* \delta_{ij}$ ;  $\sigma_{ij}^*$  is the component of stress tensor;  $\delta_{ij}$  is the Kronecker symbol.

The modified Cam-Clay model with a more comprehensive yield criterion theory and a wider application range is adopted. The yield criterion<sup>[23]</sup> is

$$f = q^{*2} + M^2 p^{*2} - M^2 p^* p_0^* = 0 \quad (18)$$

where  $q^*$  is the generalized shear stress;  $M$  is the slope of the critical state line.

Therefore, Eq. (17) can be expressed as

$$\varepsilon_{ij} = \left[ \frac{1}{2G_0} + \frac{1}{2\beta_1} \frac{t^r}{\Gamma(1+r)} \right] s_{ij} + \frac{1}{2\beta_2} \frac{t^r}{\Gamma(1+r)} \left\langle \frac{p^*}{p_0^*} \right\rangle \frac{3}{M^2} \frac{s_{ij}}{p^*} \quad (19)$$

## 5 Analysis and verification

### 5.1 Creep characteristics analysis

Combining the experimental results with the fractional creep equation considering the chemical-mechanical coupling of saturated saline soils, the study on the consolidation creep characteristics of saturated saline soils is carried out. It mainly focuses on the correlation between its osmotic suction and the ratio of secondary consolidation coefficient to compression index  $C_a/C_c$ , initial shear modulus and initial shear strain. At the same time, the prediction ability of the fractional creep equation is analyzed and verified in combination with the

triaxial shear creep test, and the formula of viscosity coefficient is given. The material parameters of the saturated sulphate saline soil used in the numerical calculation are shown in Table 2.

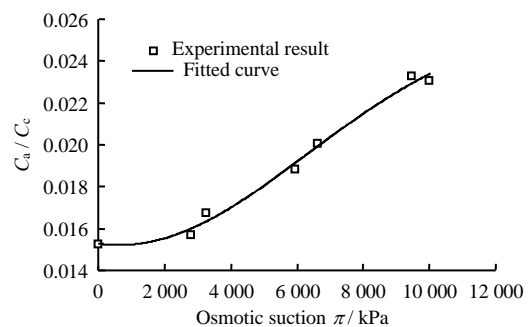
**Table 2 Model parameters of saturated sulfuric saline soil**

$m$ /(mol·kg <sup>-1</sup> )	$\lambda$	$C_\phi^{MX}$ /(kg <sup>2</sup> ·mol <sup>-2</sup> )	$f_\phi$	$B_\phi^{MX}$ /(kg·mol <sup>-1</sup> )
0.0		0.000 00	0.000 00	0.000 0
0.6	0.15	-0.004 83	-0.201 2	0.109 8
1.2		-0.004 83	-0.226 7	0.067 0
1.8		-0.004 83	-0.240 1	0.055 0

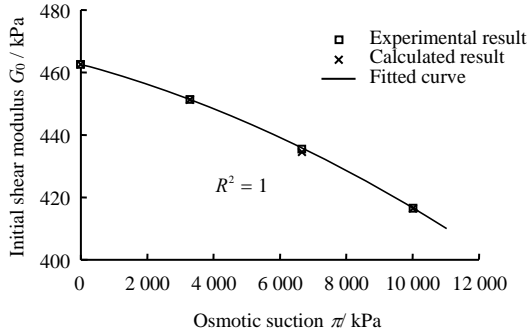
The slope  $M$  of the critical state line and the shear modulus  $G_0$  are obtained by a triaxial shear test. This paper obtains  $M = 1.2$  based on the internal friction angle, without considering the effect of salt content. The shear modulus can be determined by triaxial tests, that is, transient deformation can be considered as purely elastic. Therefore, the equation  $q^* = 3G_0\varepsilon_0$  is valid, where  $\varepsilon_0$  is the initial shear strain. Combined with the triaxial test results, the shear modulus at 0.0, 0.6, 1.2, and 1.8 mol/kg were 5 021.2, 4 886, 4 737.8, and 4 432.7 kPa, respectively.

In order to study the influence of salt content in saturated saline soil on the ratio of secondary consolidation coefficient to compression index  $C_a/C_c$ , initial shear modulus and initial shear strain, the osmotic suction calculated by Eq. (1) is used to quantify the correlation between the consolidation creep characteristics and the salt content of the saturated saline soil. Figures 4 to 6 respectively show the relationship curves between the osmotic suction and the ratio  $C_a/C_c$ , the initial shear modulus  $G_0$ , and the initial shear strain  $\varepsilon_0$ .

