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Keywords

rock tunnel, structural system reliability, interval non-probability, multiple failure modes, sensitivity analysis

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Abstract: For the reliability analysis of rock tunnel stability, considering the difficulty in obtaining the statistical data on rock mass parameters, there is the potential inadequacy when using the probabilistic reliability method; on the other hand, the rock tunnel stability is intimately pertinent to a variety of failure modes, meaning it is necessary to consider the issue on structural system reliability. First, based on the interval theory, the uncertainty parameters were represented by adopting the interval variables. Subsequently, aiming at the coexistence of multiple failure modes in rock tunnel engineering, the concept of structural system reliability was introduced, and then a structural system reliability method to calculate the reliability index and assess the rock tunnel stability based on the non-probabilistic interval theory was established. On this basis, the rationality of the proposed method was verified via the engineering example. Finally, the fluctuation range of the uncertainty parameters was defined to further analyze the influence of different parameters in each failure mode on the corresponding reliability index and the system reliability index. The results show that the non-probabilistic reliability index of each failure mode decreases with the increase of the range of interval variables, and the same parameter in various failure modes causes different effects. In addition, the change of uncertainty parameters also leads to the variations in the main failure modes that affect the structural system stability of rock tunnel.

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1 Introduction

In underground engineering such as rock tunnels, traditional probabilistic reliability methods occupy center stage in the existing reliability research, and the reliability is mainly calculated based on classical mathematical statistics, stochastic process, and probability theory[1]. However, the traditional method has many limitations, and the primary one is that it is highly dependent on the amount of statistical data, that is, the basic premise of the traditional method is that enough statistical sample information of random parameters must be known to construct the probability density distribution function. But it is difficult to obtain a large amount of statistical information about random parameters in practical engineering, leading to the incompleteness of the uncertainty parameter information obtained under such limited conditions, so it is difficult to ensure the calculation accuracy that the reliability analysis should have, which results in significant deviations between final evaluation results and real engineering conditions, and even misjudgments[2]. To avoid abovementioned problems, a non-probabilistic reliability analysis method is expected to be proposed from the perspective of "non-probability". Non-probabilistic reliability methods have been gradually applied to geotechnical engineering related fields, such as retaining walls[3], karst cave roof under pile[4–5], and deep foundations[6]. In underground engineering, Cao and Zhang[7] established a fuzzy reliability model for underground structures using interval-truncation approach, Dong and Li[8] constructed an interval non-probabilistic reliability model for jointed rock masses in tunnels using interval mathematical theory, and Zhai et al.[9] established a non-probabilistic analysis method for the safety thickness interval to prevent karst water inrush based on the distribution characteristics of tunnel parameters. Zhu[10] employed the non-probabilistic interval reliability theory to calculate the factor of safety to evaluate the tunnel lining structure safety when the statistical parameters of random variables are given. Su and Li[11] introduced the concept of robustness, used the Info-Gap model to quantify uncertain parameters, and constructed a robust non-probabilistic analysis model for underground structures in geotechnical engineering. Based on existing studies, Li et al.[12] further established a robust design method for underground structures considering the influence of multi-parameter uncertainty.

On the other hand, due to the complexity, diversity, and high variability of rock masses and their surroundings, as well as the differences in construction time and process form of supporting, the failure of underground structures such as rock tunnels is not only caused by a single failure mode, and their stability is often closely related to multiple failure modes, that is, the working conditions where multiple failure modes work together are common. Therefore, the rock tunnel reliability must be investigated from the prospective of structural system. Compared to the existing research mainly focusing on a single failure mode, the reliability analysis of the rock tunnel structural system considering multiple failure modes is more in line with the actual engineering situation and

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thus can more reasonably reflect the engineering safety status[13].

According to the existing research, the non-probabilistic reliability analysis in rock tunnel engineering is rarely conducted, and it is overall at the initial stage. Firstly, rock tunnels are typically located in complex geological environments, undergoing strata changes and influences of various external factors, and "the uncertainty of rock masses becomes more complex compared to soil"[14]. Therefore, the statistical data acquisition for rock tunnels is limited, making it is difficult to employ probabilistic reliability methods. Secondly, due to the interaction between surrounding rock and support, as well as the complexity and variability of the relevant parameters of the two, multiple failure modes make a significant impact in actual rock tunnel engineering. Therefore, it is necessary to conduct corresponding research on the reliability of rock tunnel structural system. Thirdly, the complexity of rock formations and external influences can further lead to more complex changes in the ranges of uncertainty parameters in rock tunnels, thereby profoundly affecting the reliability of the tunnel structural system. Consequently, in-depth and detailed analysis of distinct failure modes and the sensitivity of uncertainty parameters involved are required. The above problems are also the core contents of this research.

Accordingly, for rock tunnel engineering, a non-probabilistic reliability analysis model is firstly constructed based on the interval theory, and this model does not require a large amount of statistical data and represents uncertain parameters in the form of intervals. Then, the reliability of rock tunnel structural system under multiple failure modes is explored. Finally, the sensitivity analysis about different impacts of uncertain parameters on non-probabilistic reliability indexes and multiple failure modes of structural system is conducted. Through the above investigations, a non-probabilistic reliability analysis method for structural system that adapts to the characteristics of rock tunnel engineering is expected to be established based on existing research.

