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Bearing capacity of shallow foundations considering geological uncertainty and soil spatial variability

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Abstract

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Keywords

bearing capacity of shallow foundation, geological uncertainty, soil spatial variability, contribution indexes

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Bearing capacity of shallow foundations considering geological uncertainty and soil spatial variability

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Abstract: The bearing capacity of shallow foundations is significantly affected by stratum uncertainty, mainly including geological uncertainty and spatial variability of soil properties. The influence of geological uncertainty and soil spatial variability on the bearing capacity of shallow foundations has been separately investigated in previous studies. This study aims to develop a general probabilistic computational framework to reveal the effects of geological uncertainty and spatial variability of soil properties on the bearing capacity of shallow foundations, in which the geological uncertainty is simulated by Markov random field and the soil spatial variability is characterized using log-normal random field in different strata considering variations of the vertical correlation distance. Based on the borehole and soil data collected from Mawan, Shenzhen, shallow foundation bearing capacity analysis is performed according to the proposed computational framework. The subset simulation method is used to accelerate the calculation of the reliability of each scenario, and reduction factors are proposed to reduce the calculation results to different degrees with the aim of simplifying the consideration of spatial variability. Contribution indexes are defined to quantify the effects of the geological uncertainty and spatial variability of soil properties on the bearing capacity results of shallow foundations. The results show that the traditional deterministic bearing capacity calculation will overestimate the bearing capacity of shallow foundations without considering the stratum uncertainty. When the number of boreholes is sparse, the geological uncertainty has a greater influence on the calculation results; when the number of boreholes is sufficient, it is mainly dominated by the spatial variability of soil properties.

Keywords: bearing capacity of shallow foundation; geological uncertainty; soil spatial variability; contribution indexes

1 Introduction

Foundation engineering is the most common construction type in geotechnical engineering[1]. Foundations are of great significance to the use and safety of ground structures, among which shallow foundations are widely applied in foundation engineering because of their economy and efficiency. The bearing capacity of shallow foundations mainly depends on the stratum conditions, and the limited field measurement data can hardly describe the stratum uncertainty[2].

In traditional geotechnical engineering, the bearing capacity of shallow foundations is often calculated using simplified stratum distributions and deterministic soil parameters. It should be noted that this approach relies heavily on the personal experience of engineers and is highly subjective. The commonly used deterministic design method does not take into account the uncertainties of strata[3], and it only provides an average result. This simplification could not allow an accurate estimation of the bearing capacity for shallow foundations, which might pose a threat to the use and safety of structures.

In general, natural soils exhibit strong spatial variability in their properties, and the uncertainty in the stratum has a significant impact on the bearing capacity and stability of shallow foundations[4]. This uncertainty is mainly divided into two types[5]: type 1 is the spatial variability of parameters, which is caused by different loading histories of soil and is represented by the differences of soil parameters within the same stratum type; type 2 is the geological uncertainty[6], which is caused by the sedimentary movement and tectonic history of geology and is manifested in the randomness of stratum distribution.

Vanmarcke[7] introduced a random field model to depict the spatial variability of soil parameters, utilizing correlation distances. Subsequently, some scholars employed random finite element methods or random finite difference methods to analyze the deformation and stability of geotechnical structures[8–9]. However, most studies have assumed deterministic stratum distribution, overlooking geological uncertainty, which is particularly significant due to the pronounced variability of soil properties between different soil layers. Few studies have simultaneously considered the effects of geological uncertainty and spatial variability of soil parameters on underground geotechnical structures[8]. Regarding shallow foundations, Wu et al.[10] conducted an experimental study on their bearing capacity while considering the spatial variability of soil parameters. The physical model tests under different random fields exhibited substantial differences compared to the

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calculation results of the homogeneous soil model, with a relative error in the maximum bearing capacity of up to 70.10%[10]. This highlights the significant impact of the spatial distribution of soil parameters c and φ on shallow foundation bearing capacity. Nevertheless, most studies have only separately explored the effects of geological uncertainty and spatial variability of soil parameters on shallow foundation bearing capacity. Moreover, there has been limited analysis of the proportion of influence of geological uncertainty and spatial variability of soil parameters on shallow foundation bearing capacity. Therefore, there is a need to develop a quantitative assessment method to investigate the influence of these two types of variability on calculation results, which will contribute to enhancing the safety performance of shallow foundation structures under adverse geological conditions.

The objective of this study is to investigate the effects of geological uncertainty and spatial variability of soil parameters on the bearing performance of shallow foundations. A framework that considers both the geological uncertainty and spatial variability of soil parameters is proposed for calculating the bearing capacity of shallow foundations. The rest of the paper is organized as follows. Firstly, the Markov random field (MRF) is employed to simulate the geological uncertainty of the Mawan site. Secondly, based on the stratum distribution generated from the simulation, the random field theory is introduced to characterize the spatial variability of soil parameters under each simulation result. The random samples obtained from the simulations are input into FLAC^{3D} numerical simulation software to calculate the bearing capacity for shallow foundations, followed by reliability analysis using the subset simulation method. As an illustrative example, the shallow foundation construction of the liquefied natural gas power station in Mawan, Shenzhen is utilized to conduct a comprehensive assessment of the site's stratum uncertainty. Additionally, a reduction factor is introduced to streamline the consideration of soil spatial variability, and contribution indexes are formulated to quantitatively evaluate the proportion of geological uncertainty and spatial variability of soil parameters on the bearing capacity of shallow foundations. After that, the random samples obtained from the simulations are mapped into FLAC^{3D} numerical simulation software to calculate the bearing capacity for shallow foundations, followed by reliability analysis using the subset simulation method. As an illustrative example, the shallow foundation construction of the liquefied natural gas power station in Mawan, Shenzhen is utilized to conduct a comprehensive assessment of the site's stratum uncertainty. Additionally, a reduction factor is introduced to streamline the consideration of soil spatial variability, and contribution indexes are formulated to quantitatively evaluate the impact proportion of geological uncertainty and spatial variability of soil parameters on the bearing capacity of shallow foundations.

