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Study on the mechanism of coal bursts induced by quasi-resonance of roof-support system

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Abstract: The mechanism and prevention of coal burst is an important issue in the study of coal mine dynamic disasters. The disturbance of surrounding rocks can trigger the quasi-resonance phenomena of roadway roof support system in close proximity to its resonant state, ultimately inducing impact disasters. This is manifested in three strong quasi-resonant responses, including roof displacement, velocity and acceleration. Based on the support instability and plastic buckling criteria, three safety factors and the corresponding dangerous disturbance frequency ratio interval for the dynamic failure of support are proposed, and the support damping control strategy is given, aiming at the impact disasters induced by the quasi-resonance of the roof support system. The results show that when the amplitude of disturbing force is lower than the critical failure force of the support under static loads, the quasi-resonant phenomena of roof support system is the main reason for the dynamic failure of support. The frequency ratio range of quasi-resonant dangerous disturbance for support failure is related to the damping ratio and force-amplitude ratio of roof support system. With the increasing damping ratio, the displacement, velocity and acceleration response amplitudes at the roof quasi-resonance will decrease. The support damping is a controlling factor sensitive to these three types of quasi-resonant responses. When the damping ratio in the roof support system increases or the force-amplitude ratio (the ratio between the amplitude of external disturbance and the critical load for static support failure) decreases, the dangerous disturbance frequency ratio interval of the support will narrow or even disappear. When the damping ratio and force-amplitude ratio are small, the dangerous disturbance frequency ratio intervals of the roof support system under three quasi-resonant conditions are close. This study will enrich the understanding of the mechanism behind coal bursts and provide valuable insights for the design of support control.

Keywords: coal bursts; quasi-resonance of roof-support system; support failure; control of preventing coal burst

1 Introduction

The severe threat of dynamic disasters caused by coal bursts poses a significant risk to the safe extraction of coal resources. Both domestically and internationally, research on the occurrence mechanisms and prevention of coal bursts continues to evolve and expand. Classic theories on the mechanism of coal bursts include the strength theory^[1–2], stiffness theory^[3], energy theory^[4], “three criteria” theory^[5], “three factors” theory^[6], instability theory^[7], and burst proneness theory for coal and rock masses^[8]. In recent years, significant progress has been made in the study of the mechanism of coal bursts. Pan et al.^[9] utilized catastrophe theory to obtain the necessary and sufficient conditions for the occurrence of coal bursts. Dou et al.^[10] proposed the theory of strength weakening and shock reduction for coal bursts. Pan et al.^[11] put forward the theory of impact energy initiation. Jiang et al.^[12] adopted a double-shear test model to elucidate the three stages of sliding instability, incubation, and initiation of bursts

in coal-rock structures. Based on the morphology of the plastic zone in rock masses, Ma et al.^[13] developed the three criteria for butterfly-shaped impacts. Wang et al.^[14] analyzed the burst mechanism induced by stress waves. Ju et al.^[15] reviewed the research progress on the qualitative analysis and quantitative evaluation of burst tendency in coal-rocks, and proposed a comprehensive evaluation index. Tan et al.^[16] defined a kinetic energy index to assess impact hazards based on the principle of mining disturbance. Wu et al.^[17] analyzed the occurrence mechanism of coal bursts and the mechanical energy of mining-induced seismic activities in the rift valley structural zone. Yang et al.^[18] conducted experimental studies on the impact failure characteristics of coal-rock specimens under true triaxial compression with one free face. These theories examine the complete process of coal bursts from the properties of coal and rock masses and geological structures, as well as mining disturbance factors, providing prominent theoretical basis for understanding the mechanism of

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coal bursts. Kurlenya et al.^[19] found that deep rock masses exhibit quasi-resonant phenomena, characterized by alternating displacement of rock blocks, when subjected to blasting disturbance, through monitoring and analyzing the response of surrounding rocks to blasting. Kurlenya et al.^[20] analyzed the release energy and energy conversion of the system during quasi-resonance of blocky rock masses. Qian^[21] pointed out the quasi-resonant phenomena in his research on the movement patterns of block rock masses, where the magnitude of the rock's vibration response varies over time, demonstrating frequency characteristics. When the frequency of the exciting dynamic impulse reaches the critical value, it can be predicted that the vibration of rock blocks will intensify. Aleksandrova et al.^[22–24] presented a dynamic model for blocky media and explored the frequency dispersion and quasi-resonant characteristics of dynamic stress propagation in blocky media. Wu et al.^[25] analyzed the quasi-resonant phenomena of rock masses under perturbation. During the coal mining process, the quasi-resonant phenomena exist in roadway surrounding rocks and support systems under surrounding rock disturbance. Currently, there is no quasi-resonance-based explanation for the mechanism of coal bursts in roof support systems. Furthermore, significant progress has been made recently in research on energy-absorbing supports for preventing and controlling coal bursts. Pan et al.^[26–28] proposed the theory of anti-impact energy-absorbing supports based on the dynamic model of overlying strata and support system, established a coupling model of impact energy absorption and supporting, and put forward a three-level energy-absorbing support system. Ouyang et al.^[29] conducted the analysis on the dynamic response laws of roadway surrounding rocks and support system under impacts at different positions above the roadway. Gao et al.^[30] regarded that the use of shock-absorbing support components with strong scattering and energy dissipation effects can improve the supporting effect of roadways. Wu et al.^[31] found that high damping can quickly suppress the impact vibration of roadway surrounding rocks. The mechanism of support damping for controlling impact failure induced by quasi-resonance in the roof support system needs further research. In summary, this study on the impact mechanism induced by quasi-resonance in the roof support system and corresponding control measures will enrich and advance the understanding of the occurrence mechanism of coal bursts, and provide guidance for the design of impact-resistant

supports under quasi-resonance in the roof.