2 Non-probabilistic reliability method for structural system based on interval theory[15–17]

Let the set of all real value intervals be IR, and the interval set contains any interval components A_i^I. The vector $\boldsymbol{a}=\{a_1,a_2,\cdots,a_n\}$ is a set of basic interval variables related to engineering structures, where $a_i \in A_i^I \in \mathrm{IR}\,(i=1,2,\cdots,n)$. According to the non-probabilistic reliability problem, let

$$M=g\left(a_i\right)=g\left(a_1,a_2,\cdots,a_n\right) \tag{1}$$

be a function determined under the structural failure criterion. When $g(\cdot)$ is a continuous function with respect to $a_i\left(i=1,2,\cdots,n\right)$, M is also an interval variable. M^{c} and M^{r} are defined as the mean and deviation of the function M. Under such conditions, the following formula is defined as the interval non-probabilistic reliability index[15].

$$\eta=\frac{M^{\mathrm{c}}}{M^{\mathrm{r}}} \tag{2}$$

When multiple failure factors work together in the structure, the incremental load method is often used to enumerate and solve the main failure modes of the structure[16]. The basic idea is that n components in the structure fail in sequence in a certain failure mode, due to the increase in load acting on the structure $S_1,\cdots,S_n$. When the load increases from 0 to S_1, Component 1 fails. Afterwards, any increase in load in the structure will cause the component to enter a critical state, until the number of components reaches a certain limit value, which is defined as the first failure mode. The internal force assigned to component 1 is $\alpha_{11}S_1$, where α_{11} is the utilization rate of S_1 by component 1. By analogy, when the load increases to S_j, the internal force distributed to the component i is $\sum_j \alpha_{ij}R_i$, where α_{ij} is the utilization rate of the load S_j by the component i. Therefore, the component strength R_i and each incremental load meet the following requirements:

$$\begin{Bmatrix} R_1 \\ R_2 \\ \vdots \\ R_3 \end{Bmatrix}=\begin{bmatrix} \alpha_{11} & 0 & 0 & 0 \\ \alpha_{21} & \alpha_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nn} \end{bmatrix}\begin{Bmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{Bmatrix} \tag{3}$$

When uncertain parameters such as material properties and geometric dimensions of the structure are not taken into account, α_{ij} is a determined parameter, and it can be obtained through general mechanical calculation methods. When the uncertain parameters in the structure are interval variables, α_{ij}, S_1, $\cdots$, S_n are also interval variables and are related to the uncertainty of α_{ij} and R_i. In this case, the incremental load method based on interval theory can be used to solve Eq. (3).

According to Eq.(3) and matrix related knowledge, the relationship between component strength and incremental load is as follows:

$$\{S\}=[D]\{R\} \tag{4}$$

where the matrix $[D]D$ is the inverse matrix of the matrix formed by a_{ij} in Eq. (3). Then the strength of the structural system R_{s} and the equation of limit state corresponding to the failure mode can be expressed as

$$R_{\mathrm{s}}=\sum_i S_i=\sum_i d_i R_i \tag{5}$$

$$M=\sum_i d_i R_i-P=0 \tag{6}$$

where d_i is the coefficient related to utilization rate of load, and P is the external load on the structure. If the uncertain variables in the structure are not taken into account, d_i is a determined parameter, and Eq. (6) is a linear equation. When the material properties and geometric parameters of the structure are interval variables, d_i is an interval variable, and Eq. (6) is a nonlinear equation. At this point, each failure mode corresponds to a reliability index $\eta_i\left(i=1,2,\cdots,n\right)$, and the reliability of the structural system is involved. Any failure mode can lead to the overall system failure. The total non-probabilistic reliability index of the structural system is[16]

$$\eta=\min\left\{\eta_1,\eta_2,\cdots,\eta_n\right\} \tag{7}$$

According to Eq. (7), the failure mode with the lowest non-probabilistic reliability index in the structure is the most dangerous failure mode, which is the key to affecting the structural system stability.

When the form of limit state function is complex, the variables are more, the monotonicity is difficult to determine, and the approximate solution method of optimization calculation can be used. Standardized transformations of interval variables and interval functions[17] are

$$a=a^{\mathrm{c}}+\delta a^{\mathrm{r}} \tag{8}$$

$$M=g\left(a_1,a_2,\cdots a_n\right)=G\left(\delta_1,\delta_2\cdots\delta_n\right)=0 \tag{9}$$

where a^{c} and a^{r} represent the mean and deviation of interval variable a, and δ is a standardized interval variable, whose range satisfies $\delta\in[-1,\ 1]$.

After obtaining the standardized form, the upper and lower boundaries of the limit state function M^{u} and M^{l} can be obtained, and they meet the following requirements:

$$M^{\mathrm{u}}=\max_{\delta_i\in\Delta I}F\left(\delta_1,\delta_2,\cdots,\delta_n\right) \tag{10}$$

$$M^{\mathrm{l}}=\min_{\delta_i\in\Delta I}F\left(\delta_1,\delta_2,\cdots,\delta_n\right) \tag{11}$$

where ΔI is the set of all standardized interval variables.

Therefore, the non-probabilistic reliability index of the structure can be expressed as

$$\eta=\frac{M^{\mathrm{c}}}{M^{\mathrm{r}}}=\frac{M^{\mathrm{u}}+M^{\mathrm{l}}}{M^{\mathrm{u}}-M^{\mathrm{l}}} \tag{12}$$

3 Interval non-probabilistic reliability analysis under multiple failure modes

In practical rock tunnel engineering, the factors affecting tunnel failure are often not singular. In most cases, multiple possible failure modes need to be considered simultaneously, which means that the reliability of the tunnel structural system needs to be studied. This section will focus on the failure modes of rock tunnels, and the widely employed anchorage-shotcrete support in practical engineering will be taken as an example. Three failure modes of insufficient support capacity, excessive surrounding rock convergence, and insufficient anchor length are specially examined. Non-probabilistic reliability indexes for different modes are established, and the reliability index of the structural system is introduced to evaluate the stability of rock tunnel engineering.

3.1 Analysis model of tunnel system

For the tunnel structural system using anchorage-shotcrete support, considering both the surrounding rock stability and the support structure stability, the following three most common failure modes in practical engineering are established[18].