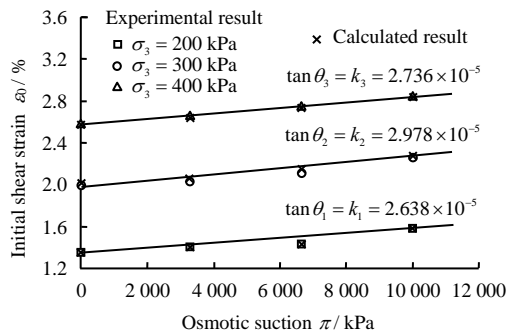
Mesri et al.<sup>[24]</sup> used the ratio of the secondary consolidation coefficient and the compression index  $C_a/C_c$  as a bridge, discussed the volume change during consolidation creep, and found that the  $C_a/C_c$  of the same soil was a fixed value. Zhang et al.<sup>[9]</sup> studied  $C_a/C_c$  of clay after saturated with sodium chloride solution. Figure 4 presents a graph of the  $C_a/C_c$  versus the osmotic suction.



**Fig.4 Relationship between  $C_a/C_c$  and osmotic suction**



**Fig.5 Relationship between initial shear modulus and osmotic suction**



**Fig.6 Relationship between initial shear strain and osmotic suction**

It can be seen from the figure that the  $C_a/C_c$  of saturated saline soil grows slowly with the increase of osmotic suction. The relationship between the two can be fitted with an exponential function.

$$\frac{C_a}{C_c} \times 10^{-2} = 2.612 - 1.089 \exp \left[ -2 \left( \frac{\pi - 621.537}{11\,280.610} \right)^2 \right] \quad (20)$$

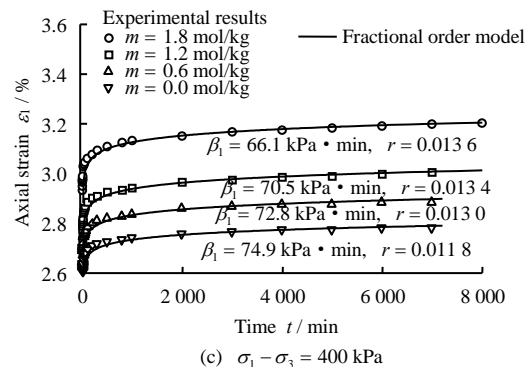
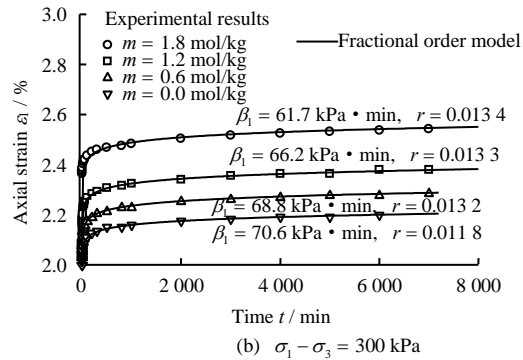
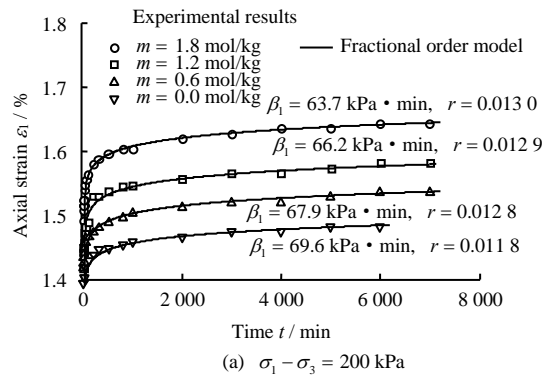
Figure 5 depicts the correlation between the initial shear modulus of the saturated sulfated soil and the corresponding osmotic suction. It is found from the figure that the test results are in good agreement with the model calculation results, and the initial shear modulus decreases slowly with the increase of osmotic suction on the original basis. This paper uses a quadratic polynomial to fit the relationship between the two, where the fitted correlation coefficient is  $R^2 \approx 1$ , and the fitting equation is