2 Simulation method

2.1 Simulation procedure

This paper introduces a shallow foundation bearing capacity calculation framework that integrates the geological uncertainty and spatial variability of soil parameters, as depicted in Fig. 1. Within this framework, the geological uncertainty is simulated using the Markov random field, while the spatial variability of soil parameters is characterized using lognormal random fields, chosen for their ability to handle non-negative data. The specific steps are outlined as follows:

(1) Divide the investigated site into suitable elements in both horizontal and vertical directions at a certain sampling distance. The initial random field and parameter β are input into the stochastic simulation algorithm and the parameter β is adjusted according to the acquired stratum information and simulation preferences.

(2) Input known borehole data for stratum modeling. Iterations are performed sequentially from the first to the last element of the random field mesh. For the *i*-th element in the *t*-th iteration, the posterior probability of the conditional probability function is computed using $P(x_i^t \left| x_{N_i}^{(t-1)} \right.)$ (N is the number of realizations of geological uncertainty). Keep iterating until the joint probability of the MRF converges. The random field result of the last iteration is saved as a single realization of the stratum distribution simulation.

(3) Once the obtained stratum distribution results are mapped to the shallow foundation numerical model and different stratum distributions are obtained, the spatial variability of soil parameters in each stratum is considered based on the consideration of geological uncertainty.

(4) Obtain the element center point coordinates of each soil layer and perform Karhunen-Loève (K-L) decomposition separately for the soil in each layer in a single stratum distribution realization, where a lognormal distribution is used here to take into account the non-negativity of the soil parameters. The discrete soil parameters are mapped into the FLAC^{3D} software according to the corresponding stratum distribution. Displacement loads are then applied to obtain a stable numerical solution for the bearing capacity of the shallow foundation at each time.

(5) Repeat step (4) until the spatial variability samples of the soil parameter reach M. M is the number of realizations of the spatial variability of soil parameters. Finally, the bearing capacity values of the stratum distribution samples considering the $1\times M$-th realization of the spatial variability of soil parameters are extracted and statistically analyzed accordingly.

The above steps are repeated until N samples of the stratum distribution are generated (i.e., a total of $N \times M$ samples), using Monte Carlo simulation. In order to ensure that the bearing capacity results converge after $N \times M$ calculations the determination of the number of Monte Carlo simulations is crucial

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for stochastic probability analysis. The method for determining the convergence criteria will be described in the next section.

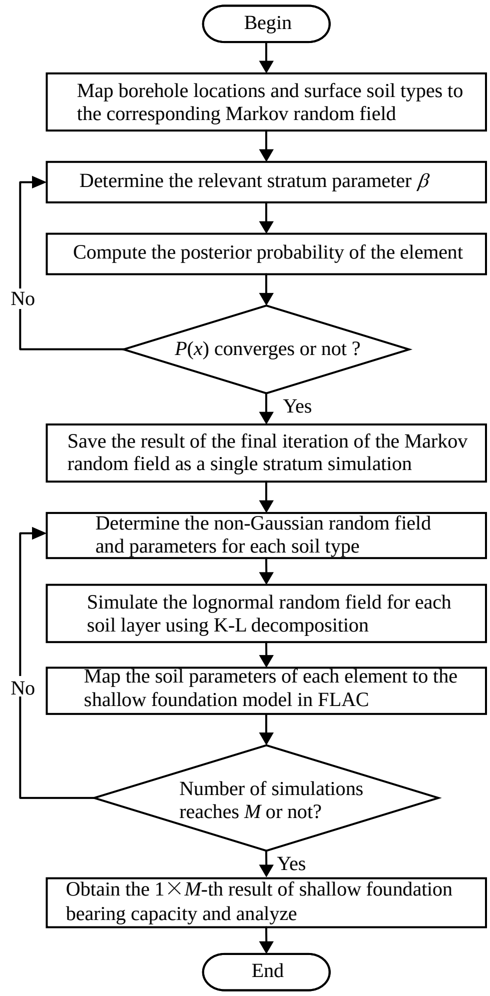


Fig. 1 Flowchart of bearing capacity calculation for shallow foundations considering geological uncertainty and soil spatial variability

2.2 Determination of number of Monte Carlo simulations

In Markov random fields, the number of geological uncertainty samples is generally determined by the information entropy, and the convergence of the information entropy is used to determine an appropriate number of samples. In past studies, the sample size of the MRF is generally set at 100. Therefore, $N = 100$ is chosen as the sample size of the stratum distribution in this paper. In the lognormal random field of soil parameters, the number of samples considering the spatial variability of soil parameters is generally determined by the convergence of the mean. Fig.2 shows the effect of the number of Monte Carlo simulations of spatial variability of soil parameters on the mean of the shallow foundation bearing capacity. From the figure, it is evident that the mean bearing capacity of the shallow foundation converges after 100 realizations; hence, $M = 100$ is used as the sample size for the spatial variability of soil parameters in this paper.

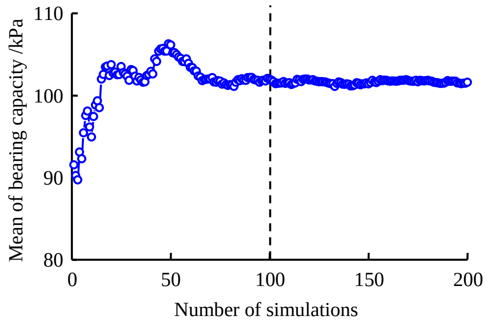


Fig. 2 Effect of soil spatial variability on the mean of bearing capacity

In this paper, $N = 100$ and $M = 100$ (i.e., $N \times M = 10\ 000$ samples) are chosen as the sample sizes for the geological uncertainty and spatial variability of soil parameters, respectively. It is worth mentioning that 10 000 is sufficient to ensure the calculation accuracy of the mean of bearing capacity and confidence interval of the shallow foundation. However, for the failure probability of the shallow foundation, the number of samples should be increased appropriately to ensure the accuracy of the reliability analysis. In order to improve the computational efficiency, this paper adopts the subset simulation method for the reliability analysis of shallow foundation bearing capacity.