(1) Consider safety of support structure

When the tunnel support structure is subjected to excessive load or surrounding rock pressure, engineering phenomena such as cracking, deformation, or local collapse of the support structure may occur. The tunnel may be further damaged and the support system may fail due to insufficient bearing capacity. This is the most basic safety and reliability problem, and its limit state function can be set as

$$g_1\left(x\right)=p_{\mathrm{s}}^{\max}-p_{\mathrm{s}}^{D} \tag{13}$$

where $p_{\mathrm{s}}^{\max}$ is the maximum bearing capacity the support structure can provide, also the peak value of the support characteristic curve, and it can be determined by the maximum support force the anchorage-shotcrete support can provide; p_{s}^{D} is the support force or surrounding rock pressure when the tunnel surrounding rock and support are in a balanced state, which is generally determined by the classical convergence-confinement method (i.e. characteristic curve method), and it is determined by the intersection point of the surrounding rock displacement characteristic curve and the support characteristic curve. This failure mode is defined as the support structure bearing capacity failure mode, and Eq. (13) represents the corresponding judgement criterion for the support bearing capacity. When Eq. (13) is greater than 0, that is, when the maximum bearing capacity provided by the support structure $p_{\mathrm{s}}^{\max}$ is greater than the support force or surrounding rock pressure at the equilibrium state p_{s}^{D}, the support structure is indicated as stable and reliable.

The stiffness coefficient and maximum bearing capacity of concrete lining support can be expressed as[19]

$$K_{\mathrm{con}}=\frac{E_{\mathrm{c}}}{1+\mu_{\mathrm{c}}}\frac{\left[R^2-\left(R-t_{\mathrm{c}}\right)^2\right]}{\left(R-t_{\mathrm{c}}\right)^2+\left(1-2\mu_{\mathrm{c}}\right)R^2}\frac{1}{R} \tag{14}$$

$$p_{\mathrm{con}}^{\max}=\frac{1}{2}\sigma_{\mathrm{cc}}\left[1-\frac{\left(R-t_{\mathrm{c}}\right)^2}{R^2}\right] \tag{15}$$

where E_{c} is the elastic modulus of concrete; μ_{c} is the Poisson's ratio of concrete; t_{c} is the thickness of concrete lining; and σ_{cc} is the compressive strength of concrete without lateral restraint.

The stiffness coefficient and maximum bearing capacity of anchor bolt support can be expressed as

$$K_{\text{bolt}} = \frac{1}{S_{\text{c}} \cdot S_{\text{l}} \cdot \left[\dfrac{4L_{\text{bolt}}}{\pi D_{\text{b}}^2 E_{\text{st}}} + Q \right]} \tag{16}$$

$$p_{\text{bolt}}^{\max} = \frac{T_{\max}}{S_{\text{c}} \cdot S_{\text{l}}} \tag{17}$$

where S_{c} is the circumferential anchor bolt spacing; S_{l} is the longitudinal anchor bolt spacing; D_{b} is the anchor bolt diameter; E_{st} is the elastic modulus of the anchor bolt; Q is a constant related to the deformation and stress of the anchor bolt, which can be determined through experiments; and $T_{\max}$ is the ultimate bearing capacity of the anchor bolt.

For the case of anchorage-shotcrete support, the stiffness coefficient and maximum bearing capacity can be considered as the following formula:

$$K_{\text{tot}} = K_{\text{con}} + K_{\text{bolt}} \tag{18}$$

$$p_{\text{tot}}^{\max} = K_{\text{tot}} u_{\text{tot}}^{\max} = K_{\text{tot}} \cdot \min\left[\frac{p_{\text{con}}^{\max}}{K_{\text{con}}}, \frac{p_{\text{bolt}}^{\max}}{K_{\text{bolt}}} \right] \tag{19}$$

(2) Consider the surrounding rock stability after excavation

When the displacement deformation of surrounding rock after excavation is too large and exceeds its allowable convergence value, the tunnel will fail accompanied by engineering phenomena including cracking, spalling, sliding, loosening of surrounding rock, and then corresponding settlement and deformation form. At this situation, the limit state function is

$$g_2(x) = \varepsilon_{\text{u}}^{\max} - \frac{u_{\text{r}}^{\text{pl}}}{R} \tag{20}$$

where $\varepsilon_{\text{u}}^{\max}$ is the maximum allowable ratio related to $\frac{u_{\text{r}}^{\text{pl}}}{R}$; u_{r}^{pl} is the final radial displacement of the tunnel surrounding rock; and R is the excavation radius of the tunnel. This failure mode is defined as the surrounding rock convergence failure mode, and Eq. (20) is the corresponding judgement criterion for the surrounding rock convergence. The plastic zone formed in surrounding rock in this mode is taken as the analysis object. When Eq. (20) is greater than 0, that is, when the ratio $\frac{u_{\text{r}}^{\text{pl}}}{R}$ is not beyond the allowed convergence value $\varepsilon_{\text{u}}^{\max}$, the tunnel is stable and reliable. $\varepsilon_{\text{u}}^{\max}$ has significant variability and should be considered as an uncertain parameter. However, it is difficult to conduct a large number of failure tests on the tunnel surrounding rock to obtain this value in practical engineering. Hence, empirical values are usually selected based on the experience summary of relevant engineering specifications in specific engineering calculations, and this calculation result often tends to be conservative and safe[20].

When the internal pressure provided by tunnel support p_i is less than the critical pressure p_{cr}, plastic yielding occurs in the rock mass, and a plastic zone begins to appear around the tunnel. At this situation, the critical pressure p_{cr} can be defined as[19]

$$p_{\text{cr}} = \frac{2p_0 - \sigma_{\text{cm}}}{1+k} \tag{21}$$

where σ_{cm} represents the uniaxial compressive strength of the rock mass, and k is the slope of the Mohr-Coulomb strength curve, which satisfies the Mohr-Coulomb criterion:

$$\sigma_1 = k\sigma_3 + \sigma_{\text{cm}} \tag{22}$$

$$k = \frac{1+\sin\varphi}{1-\sin\varphi} \tag{23}$$

$$\sigma_{\text{cm}} = \frac{2c\cos\varphi}{1-\sin\varphi} \tag{24}$$

where σ_1 and σ_3 represent the maximum and minimum principal stresses; c and φ are cohesion and internal friction angle of the rock mass.