2 Dynamic response and quasi-resonance analysis of roof support system

2.1 Dynamic response model of roof support system

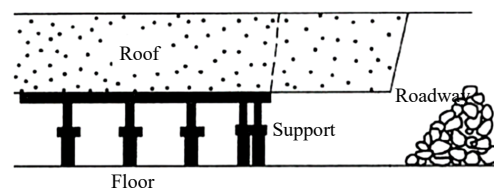
After roadway excavation, the support and the roof and floor form a mechanically integrated system for bearing loads. Qian et al.^[32] presented a schematic model of the roof-support system, as shown in Fig. 1(a). Pan et al.^[26] established a dynamic mechanical model of the roof-support system, as shown in Fig. 1(b), in which the support stiffness is denoted by k , the support damping coefficient by c , and the roof mass by m . The roof is subjected to impact loads induced by mining disturbances and other factors. According to the micro-seismic monitoring of surrounding rocks, dynamic loads propagate with certain disturbance amplitude and frequency within rock masses. For theoretical analysis purposes, the continuous disturbance load $f(t)$ acting on the roof is assumed as

$$f(t) = P \sin \omega t \quad (1)$$

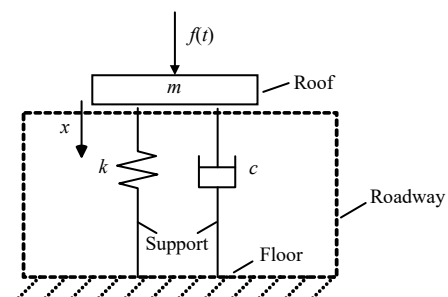
where P and ω represent the amplitude and frequency of disturbance force, respectively. Assuming that the floor remains fixed, the dynamic response equation of the roof-support system along the impact disturbance direction is

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (2)$$

where x represents the impact displacement of the roof. As indicated by Eq. (2), the impact displacement response of the roof is



(a) Interaction model between roof and support



(b) Dynamic mechanical model of the roof support system

Fig. 1 Roadway roof support system

$$x = x_p \sin(\omega t - \varphi) \tag{3}$$

where x_p represents the amplitude of the roof displacement, and φ denotes the phase difference of the vibration response.

$$x_p = \frac{P}{k\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \tag{4}$$

$$\varphi = \arctan \frac{2\zeta\lambda}{1-\lambda^2} \tag{5}$$

where $\lambda = \omega/\omega_n$ represents the frequency ratio, $\omega_n = \sqrt{k/m}$ represents the natural frequency of the system, and $\zeta = c/(2m\omega_n)$ represents the damping ratio of the roof-support system. If damping is neglected, i.e., $\zeta = 0$, the system exhibits a strictly resonant phenomenon at the frequency ratio of $\lambda = 1$, and the impact displacement of the roof becomes infinitely large. In general, the system experiences damping, and the damping ratio satisfies $0 < \zeta < 1$. At the resonant frequency, the impact displacement of the roof does not tend towards infinity, instead performs a displacement extreme value that varies with frequency. When the external disturbance frequency approaches the natural frequency of the system, the displacement amplitude of the roof gradually increases and approaches the extreme value, resulting in a quasi-resonant phenomenon close to the resonant state. This quasi-resonant phenomenon will cause impact damage to the roof support system.

2.2 Displacement quasi-resonance of the roof-support system

The quasi-resonant phenomena of the roof-support system are closely related to the occurrence of coal bursts. As indicated by Eq. (4), the frequency ω_1 at which x_p reaches the extreme value is given by

$$\omega_1 = \omega_n \sqrt{1 - 2\zeta^2} = \sqrt{\frac{k(1 - 2\zeta^2)}{m}} \tag{6}$$

When the external disturbance frequency satisfies Eq. (6), the displacement of the roof along the disturbance direction reaches the maximum value. Meanwhile, when the disturbance frequency approaches ω_1 , quasi-resonant phenomena of the roof displacement will occur. $0 < \zeta < 1/\sqrt{2}$ must be satisfied to ensure the existence of the frequency in Eq. (6), and then the maximum displacement of the roof x_{pmax} is expressed as

$$x_{pmax} = \frac{2Pm}{c\sqrt{4km - c^2}} \tag{7}$$

Here is an analysis of the effect of the parameters of the roof-support system on x_{pmax} in Eq. (7). The variations

of the maximum displacement x_{pmax} of the roof with the external disturbance amplitude P , roof mass m , support stiffness k , and support damping c are written as follows:

$$\frac{\partial x_{pmax}}{\partial P} = \frac{2m}{c\sqrt{4km - c^2}} > 0 \tag{8}$$

$$\frac{\partial x_{pmax}}{\partial m} = \frac{2P(2km - c^2)}{c\sqrt{(4km - c^2)^3}} > 0 \tag{9}$$

$$\frac{\partial x_{pmax}}{\partial k} = \frac{-4Pm^2}{c\sqrt{(4km - c^2)^3}} < 0 \tag{10}$$

$$\frac{\partial x_{pmax}}{\partial c} = \frac{4Pm(c^2 - 2km)}{c^2\sqrt{(4km - c^2)^3}} < 0 \tag{11}$$

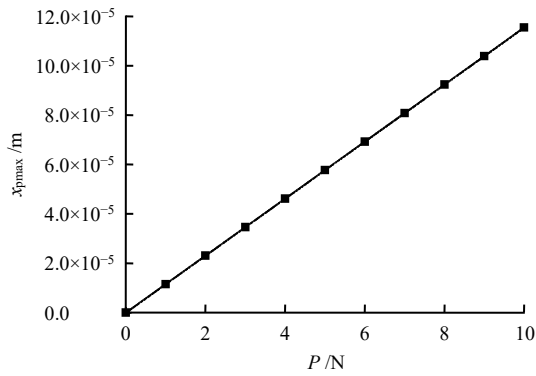
Since $0 < \zeta < 1/\sqrt{2}$ and $\zeta = c/(2m\omega_n)$, $c^2 < 2km$ can be obtained, and whether it is positive or negative can be determined.