$$G_0 = 5.021\,2 - 0.022\,6\pi - 0.003\,5\pi^2 \quad (21)$$

Figure 6 shows the relationship between initial shear strain and osmotic suction at different stress levels. It can be found from the figure that the initial shear strain and osmotic suction of saturated saline soil under different stress conditions show a good linear relationship, and the straight lines under the three stress levels are almost parallel (average slope  $\bar{k} = 2.784 \times 10^{-5}$ ). That is, the stress level only changes the intercept of the line on the strain axis and does not affect the slope of the line. In general, the initial shear strain of saturated

saline soils increases slowly with the increase of osmotic suction, and the model calculation results are in good agreement with the test results. This phenomenon is the same as the research results of Fig.4 and Fig.5 and literature[2–4]. However, the growth relationship between the secondary compression coefficient and osmotic suction shown in Figure 4 is different from the results of Zhang et al. [9].

The comparison between the fitting by the fractional creep model and the triaxial creep test results is shown in Figure 7. The fractional creep model built on the basis of the Abel damping and element model eliminates the dependence of the integer-order element model on the exponential function, improves the linear creep model under the integer-order condition, and realizes the nonlinear creep description of geotechnical materials. From Fig. 7, it can be found that the fractional creep model fits the creep behavior of saturated saline soils well under different stress states and salt contents, and reflects the nonlinear creep process of saturated saline soils.



**Fig.7 Fractional order model and test result analysis**

The correlation among the creep of saturated sulphate saline soils with different salt contents under the same stress state and the viscosity coefficient and fractional order are analyzed. The results show that the viscosity coefficient decreases with increasing salt content, and the fractional order increases with increasing salt content. Among them, the change of the viscosity coefficient with the salt content reflects the correlation between the creep variable and the salt content of the saturated saline soil. The change of fractional order with salt content indicates the mechanism of the effect of salt content on its creep strain rate. That is to say, with the increase of salt content, the degree and speed of viscoplastic readjustment and ionic interaction in saturated saline soil are accelerated, thereby increasing the creep variable and accelerating the creep rate of saline soil.

Comparing the creep conditions at three stress levels, it is found that the fractional order of the creep of saturated non-salinized soil (salt content is 0.0 mol/kg) is the same, i.e.,  $r = 0.0118$ . This is the same as the case of the secondary compression coefficient of non-salinized soil in Table 1, indicating that the effect of stress level on the creep effect is less obvious. The difference in the order  $r$  under different salt conditions reflects that the salt content has a certain effect on the creep behavior of the soil.

In order to clarify the influence of salt content on the viscosity coefficient, the method of determining the viscosity coefficient is discussed. According to the research results of existing component models, it is found that there is a clear correlation between the viscosity coefficient and factors such as stress level and temperature [25–26]. Based on the results of the study in Figure 7, a solution formula for the viscoplastic coefficient is constructed by combining the deviatoric stress and osmotic suction:

$$\beta_1 = \beta_1^0 + b_{11} \exp(b_{12} q_a^*) - c_{11} \pi - c_{12} \pi^2 \quad (22)$$

where  $q_a^*$  is the shear stress without considering effective seepage stress;  $\beta_1^0$  is the initial viscosity coefficient;  $b_{11}$  and  $b_{12}$  are the coefficients of  $\beta_1$  interaction with stress levels in different soil structures;  $c_{11}$  and  $c_{12}$  are the coefficients of influence of osmotic suction on parameters  $\beta_1$ .

Figure 8 shows the comparison between the calculation result using Eq. (22) and the viscosity coefficient  $\beta_1$  in Figure 7. The value of the parameter is obtained by solving the equation with the fitting result. According to the results shown in the figure, only large deviations occur at  $\sigma_1 - \sigma_3 = 200$  kPa,  $m = 1.2$  mol/kg and  $m = 1.8$  mol/kg, while the calculation results in other cases are very close to the fitting results. Therefore, it is shown that Eq. (22) is effective for calculating the viscosity coefficient.