The plastic zone radius can be expressed as

$$R_{\text{pl}} = R\left\{ \frac{2\left[p_0(k-1)+\sigma_{\text{cm}}\right]}{(k+1)\left[(k-1)p_i+\sigma_{\text{cm}}\right]} \right\}^{\frac{1}{k-1}} \tag{25}$$

The total inward radial displacement of the tunnel wall corresponding to plastic failure can be written as

$$u_{\text{r}}^{\text{pl}} = \frac{R(1+\mu)}{E}\left[2(1-\mu)(p_0 - p_{\text{cr}})\left(\frac{R_{\text{pl}}}{R}\right)^2 - (1-2\mu)(p_0 - p_i) \right] \tag{26}$$

(3) Consider the reliability of anchor bolt in anchorage-shotcrete support

When the anchorage-shotcrete support is adopted for tunnels, the anchor bolt must meet a certain anchorage length requirement, that is, the effective length of the anchorage part of the anchor bolt inserted into the rock layer, and it is generally related to the plastic zone in the surrounding rock. When the plastic zone in the surrounding rock is too large, the anchorage length may be insufficient, and the anchorage-shotcrete support structure stability is destroyed because the support structure fails to meet the conditions, which then causes the tunnel failure. At this situation, the limit state function is

$$g_3(x) = L_{\text{bolt}} - \left(R_{\text{pl}} - R + l_{\min}\right) \tag{27}$$

where L_{bolt} is the anchor bolt length; R_{pl} is the plastic zone radius in the surrounding rock; and $l_{\min}$ is the minimum anchorage length required in support. It is generally specified that 40% of the anchor bolt length needs to be anchored in the elastic zone of stable surrounding rock, which can be expressed as $l_{\min} = 0.4L_{\text{bolt}}$. This failure mode is defined as the anchor bolt length failure mode, and Eq. (27) is the corresponding judgement criterion for the anchor bolt length. This mode also takes the plastic zone in the surrounding rock as the analysis object. Under the radial anchorage effect, the anchor bolt should have a certain length anchored in the undisturbed rock layer outside the plastic zone. When Eq. (27) is greater than 0, that is, the length of the anchor bolt meets the above requirements, the tunnel is stable and reliable.

In summary, among the three failure modes mentioned above, the support bearing capacity criterion shown in Eq. (13) reflects the bearing capacity limit state of the support structure, while the surrounding rock convergence criterion shown in Eq. (20) and anchor bolt length criterion shown in Eq. (27) reflect the normal service limit state.

Based on the above analysis, the limit state functions $g_1(x)$, $g_2(x)$, and $g_3(x)$ under three failure modes can be obtained. According to the non-probabilistic theory of structural system, the limit state function of the tunnel structural system is expressed as

$$M = g_i(a_1, a_2, \cdots, a_n)(i = 1, 2, 3) \tag{28}$$

Because there are many uncertain factors in the rock tunnel, and the rock mass itself has a complex and changeable nature, some variables on the right side of the above limit state function show uncertainty. Therefore, the interval method is used to express these uncertain parameters in terms of parameter values to reflect the interval distribution characteristics. Through research and engineering experience, the basic interval variables of the rock mass in this model mainly include the cohesion c, internal friction angle φ, elastic modulus E, and initial geostress p_0. Since the variation range of Poisson's ratio is small, it is quantitatively expressed here. The interval variables of the support mainly include the thickness of the concrete lining t_c, the circumferential and longitudinal spacing of the anchor bolt S_c and S_l, and the provided support force p_i. All other parameters are considered as fixed values. Interval calculation can be programmed using the INTLAB toolkit in MATLAB software to obtain the range $g_i = \left[g_i^{\text{l}}, g_i^{\text{u}}\right]$ of response variables $g_i(i = 1, 2, 3)$, where g_i^{l} and g_i^{u} are the lower and upper limits of the interval.

According to the aforementioned interval non-probabilistic reliability theory, the non-probabilistic reliability index of each objective variable can be solved by the following equation[15, 17]:

$$\eta_i = \frac{g_i^{\text{u}} + g_i^{\text{l}}}{g_i^{\text{u}} - g_i^{\text{l}}} \tag{29}$$

Based on the interval reliability theory, when $\eta_i > 1(i = 1, 2, 3)$, the corresponding limit state function is constant and the structure is stable and reliable. When η_i is within the interval $[0, 1]$, there is a possibility of structural failure and the structure is unreliable. Therefore, only the requirements of support bearing capacity, radial displacement of surrounding rock, and anchor length are simultaneously met, the tunnel structural system can be considered stable. That is, the tunnel structural system is safe and reliable when the non-probabilistic reliability index of the tunnel structural system $\eta = \min\{\eta_1, \eta_2, \eta_3\} > 1$ is met.

3.2 Reliability analysis of engineering example

The Tongshuxi Tunnel in Hunan Province has a total length of 6 672 m and an excavation radius of 5.33 m. The topography of the tunnel site is a hilly landform, and the main stratum lithology in the survey area is composed of the overlying Quaternary overburden, the lower Cretaceous argillaceous siltstone and calcareous siltstone. The tunnel surrounding rock is mainly divided into Class IV and V, and the rock mass is in a strongly and moderately weathered state. In the specific calculation process, the engineering section from Huaihua end (stake number K4+320) to Zhijiang end (stake number K4+900) is taken as an example for analysis. Based on the information such as lithology and surrounding rock classification, relevant specifications are read, and existing engineering experience is combined. After comprehensive consideration, the range of the physical and mechanical parameters and geometric parameters of the rock mass describing the engineering section is given as Table 1.