From Eq. (8) to (11), one can see that the maximum displacement of the roof x_{pmax} is positively correlated with P and m , and negatively correlated with k and c . The variations of support damping and stiffness are compared:

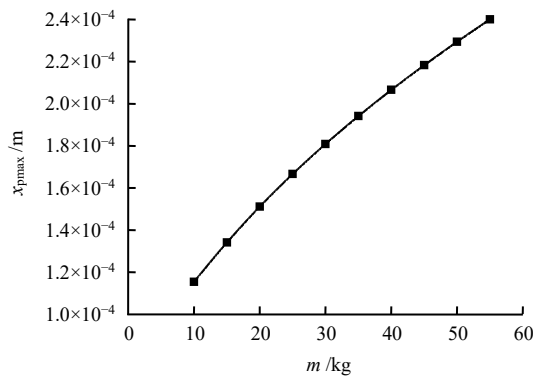
$$\frac{\frac{\partial x_{pmax}}{\partial c}}{\frac{\partial x_{pmax}}{\partial k}} = \frac{2km - c^2}{cm} \tag{12}$$

Theoretically, increasing k and c can effectively suppress the maximum displacement x_{pmax} of the roof. From Eq. (12), it can be seen that when the value of c satisfies $0 < c < \frac{-m + \sqrt{m^2 + 8km}}{2}$, thus $\frac{2km - c^2}{cm} > 1$. It can be inferred that damping has a greater effect on the decrease of x_{pmax} within this range of c . To demonstrate the effect of the parameters of the roof-support system on x_{pmax} , a calculation analysis is carried out below by taking the following parameters into account: $m = 10$ kg, $k = 1 \times 10^5$ kg /s², $P = 10$ N, and $c = 1 \times 10^3$ kg /s. The relationships between P , m , k , c and x_{pmax} are shown in Fig. 2.

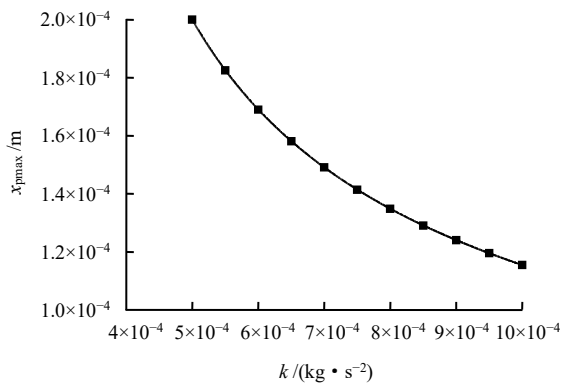
From Fig. 2, it is found that reducing the amplitude of external disturbance P or cutting roof to decrease the roof mass borne by support structures will be beneficial in reducing the maximum impact displacement x_{pmax} of the roof. This study primarily focuses on support control, without further discussion on control strategies for the roof. As x_{pmax} exhibits negative correlation with k and c , enhancing support performance (including stiffness and damping) can effectively suppress the impact displacement of the roof in the roof-support system.



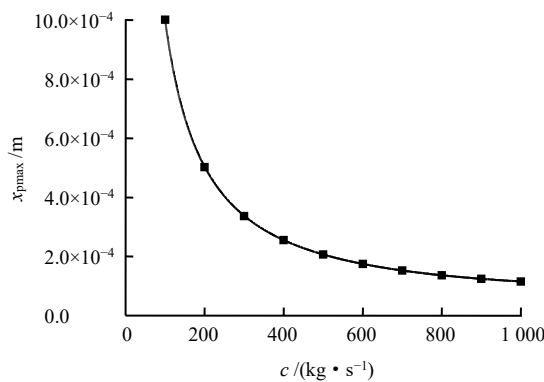
(a) Relationship between the amplitude of disturbance force P and the maximum displacement x_{pmax} of the roof.



(b) Relationship between the roof mass m and the maximum displacement x_{pmax} of the roof



(c) Relationship between the support stiffness k and the maximum displacement x_{pmax} of the roof



(d) Relationship between the support damping c and the maximum displacement x_{pmax} of the roof

Fig. 2 Relationships between maximum displacements x_{pmax} and influencing factors

2.3 Velocity quasi-resonant of the roof-support system

According to Eq. (3), the velocity response of the roof is

$$\dot{x} = v_p \cos(\omega t - \varphi) \tag{13}$$

where v_p represents the velocity amplitude of the roof, and t represents time. The calculation formula for v_p is

$$v_p = x_p \omega = \frac{P \omega}{k \sqrt{(1 - \lambda^2)^2 + (2\zeta\lambda)^2}} \tag{14}$$

The external disturbance frequency at the maximum roof velocity by differentiating Eq. (14) with respect to frequency can be obtained.

$$\omega_2 = \omega_n = \sqrt{\frac{k}{m}} \tag{15}$$

When the external disturbance frequency satisfies Eq. (15), the roof experiences velocity resonance and the velocity peaks.

$$v_{pmax} = \frac{P}{c} \tag{16}$$

When the external disturbance frequency approaches ω_2 , the velocity quasi-resonant phenomenon of the roof will occur.

According to Eq. (16), it can be seen that the maximum velocity of the roof v_{pmax} is proportional to P and inversely proportional to c . Based on the calculation parameters in Section 2.2, these relationships are illustrated in Fig. 3.

Figure 3 shows that v_{pmax} is positively correlated with P and negatively correlated with c . Therefore, the velocity response of the roof can be suppressed by increasing the damping of the support system.