2.3 Simulation of geological uncertainty

The Markov random field can be used to simulate the stratum distribution. It is assumed that a random field $X = [X_i, i \in V]$ is composed of a series of random variable X_i defined in the unit mesh system $S = [1, 2, \cdots, N]$. Each random variable X_i takes its value from the labeling space $L = [l_1, l_2, \cdots, l_n]$ using the probability $P = [X_i = x_i]$, $x_i \in L$. The random field configuration $D = [X_1 = x_1, X_2 = x_2, \cdots, X_N = x_{xN},]$, joint probability $P = [X_1 = x_1, X_2 = x_2, \cdots, X_N = x_{xN},]$, and configuration space $\Omega = [x = [x_i], i \in S, x_i \in L]$ generated by the random field X is the set of all configurations of the random field, representing all possible random field realizations. As shown in Fig. 3, the neighborhood system of the element i is defined as $N_i = [j | j \in S], N_i \in S$, where j represents the adjacent elements i. The element i has a local neighborhood system N_i containing 8 neighborhoods $[j_1, j_2, \cdots, j_n]$. In the neighborhood system, an element cannot be a neighbor of itself, and the neighborhood relations are mutual, satisfying $i \notin N_i$ and $i \in N_j \leftrightarrow j \in N_i$.

The Markov random field X and its neighbourhood system N_i need to meet the following two conditions

$$P(x) > 0,\ \forall x \in \Omega \tag{1}$$

$$P(x_i | x_j, i \neq j) = P(x_i | x_j, j \in N_i) \tag{2}$$

Equation (1) specifies that the probability of the MRF must be positive. Eq. (2) describes the local properties of the MRF, indicating that any random variable in the MRF is only affected by its neighboring variables. The Hammersley-Clifford theory establishes an equivalence between the MRF and the Gibbs random field in order to transform the probability distribution formula of the MRF into an explicit exponential function form. The equivalent exponential function of the joint probability $P(x)$ is expressed as

$$P(x)=\frac{1}{Z}\exp(-\sum_{c\in C}V_{c}(x)/T) \tag{3}$$

$$Z=\sum_{x\in\Omega}\exp(-\sum_{c\in C}V_{c}(x)/T) \tag{4}$$

where the cliques c is a subset of the neighborhood system, and any two elements comprising the group c are adjacent; C denotes the set of cliques c; the temperature constant T is generally set to be 1; and V_c, as part of the total energy of the random field, is the potential energy of the group c, reflecting the strength of the inherent spatial connections in the local neighborhood system, and it can be further expressed as

$$V_{c}(x_i,x_j)=\begin{cases}-\beta(i,j),\ j\in N_i \text{ and } x_i=x_j\\ 0, \qquad \text{others}\end{cases} \tag{5}$$

where $\beta(i,j)$ can measure the strength of the spatial connection of the neighborhood system. Fig. 3 shows a local neighborhood system and the corresponding $\beta(i,j)$. The long side, short side and angle of the ellipse are a, b and φ, which are related to the prior information of the investigated site. These prior parameters determine the value of $\beta(i,j)$. A large value of $\beta(i,j)$ results in a small potential energy for the group, which enhances the spatial connection of the neighboring system, i.e., the labels of the adjacent elements tend to be consistent. Therefore, $\beta(i,j)$ is an essential prior parameter. The value of $\beta(i,j)$ is based on the understanding and judgment of the distribution state of the spatial variables and is adjusted during the simulation according to the experience and preference. A detailed explanation of the MRF and the modeling process can be found in the relevant literature[13].

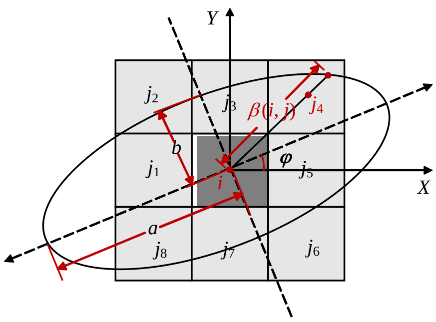


Fig. 3 Schematic diagram of the neighborhood system

2.4 Simulation of spatial variability of soil parameters

The corresponding stratum distribution, i.e., the soil type distribution, can be obtained from the simulation results of the geological uncertainty in section 2.3. The spatial variability of the parameters for each soil type can be simulated separately according to the stratum distribution using lognormal random fields. The autocorrelation function is used in random fields to describe the spatial correlation between two points. The single index autocorrelation function adopted in this study is the most extensively used autocorrelation function in geotechnical engineering[5], as shown in the following equation.

$$\rho(\tau_x,\tau_y)=\exp\left[-2\left(\frac{|\tau_x|}{\delta_{\mathrm{h}}}+\frac{|\tau_y|}{\delta_{\mathrm{v}}}\right)\right] \tag{6}$$

where $\rho(\tau_x,\tau_y)$ is the correlation coefficient between any two separated points; τ_x and τ_y are the horizontal and vertical distances between two points in space, respectively; δ_{h} and δ_{v} are the horizontal and vertical correlation distances (CD). Correlation distance is an important concept in the random field modeling for geotechnical parameters. The correlation of parameters between two points in space decreases as the distance between the two points increases, and the correlation is negligible when the distance is greater than a certain critical distance. The critical distance is called the correlation distance[11].

Among the properties of soil, it is widely recognized that cohesion c and friction angle φ are important parameters affecting the bearing capacity of shallow foundations. Therefore, in this paper, the cohesion and friction angle are discretized by random fields, respectively. Due to the fact that the bearing capacity is not sensitive to other parameters, the bulk modulus K and shear modulus G are considered as constant parameters in this study. The K-L decomposition has high accuracy for random fields. The K-L expansion method is hence used for the discrete random field, and the corresponding lognormal random field $H_i(x,\theta)$ can be written as

$$\left.\begin{array}{l}H_i(x,\theta)=\exp[\mu_i+\sum_{j=1}^{M}\sigma_i\sqrt{\lambda_j}\varphi_j(x)\chi_{i,j}^{D}(\theta)]\\ \qquad (\text{for } i=c,\varphi)\end{array}\right\} \tag{7}$$

$$\mu_i=\ln\mu_i-0.5\sigma_i^2(\text{for } \ i=c,\varphi) \tag{8}$$

$$\sigma_i=\sqrt{\ln(1+\mathrm{COV}_i^2)} \tag{9}$$

where μ_i and σ_i are the mean and standard deviation of the corresponding soil parameters; λ_j and $\varphi_j(x)$ are the discrete eigenvalue and eigenvector; and $\chi_{i,j}^{D}(\theta)$ ($i=c,\varphi$) denotes the random vector with D independent random variables, where D is the number of K-L expansion terms and is taken as 700 in this study. The coefficient of variation (COV) of the soil parameters is set as 0.3.