The project adopts anchorage-shotcrete combined support, and the corresponding relevant parameters are as follows: Poisson's ratio of rock mass $\mu = 0.316$; the allowable displacement convergence value generated by surrounding rock excavation $\varepsilon_{\text{u}}^{\max} = 2\%$; the shotcrete grade is C30, whose elastic modulus $E_c = 30 \times 10^3$ MPa, Poisson's ratio μ_c = 0.2, and uniaxial compressive strength σ_{cc} = 30 MPa; the anchor bolt length L_{bolt} = 4 m, the anchor bolt diameter $D_b = 0.021\ 6$ m, and the elastic modulus of the anchor bolt $E_{st} = 200$ GPa; the deformation–load constant of the anchor bolt $Q = 0.143$ m/MN, and the ultimate bearing capacity $T_{\max} = 454$ kN. The other uncertain parameters are shown in Table 1.

Table 1 Interval variables and their value ranges

Interval variable	Cohesion of rock mass c /MPa	Internal friction angle φ /(°)	Initial geostress p_0 /MPa	Elastic modulus E /MPa	Concrete lining thickness t_c /m	Circumferential anchor bolt spacing S_c /m	Longitudinal anchor bolt spacing S_l /m
Value range	[0.585, 0.715]	[29.45，32.55]	[2.52, 3.48]	[3 807.4, 4 124.85]	[0.17, 0.23]	[0.8, 1.2]	[0.8, 1.2]

The value of each parameter is substituted into the Eqs.（13）–（27）, and the INTLAB toolbox is used to calculate the response variable ranges of the three failure mode functional functions, which are $g_1 = [302.7, 1\ 380.6]$, $g_2 = [0.017\ 7, 0.020\ 0]$, and $g_3 = [0.681\ 0, 3.493\ 4]$. Then, according to Eq. (29), the non-probabilistic reliability indexes for the three failure modes are calculated as $\eta_1 = 1.561\ 8$, $\eta_2 = 17.011\ 5$, and $\eta_3 = 1.484\ 3$.

From the above calculation results, the non-probabilistic reliability index of the rock tunnel structural system can determined as $\eta = \min\{\eta_1, \eta_2, \eta_3\} = \min\{1.561\ 8,\ 17.011\ 5,\ 1.484\ 3\} = 1.484\ 3$, and $\eta > 1$. Therefore, the tunnel is considered as stable and reliable, and the design of the tunnel structural system meets the requirements.

To verify the non-probabilistic method, the classical Monte-Carlo importance sampling method[21] is adopted to calculate the failure probability of each failure mode in the rock tunnel structural system. The mean of random variables is taken as the midpoint value of the interval (the value near or within the interval). The variation coefficient shall be determined based on engineering codes, and the probability distribution is regarded as normal distribution. The calculation situations are summarized as follows:

For the first failure mode, the concrete lining thickness t_c, the circumferential anchor bolt spacing S_c, and the longitudinal anchor bolt spacing S_l are considered as random variables. For convenience, the two parameters corresponding to the anchor bolt are merged into one for analysis, that is, the anchor bolt arrangement spacing S. The average concrete lining thickness μ_{t_c} = 0.2 m, the average anchor bolt arrangement spacing μ_S = 1.0 m, the variation coefficient of concrete lining thickness $V_{t_c} = 0.15$, and the variation coefficient of anchor bolt arrangement spacing V_S = 0.15. The failure probability in this mode is calculate as $P_{f1} = 1.306\ 9 \times 10^{-11}$.

For the second failure mode, the cohesion c, internal friction angle φ, elastic modulus E, and initial geostress p_0 are deemed as random variables. The average cohesion $\mu_c = 0.65$ MPa, the average internal friction angle $\mu_\varphi = 31$ °, the average elastic modulus $\mu_E = 4$ GPa, and the average initial geostress $\mu_{p_0} = 3$ MPa. The variation coefficient of cohesion $V_c = 0.4$, the variation coefficient of internal friction angle $V_\varphi = 0.15$, the variation coefficient of elastic modulus V_{p_0} = 0.1, and that of initial geostress V_E = 0.3. The failure probability in this mode is calculate as $P_{f2} = 6.8 \times 10^{-5}$.

For the third failure mode, the cohesion c, internal friction angle φ, and initial geostress p_0 are regarded as random variables. The average cohesion $\mu_c = 0.65$ MPa, the average internal friction angle $\mu_\varphi = 31$ °, and the average initial geostress $\mu_{p_0} = 3$ MPa. The variation coefficient of cohesion $V_c = 0.4$, the variation coefficient of internal friction angle $V_\varphi = 0.15$, and that of initial geostress $V_{p_0} = 0.1$. The failure probability in this mode is estimated as $P_{f3} = 4.0 \times 10^{-6}$.

Through the three failure mode calculations using the above probabilistic method, it is found that the failure probability corresponding to each failure mode is very small, indicating that the tunnel is stable and reliable, and this also verifies the conclusion that the interval non-probabilistic reliability method can distinguish the stability and reliability of this tunnel. In fact, the Tongshuxi tunnel has remained in normal working condition since its opening, further confirming the effectiveness and rationality of the proposed non-probabilistic reliability method.

It is worth pointing out that the probabilistic method is undoubtedly an ideal means of engineering safety assessment when sufficient statistical data are available. However, the proposed non-probabilistic method can be attempted for reliability evaluation and analysis when there is limited statistical data and insufficient data information as a prerequisite for the probabilistic method application.

4 Sensitivity analysis

According to the characteristics of intervals, the fluctuation range λ is defined as the change in the interval range of uncertain parameters under non-probabilistic situation, which can be expressed by the ratio of interval radius a^r to interval mean a^c

$$\lambda = \frac{a^r}{a^c} \tag{30}$$

As λ approaches 0, the range of uncertain parameter becomes smaller and it approaches a fixed value. When λ is larger, the range of uncertain parameter is larger, and more values can be selected for the uncertain parameter.