2.4 Acceleration quasi-resonance of the roof-support system.

The acceleration response of the roof can be obtained from Eq. (3) as follows:

$$\ddot{x} = -a_p \sin(\omega t - \varphi) \tag{17}$$

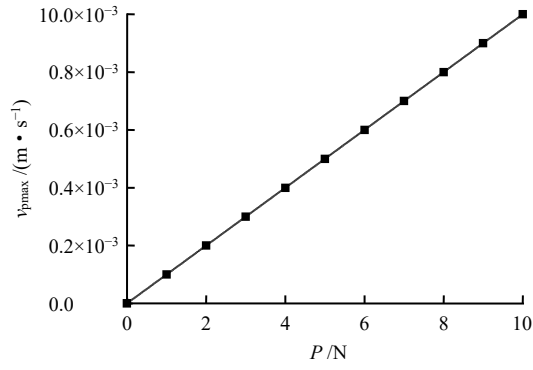
where a_p is the amplitude of the roof acceleration, it can be acquired from the following equation:

$$a_p = x_p \omega^2 = \frac{\lambda^2 P}{m \sqrt{(1 - \lambda^2)^2 + (2\zeta\lambda)^2}} \tag{18}$$

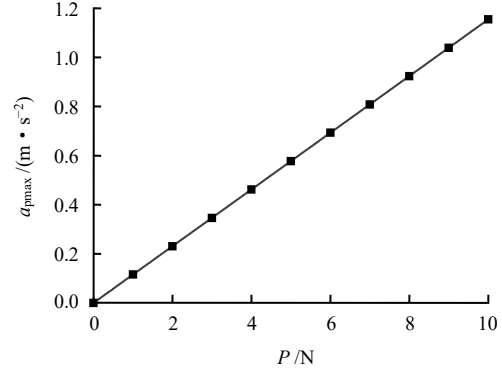
The external disturbance frequency at which Eq. (18) peaks satisfies:

$$\omega_3 = \frac{\omega_n}{\sqrt{1 - 2\zeta^2}} = \sqrt{\frac{k}{m(1 - 2\zeta^2)}} \tag{19}$$

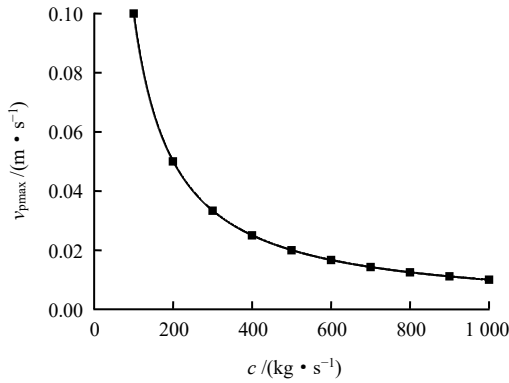
As the external disturbance frequency satisfies Eq. (19), the roof undergoes acceleration resonance with



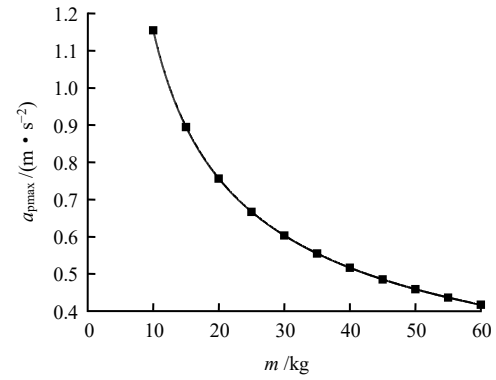
(a) Relationship between the amplitude of disturbance force P and the maximum velocity v_{pmax} of the roof.



(a) Relationship between the amplitude of disturbance force P and the maximum acceleration a_{pmax} of the roof.



(b) Relationship between the damping of the support system c and the maximum velocity v_{pmax} of the roof



(b) Relationship between the roof mass m and the maximum acceleration a_{pmax} of the roof

Fig.3 Relationship between maximum velocities v_{pmax} and influencing factors

the maximum acceleration a_{pmax} :

$$a_{pmax} = \frac{2Pk}{c\sqrt{4km - c^2}} \quad (20)$$

When the external disturbance frequency approaches ω_b , the acceleration quasi-resonant phenomenon of the roof will occur. The maximum acceleration of the roof a_{pmax} varies with P , m , k , and c as follows:

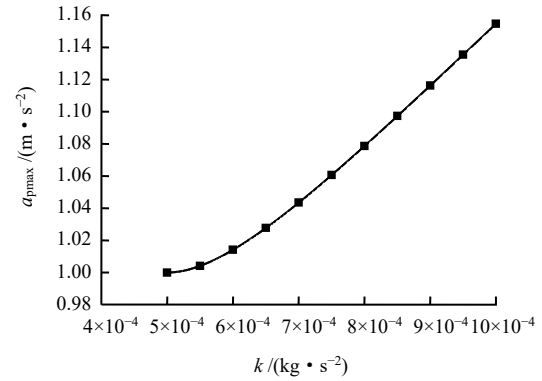
$$\frac{\partial a_{pmax}}{\partial P} = \frac{2k}{c\sqrt{4km - c^2}} > 0 \quad (21)$$

$$\frac{\partial a_{pmax}}{\partial m} = \frac{-4Pk^2}{c\sqrt{(4km - c^2)^3}} < 0 \quad (22)$$

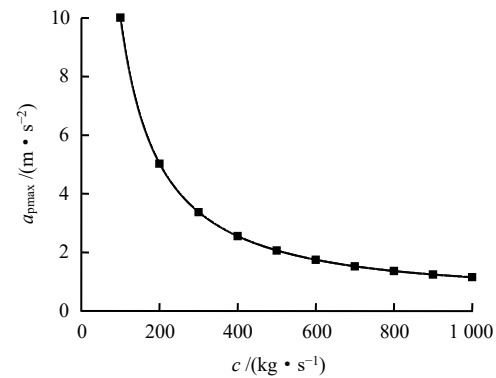
$$\frac{\partial a_{pmax}}{\partial k} = \frac{2P(2km - c^2)}{c\sqrt{(4km - c^2)^3}} > 0 \quad (23)$$

$$\frac{\partial a_{pmax}}{\partial c} = \frac{4Pk(c^2 - 2km)}{c^2\sqrt{(4km - c^2)^3}} < 0 \quad (24)$$

Based on the calculated parameters in Section 2.2, the variation of a_{pmax} with system parameters is presented in Fig. 4. It can be seen that a_{pmax} is positively correlated with P and k , and negatively correlated with m and c .