2.5 Subset simulation

The subset Monte Carlo simulation can accelerate the simulation of spatial random fields[8, 12]. Monte Carlo subset algorithm is a type of Markov chain, which mainly consists of the following steps.

(1) Calculate the initial failure probability P_{f}. The Monte Carlo method is used to generate n samples to obtain the value of the limit state function $G=[N]-N_i$, where N_i is the calculated bearing capacity for each sample and $[N]$ is the allowable bearing capacity. The values of G are then arranged in

increasing order and used. The value of c_i is equal to the limit state function value G_0 corresponding to the failure proportion p_0 of the subset Monte Carlo simulation method, that is, the c_i is equal to the $[(1-p_0)n]$-th value from the largest to the smallest in $[G_i: i=1, 2, \cdots, n]$. p_0 is the specific conditional probability under each layer during the subset simulation, i.e., the failure proportion of each layer.

(2) $i > p_0 n$ obeys $G_i < c_1$ when calculating the initial failure probability. The above $p_0 n$ samples are retained as the subset seeds and are used as the samples for calculating the failure probability $P_f(F_2)$. Based on the Markov property, the previously generated samples are used as conditional data to generate other $(1-p_0)n$ samples.

(3) Repeat the above steps, and calculate sequentially $F_3, F_4, \cdots, F_m$ where F_m is the m-th intermediate failure event, until $C_m = 0$. Then the system failure probability is

$$P_f(F) \approx P_f(F_0)^{m-1} P_f(F_m | F_{m-1}) \quad (10)$$

The total number of samples N_{tot} required for the subset simulation method is

$$N_{tot} = M \times (1-p_0)n + n \quad (11)$$

3 Shallow foundation example

3.1 Site information

Zhang et al.[13] collected the borehole and soil data of Mawan, Shenzhen and carried out relevant studies, as shown in Fig.4. For convenience, the simplified engineering geological survey map is taken as the real stratum in this paper. The main soil types within the stratum distribution range are Plain fill, silt, clay, sand, and weathered rock, showing soft characteristics at the upper and hard at the lower. In the figure, *B* is the width of the shallow foundation.

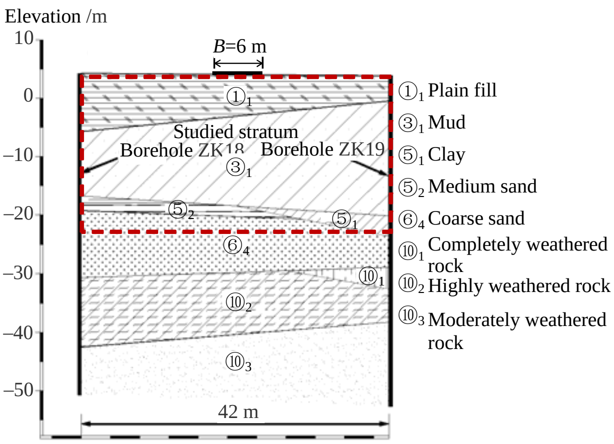


Fig. 4 Stratum profile and soil types

In this paper, four types of near-surface soils adjacent to the shallow foundation are selected for analysis, i.e., plain fill, mud, clay and coarse sand. The soils are characterized by low strength and poor stability. This may result in inadequate bearing capacity of the shallow foundation. In addition, the soil cohesion and friction angle have strong spatial variability[10]. Since the plain fill, mud and clay layers at the site have a large influence on the shallow foundation bearing capacity, cone penetration tests (CPT) were conducted for the three soil layers. The COVs of the CPT tip resistance measured is 0.54, 0.10, and 0.28 for the Plain fill, mud, and clay, respectively.

Equidistant virtual boreholes, as illustrated in Fig. 5, refer to additional boreholes placed at different locations within the stratum distribution based on various soil types. For simplicity, the virtual boreholes in this paper are equally spaced, and the impact of borehole locations on the modeling is not addressed. Furthermore, it is noted that the depth of the boreholes should extend beyond the depth of the geotechnical structure's influence area. In this study, the depth of the boreholes is considered to be three times the width of the foundation.

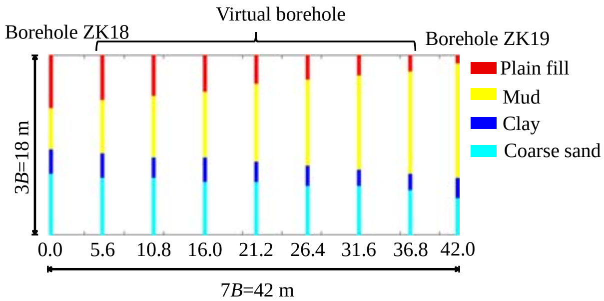


Fig. 5 Layout of the equidistant 9 borehole scheme

3.2 Finite difference modeling

The statistical characteristics of the soil parameters for each soil layer, including the cohesion, friction angle, Poisson's ratio, Young's modulus and dilatancy angle, are shown in Table 1. Among the soil parameters, cohesion and friction angle are the parameters that significantly affect the bearing capacity of shallow foundations[10]. Therefore, this paper focuses on the effect of the spatial variability of cohesion and friction angle. Additionally, relevant literature has reported that the vertical correlation distance has a greater impact on geotechnical engineering than the horizontal correlation distance[10]. For this reason, only the variation of the vertical correlation distance δ_v is concerned in this paper. This paper also investigates the effect of geological uncertainty under different borehole schemes, and four borehole schemes are set up, namely the equidistant 3 borehole scheme (BS3), equidistant 5 borehole scheme (BS5), equidistant 7 borehole scheme (BS7), and equidistant 9 borehole scheme (BS9). Five random field simulation scenarios (ANI-1 to ANI-5) with vertical correlation distances are also set up for each borehole scheme, and the detailed parameters are shown in Table 2.