In addition, to compare the impact of various parameters in the same failure mode on the reliability index of the corresponding mode, the absolute of the curve secant slope K is defined. The larger the slope K, the greater the variation of the corresponding non-probabilistic reliability index with the fluctuation of interval variables, indicating a higher sensitivity.

4.1 Sensitivity analysis for a single failure mode

4.1.1 Support bearing capacity criteria

In this mode, the main uncertainty parameters considered include the concrete lining thickness t_c and the anchor bolt spacing S including circumferential and longitudinal spacings. The fluctuation range increases from 0.06 to 0.25, and the values and trend of the non-probabilistic reliability index of the tunnel under the first failure mode η_1 when the two parameters change separately are shown in Table 2 and Fig. 1. The relationship between the calculated absolute secant slopes K_1 (corresponding to t_c) and K_2 (corresponding to S) is shown as: $K_1 \sim t_c(12.41) > K_2 \sim S(0.12)$. In the failure mode considering the support bearing capacity, the non- probabilistic index η_1 will decrease as the fluctuation ranges of the intervals of the concrete lining thickness t_c and the anchor bolt spacing S increase separately. Moreover, the concrete lining thickness t_c has a much greater impact on the index η_1 than the anchor bolt spacing S, and the anchor bolt spacing S has a very small impact on the index η_1 with the relationship curve approaching the horizontal line.

Table 2 Variations of non-probabilistic reliability index for the first failure mode

Fluctuation range	η_1 when concrete lining thickness changes	η_1 when anchor bolt spacing changes
0.06	3.466 5	1.578 1
0.07	3.012 1	1.576 9
0.08	2.669 5	1.575 7
0.09	2.402 7	1.574 6
0.10	2.189 6	1.573 4
0.11	2.016 1	1.572 3
0.12	1.872 4	1.571 1
0.13	1.751 8	1.569 9
0.14	1.649 5	1.568 8
0.15	1.561 8	1.567 6
0.16	1.486 2	1.566 5
0.17	1.420 4	1.565 3
0.18	1.362 9	1.564 2
0.19	1.312 4	1.563 0
0.20	1.267 8	1.561 8
0.21	1.223 8	1.560 6
0.22	1.193 2	1.559 4
0.23	1.161 9	1.558 2
0.24	1.134 0	1.557 0
0.25	1.109 0	1.555 8

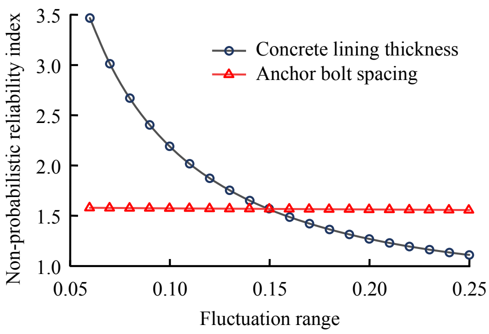

Fig. 1 Comparison of sensitivity analysis of variables for the first failure mode

4.1.2 Surrounding rock convergence criteria

In this mode, the main uncertainty parameters considered include initial geostress p_0, internal friction angle φ, cohesion c, and elastic modulus E. The fluctuation range increases from 0.01 to 0.25, and the values and trend of the non-probabilistic reliability index of the tunnel under the second failure mode η_2 when the four parameters individually change are shown in Table 3 and Fig. 2. The relationship between the calculated absolute secant slopes K_1 (corresponding to p_0), K_2 (corresponding to φ), K_3 (corresponding to c) and K_4 (corresponding to E) is written as: $K_1 \sim p_0$ (89.01)> $K_2 \sim \phi(86.95) > K_3 \sim c(24.47) > K_4 \sim E$ (18.88). In the failure mode considering the surrounding rock convergence, the non-probabilistic index will decrease as the fluctuation range of the four parameter intervals alone increase. In terms of the influence degree, the initial geostress p_0 and internal friction angle φ have a greater impact on η_2, while the cohesion c and elastic modulus E have a smaller impact on η_2, and the impact of elastic modulus E is the smallest.

4.1.3 Anchor bolt length criterion

The uncertainty parameters mainly considered in this mode include cohesion c, internal friction angle φ, and initial geostress p_0. The fluctuation range increases from 0.01 to 0.2, and the values and trend of the non-probabilistic reliability index of the tunnel under the third failure mode η_3 when the three parameters individually change are shown in Table 4 and Fig. 3. The relationship between the calculated absolute

Table 3 Variations of non-probabilistic reliability index for the second failure mode

Fluctuation range	η_2 when cohesion changes	η_2 when internal friction angle changes	η_2 when elastic modulus changes	η_2 when initial geostress changes
0.01	19.490 3	23.182 8	17.579 6	34.165 8
0.02	19.193 4	21.438 1	17.390 1	32.091 5
0.03	18.902 4	19.840 3	17.200 8	30.245 4
0.04	18.617 0	18.370 1	17.011 5	28.591 6
0.05	18.336 9	17.011 5	16.822 2	27.101 6
0.06	18.062 1	15.745 0	16.633 0	25.752 1
0.07	17.792 3	14.563 2	16.443 9	24.524 2
0.08	17.527 4	13.462 4	16.254 8	23.402 0
0.09	17.267 1	12.434 0	16.065 8	22.372 5
0.10	17.011 5	11.470 9	15.876 9	21.424 7
0.11	16.760 2	10.567 2	15.688 0	20.549 1
0.12	16.513 2	9.717 8	15.499 2	19.737 7
0.13	16.269 1	8.918 4	15.310 4	18.983 8
0.14	16.028 0	8.165 4	15.121 7	18.281 4
0.15	15.790 9	7.455 7	14.933 0	17.625 5
0.16	15.557 8	6.786 8	14.744 4	17.011 5
0.17	15.328 4	6.156 4	14.555 8	16.430 9
0.18	15.102 7	5.562 6	14.367 3	15.880 0
0.19	14.880 6	5.003 8	14.178 8	15.362 0
0.20	14.661 9	4.478 6	13.990 4	14.874 1
0.21	14.446 6	3.985 5	13.802 0	14.413 6
0.22	14.234 5	3.523 6	13.613 6	13.978 3
0.23	14.025 6	3.091 7	13.425 3	13.566 2
0.24	13.819 8	2.688 9	13.237 0	13.175 5
0.25	13.617 0	2.3141	13.048 8	12.804 5