(c) Relationship between the stiffness k of the support and the maximum acceleration a_{pmax} of the roof



(d) Relationship between the support damping c and the maximum acceleration a_{pmax} of the roof

Fig. 4 Relationship between maximum accelerations a_{pmax} and influencing factors

In summary, three different quasi-resonant responses will cause impact damage to the roof-support system. The damping of the support system exerts a repressive effect on the amplitude of these three quasi-resonant responses.

3 Assessment and controls of quasi-resonant impact hazards in the roof-support system

It is clear from Section 2 that the displacement, velocity, and acceleration of the roof peak at their respective resonant frequencies of the system, and when the external disturbance frequency is close to the resonant frequency of the system, the amplitudes of these parameters increase sharply, resulting in a quasi-resonant phenomenon that approximates the resonance state. Based on the three quasi-resonant response characteristics of the roof in the roof-support system, the impact failure conditions and risk assessment of the support in the roof-support system are analyzed. Figure 5 depicts a site of damage to tunnel support induced by roof impact during underground mining, which may be resulted from support instability or plastic buckling. Therefore, based on the member buckling criterion, Section 3.1 will provide the safety factor of support structures under the conditions of roof displacement resonance and acceleration resonance through a mechanical analysis on support. Additionally, Section 3.2 will provide a safety factor for the occurrence of buckling failure induced by plastic energy absorption in support structures under the condition of velocity resonance, based on the energy destruction criterion of support.

3.1 Support instability induced by roof quasi-resonance and the controls against coal bursts

3.1.1 Support instability induced by the displacement quasi-resonance of the roof

Under the roof impact, the force F_s exerted on the support structure is

$$F_s = k\Delta x \quad (25)$$



Fig. 5 Scene of roadway support failure impacted by the roof

where Δx is the deformation of the support. Due to the roof being disturbed by the impact and the floor being assumed to be fixed, it can be inferred from Eq. (3) that $\Delta x = x_p \sin(\omega t - \varphi)$.

When roof displacement reaches a quasi-resonant state, the deformation Δx of the supporting structure will rapidly increase and approach Eq. (4). If the support structure is subjected to a critical force F_{cr} that exceeds its stability limit, it will undergo unstable failure.

$$F_s = kx_p \geq F_{cr} \quad (26)$$

where F_{cr} represents the critical force at which the support structure becomes unstable. If the individual pillar support in the tunnel can be regarded as a compression rod fixed at both ends, the critical force at the instability of the compression rod is

$$F_{cr} = (\pi^2 EI) / l^2 \quad (27)$$

where E represents the support stiffness, I denotes the inertia moment of the cross section of support structure, and l is the support structure length. Substituting x_p of Eq. (4) into Eq. (26) gives

$$\frac{P / F_{cr}}{\sqrt{(1 - \lambda^2)^2 + (2\zeta\lambda)^2}} \geq 1 \quad (28)$$

The amplitude ratio of the force is defined as $\beta = P / F_{cr}$. Only the case where $\beta < 1$ is discussed herein, as if $\beta \geq 1$, the initial load P has already reached the critical value F_{cr} for support instability, resulting in natural instability and failure of the support. Additionally, the safety factor α_1 for the quasi-resonance of roof displacement is defined as

$$\alpha_1 = \frac{\beta}{\sqrt{(1 - \lambda^2)^2 + (2\zeta\lambda)^2}} \quad (29)$$

Therefore, the support will experience impact instability failure when the safety factor α_1 is greater than or equal to 1. The resonance of the roof is closely related to the frequency ratio λ . When the external disturbance frequency approaches the natural frequency of the system, and quasi-resonance occurs, λ approaches 1.

Based on the safety factor for support impact instability failure given by Eq. (29), the assessment of the impact risk of support based on the external disturbance frequency ratio λ will be conducted below. If the damping ratio ζ satisfies:

$$\zeta > \sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \beta^2}} \quad (30)$$

for $\alpha_1 \geq 1$, there is no solution for λ , indicating that the roof support system remains consistently stable with

the variations in the external disturbance frequency. If the damping ratio ζ satisfies

$$\zeta \leq \sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \beta^2}} \quad (31)$$

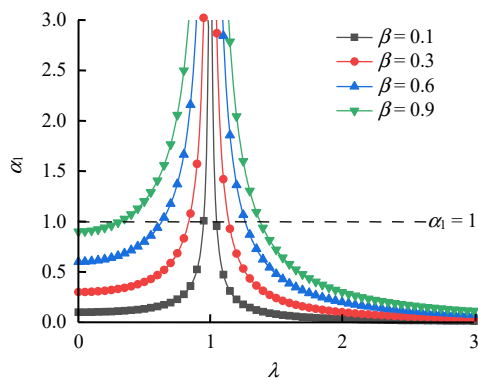
there exists a solution for λ when $\alpha_1 \geq 1$.

At this moment, the impact risk frequency ratio interval of the support under the quasi-resonant response of the roof displacement exists:

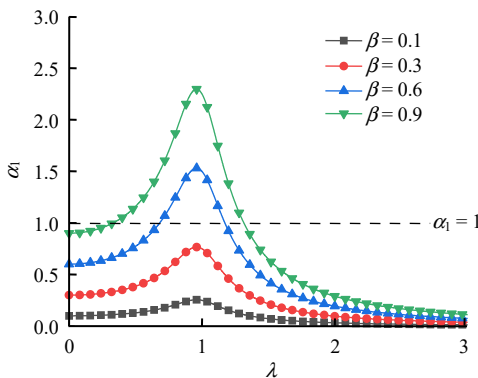
$$\lambda_1 \leq \lambda \leq \lambda_2 \quad (32)$$

where, $\lambda_{1,2} = \sqrt{(1 - 2\zeta^2) \mp \sqrt{(2\zeta^2 - 1)^2 - (1 - \beta^2)}}$.