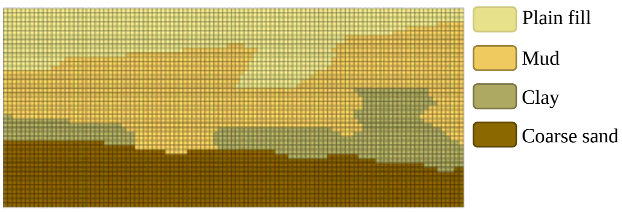
Figure 6 illustrates the results of one simulation of the stratum distribution, cohesion and friction angle for the case ANI-2 under the BS5 scheme. Different colors in the figure represent different parameter values, and the specific legend is shown on the right side of the figure. Blue represents areas with small values, and red represents areas with large values.

Table 1 Soil properties of each stratum

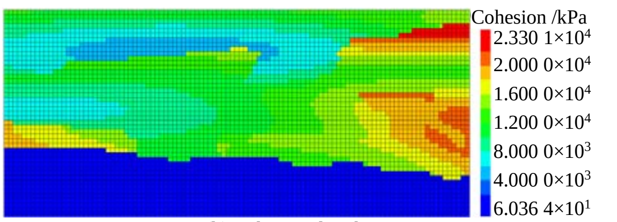
Soil type	Cohesion		Friction angle		Poisson's ratio	Young's modulus /MPa	Dilatancy angle /(°)
	Mean /kPa	COV	Mean /(°)	COV			
Plain fill	15	0.3	12	0.3	0.30	15.64	0
Mud	16	0.3	2	0.3	0.35	9.20	0
Clay	30	0.3	10	0.3	0.30	20.00	0
Coarse sand	10	0.3	35	0.3	0.25	50.00	5

Table 2 Correlation distances in anisotropic random fields

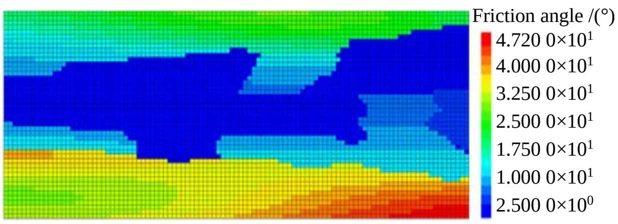
Case	Correlation distance /m		Anisotropy ratio	δ_h/B	δ_v/B	COV
	δ_h	δ_v	δ_h/δ_v			
ANI-1	60	3.0	20.00	10.0	0.5	0.3
ANI-2	60	6.0	10.00	10.0	1.0	0.3
ANI-3	60	15.0	4.00	10.0	2.5	0.3
ANI-4	60	30.0	2.00	10.0	5.0	0.3
ANI-5	60	60.0	1.00	10.0	10.0	0.3



(a) Stratum distribution



(b) Cohesion distribution



(c) Friction angle distribution

Fig. 6 Random field realizations of the equidistant 5 borehole scheme

The shallow foundation model depicted in Figure 7 was developed using FLAC^{3D} software, with the stratum derived from a simplified engineering geological map. In present study, the shallow foundation width B is set to be 6 m. To account for the boundary effect of the model and the influence range of the shallow foundation, the length of the finite difference model is $7B = 42$ m, and the depth is $3B = 18$ m, and the plane strain condition is adopted. The model mesh is composed of rectangular elements with a side length of 0.4 m, totaling $105 \times 45 = 4\ 725$ elements. The soil is represented by an elasto-plastic model obeying the Mohr-Coulomb yield criterion. The bottom boundary of the model is fixed, the left and right boundaries are constrained horizontally, and the top of the model is set as a free boundary. The bottom of the shallow foundation is assumed to be in hard contact with the soil, and the shallow foundation is placed in the middle of the model top. A velocity load of 0.5×10^{-4} m/step is applied to the shallow foundation model nodes, with 10,000 calculation steps set. The calculation results of the shallow foundation displacement and maximum shear strain under the simplified stratum condition in Fig.7 are plotted in Fig. 8. The results exhibit asymmetry due to a dip angle in the strata strike. In the displacement calculation results, the strata under the shallow foundation are depressed, while the soil at both sides is uplifted. The corresponding failure plane can be seen in the maximum shear strain contour.

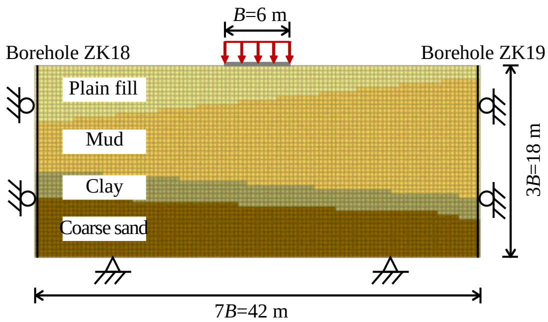
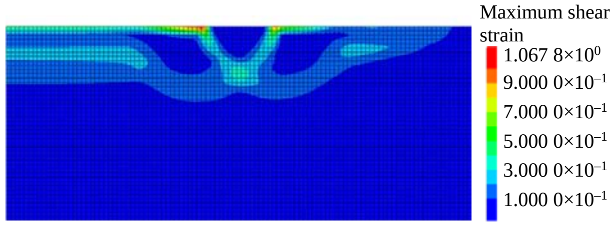
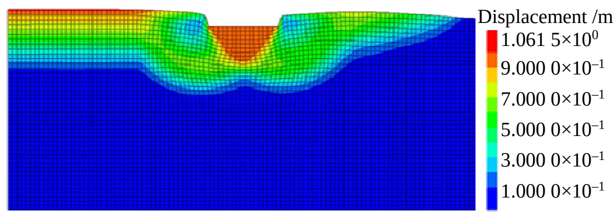


Fig. 7 Finite difference model of the simplified stratum



(a) Displacement



(b) Maximum shear strain

Fig. 8 Calculation results for the simplified stratum

3.3 Validation of the computational framework

To validate the present model in this study, a comparison of the shallow foundation bearing capacity is made with Zhang et al.[13] under the consideration of geological uncertainty. The histogram in Fig. 9 shows the calculation results of the stratum realization under the 3 borehole scheme.