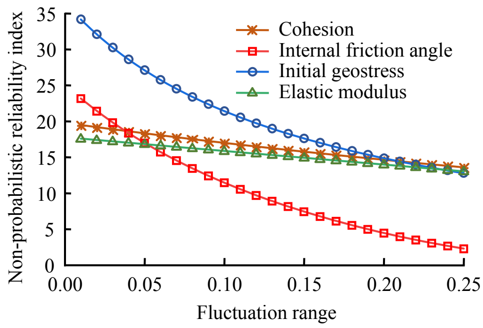

Fig. 2 Comparison of sensitivity analysis of variables for the second failure mode

Table 4 Variations of non-probabilistic reliability index for the third failure mode

Fluctuation range	η_3 when cohesion changes	η_3 when internal friction angle changes	η_3 when initial geostress changes
0.01	1.816 7	2.581 4	1.9579
0.02	1.774 8	2.217 2	1.9174
0.03	1.734 3	1.925 3	1.8784
0.04	1.695 1	1.685 5	1.8410
0.05	1.657 2	1.484 3	1.8050
0.06	1.620 5	1.312 6	1.7704
0.07	1.584 9	1.163 7	1.7370
0.08	1.550 4	1.033 0	1.7048
0.09	1.516 9	0.916 9	1.6738
0.10	1.484 3	0.812 6	1.6439
0.11	1.452 7	0.718 1	1.6150
0.12	1.421 9	0.631 8	1.5871
0.13	1.392 0	0.552 2	1.5601
0.14	1.362 8	0.478 4	1.5340
0.15	1.334 4	0.409 5	1.5087
0.16	1.306 7	0.344 7	1.4843
0.17	1.279 6	0.283 6	1.4607
0.18	1.253 2	0.225 5	1.4377
0.19	1.227 5	0.170 1	1.4155
0.20	1.2023	0.117 1	1.3940

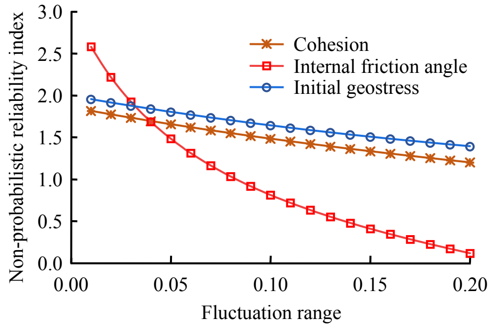

Fig. 3 Comparison of sensitivity analysis of variables for the third failure mode

secant slopes K_1 (corresponding to φ), K_2 (corresponding to c) and K_3 (corresponding to p_0) is expressed as: $K_1 \sim \varphi(12.97) > K_2 \sim c(3.23) > K_3 \sim p_0(2.97)$. In the tunnel failure mode considering the anchor bolt length, increasing the fluctuation range of the three parameter intervals alone will reduce the non-probabilistic index η_3 in this mode, and the sensitivity of η_3 to internal friction angle φ is high, while the sensitivity of η_3 to cohesion c and initial geostress p_0 is relatively small.

4.2 Influence of the same parameters on reliability of different failure modes

In the rock tunnel structural system, there are significant differences in the effects of the same parameters on different failure modes. The specific situations are as follows:

As shown in Fig.4, the initial geostress p_0 is the most influential variable in the second failure mode, while it is the least influential variable in the third failure mode. As shown in Fig.5, the variations of cohesion c in the second and third failure modes have less impact on the corresponding non-probabilistic reliability index of each mode compared to other uncertain parameters, and the impact on the reliability index in the second failure mode is slightly greater than that in the third failure mode. As shown in Fig. 6, the internal friction angle φ is the most influential variable in the second and third failure modes, and the impact on the reliability index in the second failure mode is also greater than that in the third failure mode.

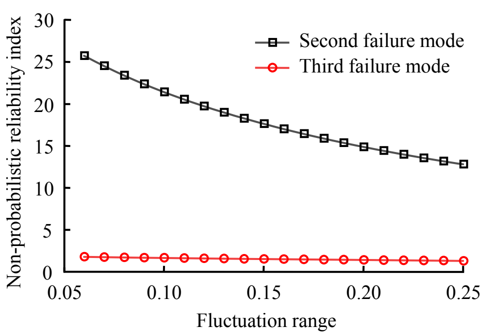

Fig. 4 Influence of variations for initial in-situ stress on reliability index under different failure modes

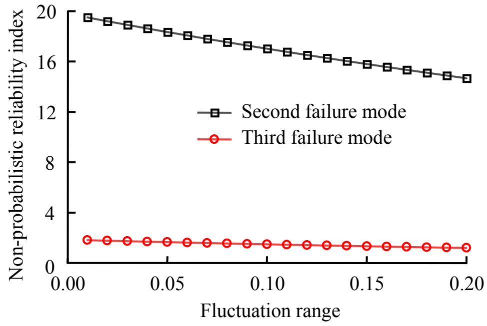

Fig. 5 Influence of variations for cohesion on reliability index under different failure modes

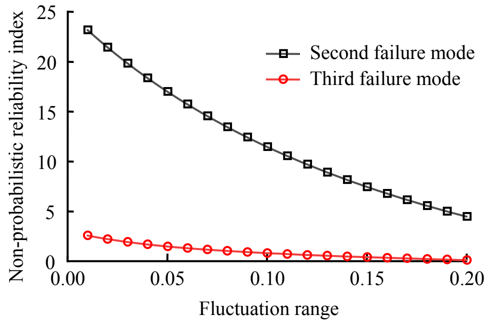

Fig. 6 Influence of variations for internal friction angle on reliability index under different failure modes