The aforementioned analysis indicates that the instability and failure of the support induced by the displacement quasi-resonance of the roof not only depends on the frequency ratio λ but also on the damping ratio ζ of the system. Increasing damping can suppress the instability and failure of the support induced by the displacement quasi-resonance of the roof. Meanwhile, it can be found from Eq. (32) that in case of unstable failure of the support, the interval of disturbance frequency ratio for rock masses with potential impact risks is related to ζ and β . The relationship between safety factor α_1 and disturbance frequency ratio λ is calculated at varying ζ and β using Eq. (29), as displayed in Fig. 6.



(a) $\zeta = 0.01$



(b) $\zeta = 0.20$

Fig. 6 Relation curves between the impact risk frequency ratio and safety coefficient of roof displacement quasi-resonance

From Fig. 6, it can be observed that the corresponding external disturbance frequency at $\lambda = 1$ is equal to the natural frequency of the system. When the frequency ratio λ is close to 1, quasi-resonance will be generated, leading to a significant increase in safety factor α_1 , and the instability failure of the support ($\alpha_1 \geq 1$) will occur. The disturbance frequency ratio for support instability failure is related to ζ and β . For $\zeta = 0.01$, the values of β (0.1, 0.3, 0.6, 0.9) in Fig. 6 and their relationships with ζ satisfy Eq. (31). Therefore, it can be inferred that the interval of the disturbance frequency ratio for rock masses with potential impact risks satisfies Eq. (32), as listed in Table 1. For $\zeta = 0.2$, the relationship between β and ζ in Fig. 6 satisfies Eq. (30) for $\beta = 0.1$ and 0.3. The roof-support system is consistently stable under displacement quasi-resonance ($\alpha_1 < 1$). However, the relationship for $\beta = 0.6$ and 0.9 satisfies Eq. (31), and it can be inferred that the interval of the disturbance frequency ratio for rock masses with potential impact risks satisfies Eq. (32), as presented in Table 1. Therefore, increasing support damping can narrow and even eliminate the interval of the risk frequency ratio. Moreover, when ζ is constant and β decreases, the endpoint of the interval of the disturbance frequency ratio for rock masses with support instability failure moves closer to $\lambda = 1$.

Table 1 Disturbance frequency ratio intervals of support instability failure under roof displacement quasi-resonance

β	$\zeta = 0.01$			$\zeta = 0.20$		
	λ_1	λ_2	$\Delta\lambda$	λ_1	λ_2	$\Delta\lambda$
0.1	0.95	1.05	0.10	—	—	0
0.3	0.84	1.14	0.30	—	—	0
0.6	0.63	1.27	0.64	0.68	1.17	0.49
0.9	0.31	1.38	1.07	0.33	1.32	0.99

3.1.2 Support instability induced by acceleration quasi-resonance of the roof

The impact response of the roof under the perturbation load $f(t)$ is

$$ma = f(t) - F_s \quad (33)$$

where a represents the roof impact acceleration. According to Eq. (2), F_s includes elastic and damping forces. If the supporting force satisfies a condition of $F_s \geq F_{cr}$, then the support will experience unstable failure.

$$F_s = f(t) - ma \geq F_{cr} \quad (34)$$

Substituting the roof acceleration in Eq. (17) and $f(t) = P \sin \omega t$ into Eq. (34) yields

$$P \sin \omega t + m a_p \sin(\omega t - \varphi) \geq F_{cr} \quad (35)$$

By substituting a_p in Eq. (18) and φ in Eq. (5) into

Eq. (35), the discriminant for determining the destabilization failure of the support induced by acceleration quasi-resonance of the roof can be given:

$$\beta^2 + \frac{2\beta^2\lambda^2}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \cos\left(\arctan \frac{2\zeta\lambda}{1-\lambda^2}\right) + \left(\frac{\beta\lambda^2}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}}\right)^2 \geq 1 \quad (36)$$

A support safety factor α_2 for acceleration quasi-resonance of the roof is defined as

$$\alpha_2 = \beta^2 + \frac{2\beta^2\lambda^2}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \cos\left(\arctan \frac{2\zeta\lambda}{1-\lambda^2}\right) + \left(\frac{\beta\lambda^2}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}}\right)^2 \quad (37)$$

If the safety factor α_2 is greater than or equal to 1, the support will experience impact instability. This can be deduced from Eqs. (18) and (5):

$$\lim_{\lambda \rightarrow \infty} a_p = \frac{P}{m}, \quad \text{且} \quad \lim_{\lambda \rightarrow \infty} \varphi = 0 \quad (38)$$

From Eq. (38), it can be inferred that as the disturbance frequency on the roof increases ($\lambda \rightarrow \infty$), the amplitude of acceleration a_p gradually approaches a constant value rather than decaying to zero. Substituting Eq. (38) into Eq. (35) and through Eq. (36), it can be determined that when force-amplitude ratio β satisfies

$$\beta \geq 0.5 \quad (39)$$

If the disturbance frequency ratio exceeds or equals 1, the support will experience unstable failure, indicating that high-frequency disturbances pose a risk of impact.

Meanwhile, it can be found from the safety factor α_2 of Eq. (37) that:

$$\lim_{\lambda \rightarrow \infty} \alpha_2 = 4\beta^2 \quad (40)$$

Equation (40) also indicates that when $\beta \geq 0.5$ and $\alpha_2 > 1$, there is a risk of impact. However, when $\beta < 0.5$, high-frequency disturbances will no longer pose a risk of impact. It can be inferred from Eq. (37) that under the condition of acceleration quasi-resonance of the roof, the endpoints of the disturbance frequency ratio (λ) interval that causes instability on the support satisfies Eq. (41):

$$\left. \begin{aligned} &\beta^2 + \frac{2\beta^2\lambda^2}{\sqrt{\lambda^4 + (4\zeta^2 - 2)\lambda^2 + 1}} \cos\left(\arctan \frac{2\zeta\lambda}{1-\lambda^2}\right) + \left(\frac{\beta\lambda^2}{\sqrt{\lambda^4 + (4\zeta^2 - 2)\lambda^2 + 1}}\right)^2 \\ &\alpha_2 = 1 \end{aligned} \right\} \quad (41)$$

From Eq. (41), it can be inferred that the value of λ is related to ζ and β . The relationships between the disturbance frequency ratio λ and the support impact safety factor α_2 of the roof acceleration quasi-resonance under varying ζ and β are derived, as shown in Fig. 7.