In the figure, GU and SV are the abbreviations of geological uncertainty (GU) and spatial variability of soil parameters (SV), respectively. The figure statistically shows the bearing capacity results of this paper considering both the geological uncertainty and spatial variability of soil parameters and the shallow foundation bearing capacity results under considering only geological uncertainty, as well as the comparative analysis with the calculation results in the literature[14]. As can be seen from Fig. 9, the mean of the shallow foundation bearing capacity obtained in the literature[14] only considering the geological uncertainty is 106.98 kPa, and the coefficient of variation (COV) is 0.102. the mean of the shallow foundation bearing capacity obtained by considering only the geological uncertainty is 107.13 kPa, and the COV is 0.1. The results of the two are basically consistent. The good agreement verifies the feasibility of the proposed finite difference model in the geological uncertainty analysis.

The effects of both the geological uncertainty and spatial variability of soil parameters on the bearing capacity of shallow foundations are also considered in this study. The grey part in Fig. 9 is the histogram of the frequency distribution of the 100 calculation results. The distribution of the calculation results has a greater variability compared to the results of considering only the geological uncertainty, and the mean of the calculation result is 98.2 and the COV is 0.232. The phenomenon indicates that more unfavorable distributions of the strata and soil parameters can be taken into account when considering simultaneously the two variability types compared to considering only the geological uncertainty. This allows for a more comprehensive understanding of the bearing capacity of shallow foundations.

Case	Mean /kPa	COV	Number of samples
This study, GU&SV	98.20	0.232	100
Zhang et al.[13], GU	106.98	0.102	100
This study, GU	107.13	0.100	100

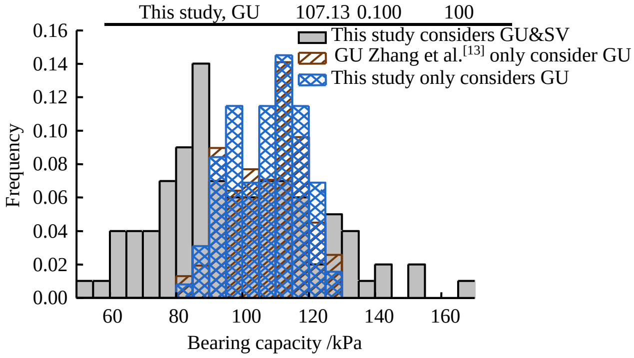


Fig. 9 Comparison of result histograms of shallow foundation bearing capacity models

3.4 Analysis of simulation results

In this study, the calculation result of 109.68 kPa for simplified stratum under the deterministic condition is used as a deterministic solution for the shallow foundation bearing capacity. In this section, the effects of geological uncertainty and spatial variability of soil parameters on the shallow foundation bearing capacity are considered under different borehole schemes. When the spatial variability of soil parameters is considered and the vertical correlation distance remains unchanged for the case ANI-2, i.e., $\delta_v = B = 6$ m. The frequency distribution of the shallow foundation bearing capacity results for the four borehole schemes is shown in Fig. 10. The black solid line in the figure indicates the deterministic result and the dashed line indicates the mean of the Monte Carlo results. As the number of boreholes increases, the mean of the Monte Carlo calculation result gradually approaches the deterministic result of the simplified stratum. As an example, the deterministic result for bearing capacity is slightly higher than the mean bearing capacity for the BS5 scheme. That is, the failure probability of the shallow foundation will be overestimated if the load of the deterministic scheme is applied to the shallow foundation. The deterministic scheme and the mean of each scheme cannot well reflect the actual situation of stratum variation and fully evaluate the bearing capacity of shallow foundations. Zhang et al.[12] and Zhang et al.[15] pointed out that the use of the confidence interval limit value can better evaluate the impact of uncertainty on underground structures than the mean value. The paper adopts the lower bound of 95% confidence

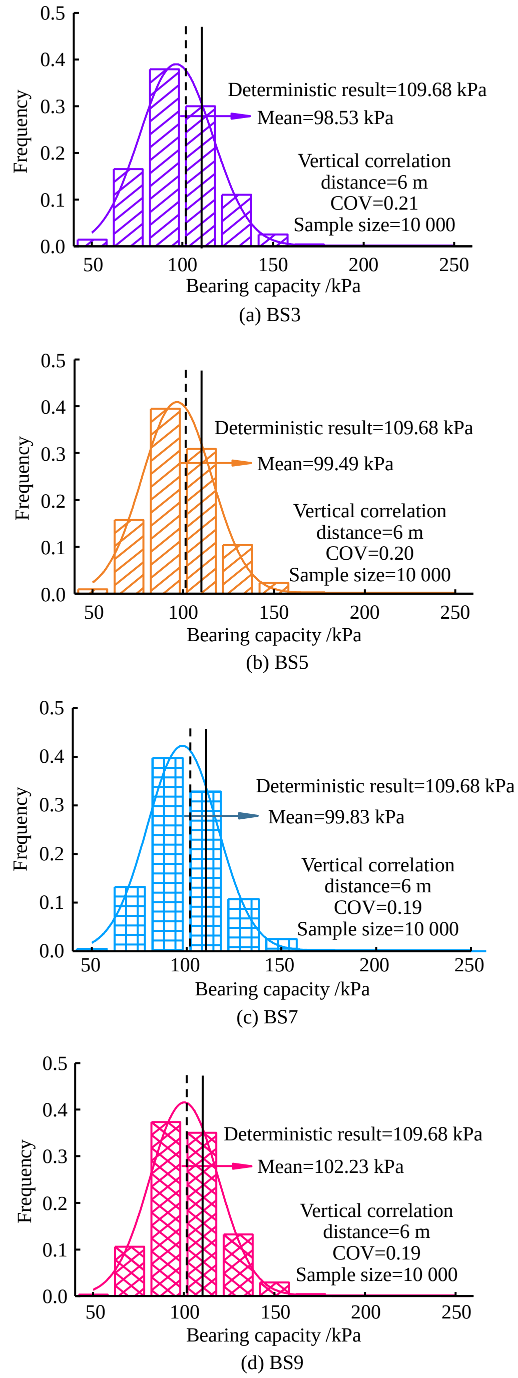
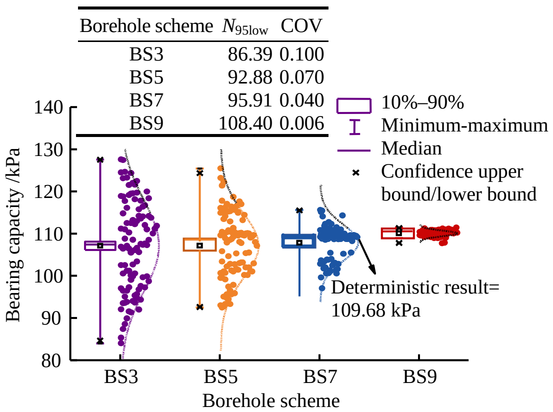