4.3 Reliability of structural system

Table 5 lists the comparison of non-probabilistic reliability indexes for three failure modes when the fluctuation range of the concrete lining thickness t_c is 0.05, 0.15, and 0.25, that is, the corresponding range of the concrete lining thickness t_c is [0.19 m, 0.21 m], [0.17 m, 0.23 m], and [0.15 m, 0.25 m]. When the fluctuation range is 0.05 (corresponding to the t_c range of [0.19 m, 0.21 m]) and 0.15 (corresponding to the t_c range of [0.17 m, 0.23 m]), the non-probabilistic reliability index of the rock tunnel system η is 1.484 3, indicating that the main failure mode affecting the tunnel system is the third failure mode. When the fluctuation range is 0.25 (corresponding to the t_c range of [0.15 m, 0.25 m]), the non-probabilistic reliability index of the rock tunnel system η is 1.109, which means that the main failure mode affecting the tunnel system becomes the first failure mode.

Table 6 lists the comparison of non-probabilistic reliability indexes for the three failure modes when the fluctuation range of the internal friction angle φ is 0.01, 0.10, and 0.20. The corresponding range of the internal friction angle is [30.69°, 31.31°], [27.9°, 34.1°], and [24.8°, 37.2°]. When the fluctuation range is 0.01 (corresponding to the φ range of [30.69°, 31.31°]), the non-probabilistic reliability index of the rock tunnel system η is 1.5618, which means that the main failure mode affecting the tunnel system is the first failure mode. When the fluctuation range is 0.1 (corresponding to the φ range of [27.9°, 34.1°]) and 0.2 (corresponding to the φ range of [24.8° , 37.2°]), the non-probabilistic reliability indexes of the rock tunnel system η are 0.812 6 and 0.117 1, and the main failure mode that affects the tunnel system is the third failure mode.

Table 5 Non-probabilistic reliability index for each failure mode with change of concrete lining thickness

Fluctuation range	Reliability index for the first failure mode η_1	Rliability index for the second failure mode η_2	Reliability index for the third failure mode η_3
0.05	4.095 9	17.011 5	1.484 3
0.15	1.561 8	17.011 5	1.484 3
0.25	1.109 0	17.011 5	1.484 3

Table 6 Non-probabilistic reliability index for each failure mode with change of internal friction angle

Fluctuation range	Reliability index for the first failure mode η_1	Reliability index for the second failure mode η_2	Reliability index for the third failure mode η_3
0.01	1.561 8	23.182 8	2.581 4
0.10	1.561 8	11.470 9	0.812 6
0.20	1.561 8	4.478 6	0.117 1

According to this example, the second failure mode is relatively stable and not easy to cause failure of the rock tunnel structural system, while both the first and third failure modes may cause damage to the rock tunnel structural system.

The above results indicate that the changes in the range of a single uncertainty parameter not only causes changes in the non-probabilistic reliability index corresponding to its failure mode, but also causes changes in the non-probabilistic reliability index of the tunnel structural system, thereby causing changes in the main failure modes that affect the structural system stability.

5 Conclusions

In the practical situation, there is a lack of uncertain data in rock tunnel engineering and it is difficult to obtain sufficient statistical information, which makes probabilistic reliability methods difficult to apply. Therefore, an interval non-probabilistic reliability model was developed based on the interval distribution characteristics of rock mass and support parameters. Meanwhile, considering the multiple failure modes in the rock tunnel stability analysis, a reliability analysis method for rock tunnel structural systems based on interval non-probability was established. On this basis, sensitivity analysis about the different effects of uncertain parameters on non-probabilistic reliability index and various failure modes was conducted. The main work and research conclusions are as follows:

(1) Based on interval theory, non-probabilistic reliability indexes and structural system reliability indexes were constructed, and an analysis method for solving non-probabilistic reliability index using interval theory was developed. Through example analysis, it is shown that the interval form can better represent the uncertain parameter characteristics, more accurately and effectively reflect the actual existence and changes of these parameters, indicating the effectiveness and rationality of this non-probabilistic analysis method.

(2) By constructing a rock tunnel model with multiple failure modes and considering the surrounding rock and support stability, a more practical functional function of the rock tunnel structural system was obtained. The reliability analysis method based on interval non-probability was used, and the non-probabilistic reliability index was solved and stability evaluation was carried out combining a rock tunnel engineering example. The final evaluation result is consistent with the actual engineering results, proving the rationality and feasibility of this analysis method.

(3) The sensitivity analysis of the non-probabilistic reliability index corresponding to each failure mode of the rock tunnel structural system shows that the non-probabilistic reliability index will decrease with the increasing range of uncertainty parameter intervals in different failure modes. The corresponding non-probabilistic reliability index has different sensitivities to each parameter. In the support bearing capacity mode, the concrete lining thickness has a significant impact on the rock tunnel stability. In the surrounding rock convergence mode, the initial geostress and internal friction angle have a significant impact on the rock tunnel stability. In terms of anchor bolt anchorage capacity, the internal friction angle has a significant impact on the rock tunnel stability.

(4) In the rock tunnel structural system, there are significant differences in the effects of the same parameters on different failure modes. The initial geostress is the most influential parameter in the second failure mode, and the least influential parameter in the third failure mode. Compared to other parameters, the variations of cohesion in the second and third failure modes have little impact on the corresponding non-probabilistic reliability indexes. The internal friction angle has the greatest impact on the reliability index compared to other parameters in the second and third failure modes.

(5) The change in the single uncertainty parameter range will lead to a change in the non-probabilistic reliability index of its corresponding failure mode, and also lead to a change in the non-probabilistic reliability index of the rock tunnel structural system, resulting in corresponding change in the main failure mode of the system.

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