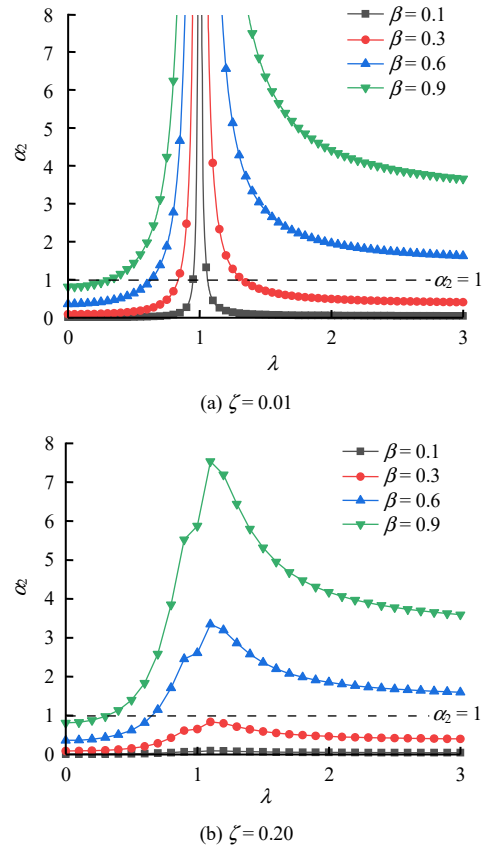


Fig. 7 Relation curves between the impact risk frequency ratio and safety factor of roof acceleration quasi-resonance

Figure 7 illustrates that quasi-resonance occurs when the disturbance frequency ratio λ is close to 1, and the safety factor α_2 increases significantly. As $\alpha_2 \geq 1$, the support will experience instability and failure. The disturbance frequency ratio λ at the instability and failure of support is related to the damping ratio ζ and force amplitude ratio β . According to Eq. (39), when $\beta \geq 0.5$, instability and failure of support occur in the interval of $\lambda \geq 1$. In Fig. 7, for $\beta = 0.6$ and 0.9 , the right end of the interval for instability and failure of support tends towards infinity, while the left end satisfies Eq. (41). As ζ increases or β decreases, the bounded ends of the interval approach $\lambda = 1$, as shown in Table 2. When $\beta < 0.5$, the end points λ_1 and λ_2 of the frequency ratio interval satisfy Eq. (41), where $\Delta\lambda = \lambda_2 - \lambda_1$. As ζ increases, the frequency ratio interval where failure occurs will shrink and may even disappear. When ζ is small (0.01), as β decreases, the disturbance frequency ratio causing instability and failure of support approaches $\lambda = 1$, as listed in Table 2.

Table 2 Disturbance frequency ratio intervals of support instability failure under roof acceleration quasi-resonance

β	$\zeta = 0.01$			$\zeta = 0.20$		
	λ_1	λ_2	$\Delta\lambda$	λ_1	λ_2	$\Delta\lambda$
0.1	0.95	1.06	0.11	—	—	0
0.3	0.84	1.32	0.48	—	—	0
0.6	0.64	∞	∞	0.66	∞	∞
0.9	0.32	∞	∞	0.32	∞	∞

3.2 Support buckling failure induced by roof velocity quasi-resonance and the controls against coal bursts

The analysis of the support failure based on the energy conversion between the roof and the support during the velocity quasi-resonance of the roof is presented below. During the impact response process of the roof, its kinetic energy will be converted into strain energy of the support. When the elastic energy storage limit of the support is reached, if there is still residual impact energy in the roof, plastic deformation will occur in the support to dissipate the remaining energy, and subsequently, leading to buckling failure in the support. The velocity resonance will cause a sharp increase in the kinetic energy of the roof. From an energy conversion perspective, when the kinetic energy transferred from the roof to the support exceeds the elastic strain energy limit of the support, buckling failure will occur. Therefore, the energy condition of the support failure is

$$E_k \geq E_{ce} \tag{42}$$

where E_{ce} represents the maximum value of elastic strain energy stored by the support, E_k is the kinetic energy of the roof, as indicated by Eq. (2), which takes the damping effect of the support into account. From the velocity response of the roof, i.e. Eq. (13), it can be inferred that

$$E_k = \frac{1}{2}mv_p^2 \tag{43}$$

E_{ce} can be calculated by

$$E_{ce} = \frac{F_{ce}^2}{2k} \tag{44}$$

where F_{ce} is the load when the support reaches its elastic limit. Eq. (42) can be expressed as

$$\frac{\lambda \cdot P / F_{ce}}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \geq 1 \tag{45}$$

Define the safety factor α_3 for support under the velocity quasi-resonance of the roof as

$$\alpha_3 = \frac{\lambda\beta'}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \tag{46}$$

where force-to-amplitude ratio $\beta' = P/F_{ce}$. Therefore, when the safety factor α_3 is greater than or equal to 1, the support will experience buckling failure under the velocity quasi-resonance of the roof. According to the support impact failure condition given in Eq. (45), an analysis of the impact risk assessment for support is presented below, based on disturbance frequency ratio λ .

(1) If the damping ratio ζ satisfies

$$\zeta > \frac{\beta'}{2} \tag{47}$$

There is no solution for λ when α_3 is greater than or equal to 1. This means that the roof support system remains consistently stable under various external frequency disturbances.