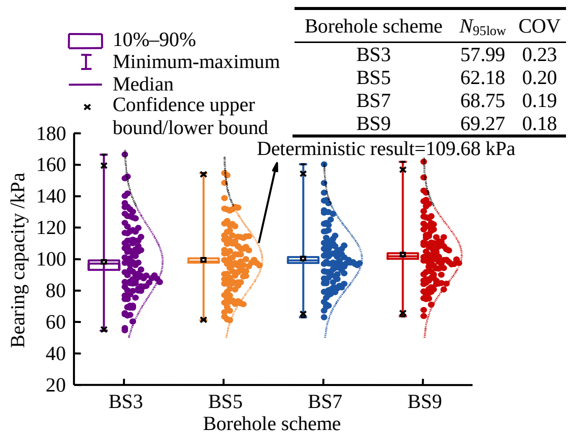
Fig. 10 Frequency distribution histogramsof calculation results for each borehole scheme

interval of the stochastic shallow foundation bearing capacity to analyze the relevant results.

Further analysis of the statistical characteristics of the Monte Carlo results of the shallow foundation bearing capacity for different borehole schemes is shown in Fig. 11, where N_{95low} represents the 95% confidence lower bound of the bearing capacity. The box represents the 80% confidence interval and the triangle represents the 95% confidence interval for the scheme. As shown in Fig. 11(a), the COV of the calculated results ranges from 0.006 to 0.100 when only geological uncertainty is considered, and the COV gradually decreases with the increase of the number of boreholes. The corresponding 100 calculation results considering both the geological uncertainty and spatial variability of soil parameters are shown in Fig.11(b). The COV of the bearing capacity of each borehole scheme are much larger than those considering only the geological uncertainty. In addition, the COV of the calculation results decreases gradually from 0.23 to 0.18. This suggests that as the number of boreholes increases, the impact of geological uncertainty on the results is relatively diminished, and the variations in the calculated results are primarily attributed to the spatial variability of soil parameters. Given the relatively uniform assumed distribution of strata in this study, the influence of geological uncertainty on the shallow foundation bearing capacity is minimal. However, it is important to note that in other sites, geological uncertainty may continue to significantly affect engineering performance.



(a) Only considering the geological uncertainty



(b) Considering both the geological uncertainty and spatial variability of soil parameters

Fig. 11 Boxplot of normal distribution of calculation results

Figure 12 illustrates the effect of different vertical correlation distances on the shallow foundation bearing capacity under the BS3 scheme. As the vertical correlation distance increases, the COV of the shallow foundation bearing capacity increases gradually. The lower bound of the 95% confidence of the bearing capacity is 58.63 kPa at $\delta_v = 60$ m.

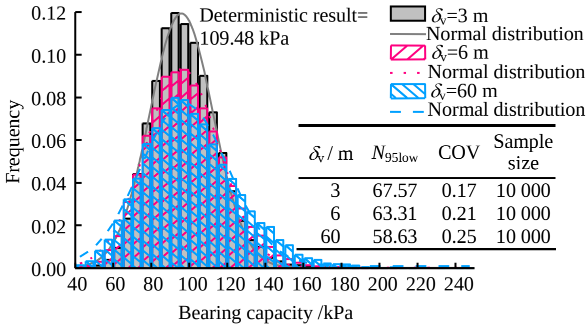


Fig. 12 BS3 frequency distribution histogram of bearing capacity at different vertical correlation distances

As more geological uncertainty is taken into account, the lower bound of the 95% confidence of the bearing capacity reduces accordingly. In addition, in the BS3 scheme plotted in Fig.13, there is a maximum COV of 0.25 at $\delta_v = 60$ m. On the contrary, in the BS9 scheme, the smallest COV of 0.15 is observed at $\delta_v = 3$ m. This is because the stratum distribution becomes more clear as the number of boreholes increases, while the spatial variability of soil parameters is weakened with the reduction of the correlation distance. The results further prove the validity and correctness of the calculation framework for the bearing capacity of shallow foundations.

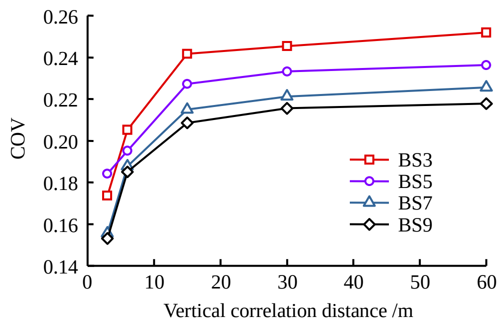


Fig. 13 Coefficients of variation under different vertical correlation distances and borehole schemes

3.5 Reliability analysis

The limit state function G of the bearing capacity of shallow foundations can be defined as

$$G = \frac{N_{de}}{F_s} - N_i \quad (12)$$

where N_{de} is the deterministic calculation result; and F_s is the safety factor. The reliability index is denoted by $\beta = \Phi^{-1} \cdot [P_f(G \leqslant 0)]$, where P_f is the failure probability for $G \leqslant 0$ and the operator Φ^{-1} represents the inverse function of the cumulative standard normal distribution.

According to the *Unified Standard for Reliability Design of Engineering Structures* (GB 50153-2008)[16], the lower and upper bounds of the reliability index should comply with the standard limits between 2.70 and 4.20 (i.e., the range of the failure probability is from 0.003 5 to 0.000 01). The result is between the limit state under normal service conditions and the bearing capacity limit state. For the subset simulations in this paper, the number of intermediate failure events is taken as $m = 3$, the number of samples per failure event is $N = 500$, and a total of 3 200 samples are calculated. The samples are randomly selected from the 10 000 samples for each of the above scenarios.