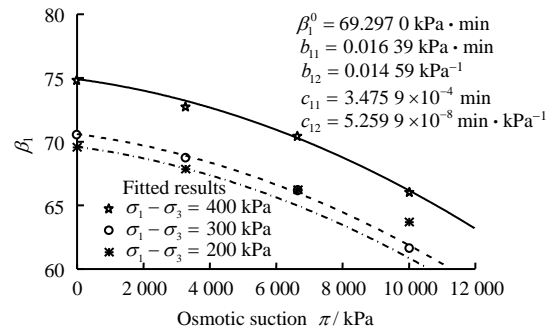
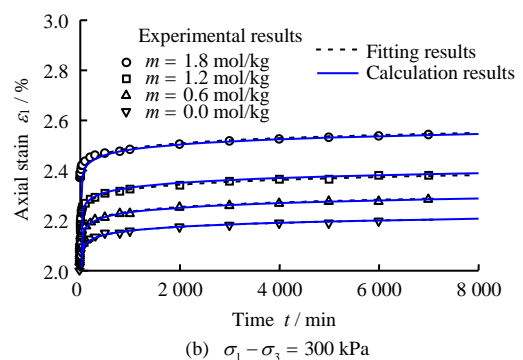
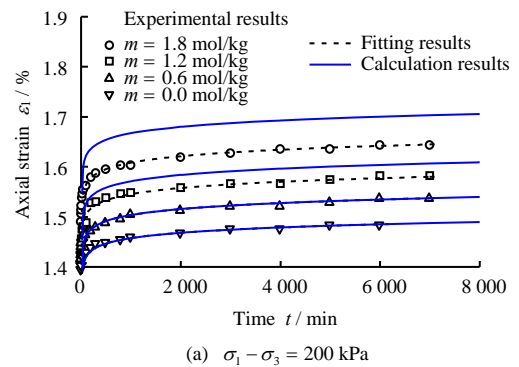


Fig.8 Comparison between  $\beta_1$  calculation results and the fitted results

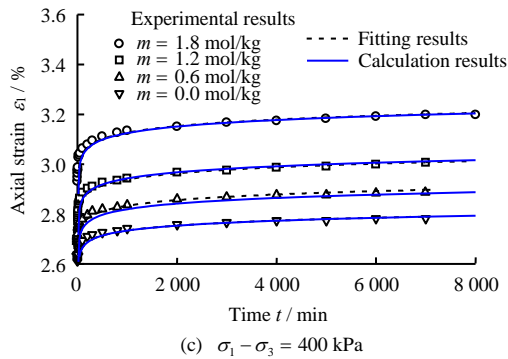
### 5.2 Model validation

Figure 9 is a comparison between the calculation result of the model after calculating the viscosity coefficient by using Eq. (22) and the fitting result of Fig. 7.

From the figure, it can be found that the calculation results, fitting results and test results under other conditions are in good agreement except that the calculation results under the action of  $\sigma_1 - \sigma_3 = 200$  kPa and  $m = 1.2$  mol/kg, 1.8 mol/kg have obvious errors. The above results are similar to the comparison results of Fig. 8. According to the comparison results of Fig. 8 and Fig. 9, it is found that the error at 200 kPa is most likely caused by the test operation (such as the difference in compaction during the sampling process), and it is unlikely to be Eq. (22) calculation error. Therefore, it can be considered that the Eq. (22) and the fractional creep model of saturated saline soil are effective.







**Fig.9 Comparison between model calculation results and fitting results**

## 6 Conclusions

In this paper, consolidation tests and triaxial shear creep tests were performed on soils saturated with three solutions. Secondly, a series of studies on the influence of osmotic suction on the consolidation creep characteristics of saturated saline soils were carried out based on the fractional creep model based on osmotic suction and Abel damper and Bingham model. Among them, the main research conclusions are summarized as follows:

The ratio of the secondary consolidation coefficient to the compression index  $C_a/C_c$  increases with the increase in osmotic suction, and the expressions of the two can be expressed by an exponential function. The initial shear modulus and initial shear strain show decreasing and increasing trends with the increase of osmotic suction, and the two and the permeability coefficient are quadratic and linear, respectively. Among them, the mechanism for  $C_a/C_c$  change with osmotic suction reflects that salt content and salt species have different degrees of promoting effect on the creep behavior of saline soil. The relationship between the initial shear modulus and the osmotic suction shows that the increase of salt content will reduce the shear strength of soil, while the increase of initial shear strain with the osmotic suction reflects the contribution of salt content to the strain effect of soil.

The fractional-order creep model of saturated saline soil is based on the improvement of the original element model to achieve the description of the creep behavior of saturated saline soil under different salt contents. By comparing with the experimental results, it is found that the model can predict the creep behavior of saturated saline soils. The viscosity coefficient formulas are given and effectiveness of fractional-order models of saturated saline soils under different stress states is verified according to the fitted results of parameters. Combining the trend of viscosity coefficient  $\beta_1$  and order  $r$  with osmotic suction, it is found that the creep and creep strain rate of saline soils increase with increasing salt content.

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