(2) If the damping ratio ζ satisfies

$$\zeta \leq \frac{\beta'}{2} \tag{48}$$

There is a solution for λ when α_3 is greater than or equal to 1. Then the interval of impact risk frequency ratio under the velocity quasi-resonance of the roof is as follows:

$$\lambda_1 \leq \lambda \leq \lambda_2 \tag{49}$$

where

$$\lambda_{1,2} = \sqrt{\left(1 - 2\zeta^2 + \frac{\beta'^2}{2}\right) \mp \sqrt{4\zeta^4 - (4 + 2\beta'^2)\zeta^2 + \beta'^2 + \frac{\beta'^4}{4}}}$$

The above analysis indicates that the support impact failure induced by the velocity quasi-resonance of the roof is closely related to the system damping ratio ζ . Increasing the support damping can suppress the support damage induced by the velocity quasi-resonance of the roof. According to Eq. (49), in case of impact failure to the support, the disturbance frequency ratio interval of rock masses with impact risks is related to ζ and β' .

Equation (46) is used to calculate the relationship between the safety factor α_3 and the disturbance frequency ratio λ under varying ζ and β' , as shown in Fig. 8.

It can be found from Fig.8 that when the disturbance frequency ratio λ is close to 1, i.e., when quasi-resonance is generated, the safety factor α_3 significantly increases, leading to the occurrence of impact damage to the support ($\alpha_3 \geq 1$). The disturbance frequency ratio for support failure depends on ζ and β' . When $\zeta = 0.01$, the relationships between ζ and varying β' (0.1, 0.3, 0.6, 0.9) in Fig. 8 satisfy Eq. (48), indicating that the disturbance frequency ratio interval for rock masses with impact risks satisfies Eq. (49), as listed in Table 3. When $\zeta = 0.2$, the relationships between ζ and β' (0.1 and 0.3) satisfy Eq. (47), indicating

that the roof-support system is always stable under velocity quasi-resonance conditions. For $\beta' = 0.6$ and 0.9 , their relationships with ζ satisfies Eq. (48), meaning that the disturbance frequency ratio interval for rock masses with impact risks satisfies Eq. (49), as listed in Table 3. Therefore, increasing support damping can narrow and even eliminate the interval of risk disturbance frequency ratios. Meanwhile, when ζ is constant, the rock disturbance frequency ratio λ of impact failure to support approaches 1 as β' decreases.

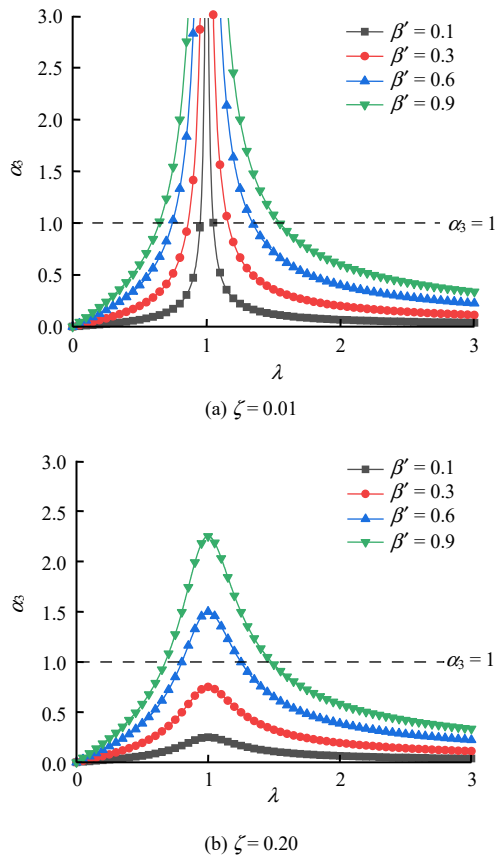


Fig. 8 Relation curves between the impact risk frequency ratio and safety factor of roof velocity quasi-resonance

Table 3 Disturbance frequency ratio intervals of support failure under roof velocity quasi-resonance

β	$\zeta = 0.01$			$\zeta = 0.2$		
	λ_1	λ_2	$\Delta\lambda$	λ_1	λ_2	$\Delta\lambda$
0.1	0.95	1.05	0.10	—	—	0
0.3	0.86	1.16	0.30	—	—	0
0.6	0.74	1.34	0.60	0.80	1.25	0.45
0.9	0.65	1.54	0.89	0.68	1.48	0.80

By comparing the calculated risk disturbance frequency ratio intervals under the three types of roof quasi-resonance, it can be seen that the interval obtained from velocity quasi-resonance is smaller than that from displacement quasi-resonance. The interval of acceleration quasi-resonance has a right endpoint that tends to infinity. As the damping ratio ζ increases, the three intervals all narrow. When both

the damping ratio and force-amplitude ratio are small, such as $\zeta = 0.01$ and $\beta = 0.1$, the ratios caused by these three types of quasi-resonances are very close. To avoid impact disasters caused by these three types of quasi-resonances, the danger disturbance frequency ratio intervals can be staggered by interfering with the disturbance frequency received by the roof or adjusting the inherent frequency of the roof-support system. Additionally, increasing support damping can reduce the impact hazard of roof quasi-resonance effects.

4 Conclusions

(1) The roof-support system is subject to disturbance from surrounding rocks, which can result in a quasi-resonant phenomenon. The roof will exhibit three types of quasi-resonant responses: displacement, velocity, and acceleration. The dynamic mechanism of the impact failure was explained from the perspective of roof-support system quasi-resonance, under the condition that the disturbance force-amplitude was less than the critical load for support failure.

(2) From the perspective of force distribution and energy conversion in support, discriminant criteria for the impact failure of the roof-support system are provided for three quasi-resonant characteristics. The safety factors α_1 , α_2 and α_3 for support under displacement, acceleration and velocity quasi-resonances are defined, respectively. A method is presented to calculate the risk disturbance frequency ratio interval at which the roof-support system is susceptible to impact failure.

(3) After analyzing the anti-burst controls of three quasi-resonance responses for the roof, it was found that support damping has a good inhibitory effect on the quasi-resonant displacement, velocity, and acceleration of the roof. Therefore, strengthening the damping energy dissipation characteristics of the support is an effective way to control the quasi-resonant effect of the roof-support system. By adjusting the support damping, the impact danger brought by quasi-resonance to the roof-support system can be effectively avoided.

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