Figure 14 plots the reliability and failure probability under different safety factors. When the safety factor of the case ANI-2 under the BS3 scheme is 1.6, the reliability index is 3.04 and the corresponding failure probability is 0.001 2, which is in line with the structural reliability design standard. The failure probability distribution under different schemes and cases with a safety factor of 1.6 is illustrated in Fig. 15. One can see from the figure that the vertical correlation distance has a significant impact on the failure probability of the shallow foundation when the number of boreholes is less. The maximum failure probability is observed when the number of boreholes is 5 and the vertical correlation distance is 25 m.

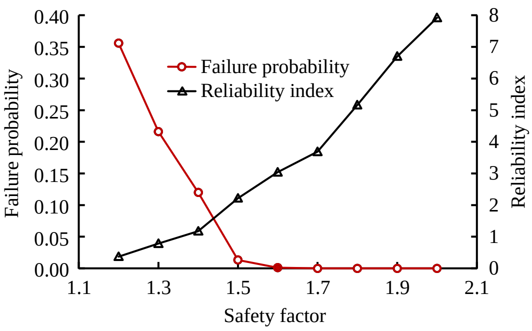


Fig. 14 Reliability indexes and failure probabilities under different safety factors

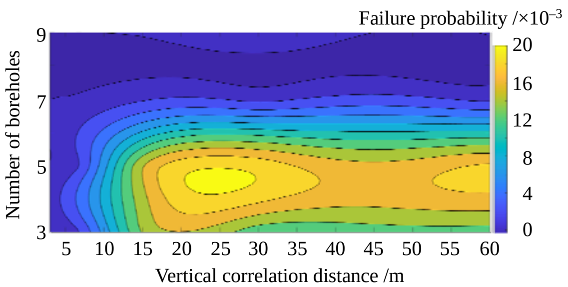


Fig. 15 Coefficients of variation under different vertical correlation distance and borehole schemes

4 Reduction factor analysis

Given that borehole arrangements at a site are generally sparse in actual engineering, employing the deterministic calculation method may lead to a high probability of failure and an overestimation of the actual bearing capacity of shallow foundations. It is therefore necessary to simultaneously consider the effects of geological uncertainty and spatial variability of soil parameters on shallow foundations. In order to simplify the practical application of engineering, the study utilizes the reduction factor method[11]. This method simplifies the consideration of the influences of geological uncertainty and spatial variability of soil parameters on the calculation by reducing the deterministic calculation results to varying degrees. Previous studies have shown that the upper or lower bound of the confidence interval of Monte Carlo calculation results is more representative for considering the geological uncertainty and spatial variability of soil parameters compared to the mean value[17–19]. Therefore, based on a large number of calculations, the 95% confidence lower bound is taken as the evaluation index, and the reduction factor is defined as follows:

$$\lambda = \frac{N_{95\mathrm{Low}}}{N_{\mathrm{De}}} \tag{13}$$

The reduction factor for the variability condition is determined by the ratio of $N_{95\mathrm{Low}}$ to N_{De}. This approach simplifies the consideration of geological uncertainty and spatial variability of soil parameters by adjusting the calculation results to different degrees based on traditional deterministic calculations.

Table 3 summarizes the reduction factors for the shallow foundation bearing capacity considering the variations of vertical correlation distances and borehole numbers. Fig. 16 displays the relationship between the vertical correlation distance and the reduction factor under different borehole numbers. The minimum reduction factor is 0.535. In the case of a small number of boreholes and a large vertical correlation distance, the reduction factor for the shallow foundation bearing capacity is relatively small. Therefore, the appropriate reduction factor should be selected according to the vertical correlation distance and the number of boreholes at the investigated site, and this should be used to take into account the influence of the geological uncertainty and spatial variability of soil parameters on the calculation of shallow foundation bearing capacity.

Table 3 Reduction factor under different schemes

Case	COV	Vertical correlation distance δ_v /m				
		3	6	15	30	60
BS3	0.3	0.616	0.577	0.547	0.541	0.535
BS5	0.3	0.606	0.599	0.563	0.558	0.561
BS7	0.3	0.648	0.617	0.590	0.590	0.588
BS9	0.3	0.666	0.635	0.616	0.607	0.615

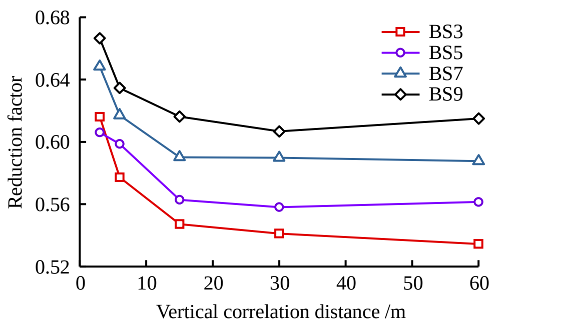


Fig. 16 Reduction factors of different vertical correlation distances and borehole schemes

5 Assessment of the impact of geological uncertainty and spatial variability of soil parameters on computational results

The analysis of the reduction factor in the previous section shows that the number of boreholes and the vertical correlation distance affect the random fields considering the geological uncertainty and spatial variability of soil parameters, and ultimately determine the bearing capacity of shallow foundations. In order to further analyze the influence of the two on the calculation, this paper defines a contribution index to quantify the proportion of influence of these two variabilities on the results.

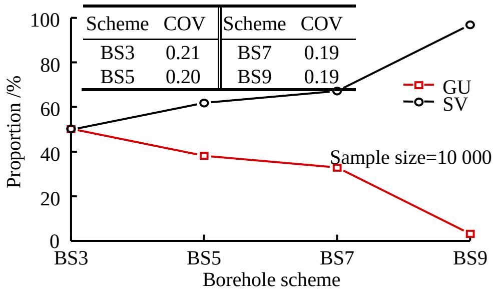
$$\alpha_{GU} = \frac{1-\lambda_{GU}}{1-\lambda_{GUSV}},\ \alpha_{SV} = 1-\alpha_{GU} \quad (14)$$

where α_{GU} represents the proportion of the influence of geological uncertainty on the calculation results, while α_{SV} represents the proportion of the influence of spatial variability of soil parameters. $1-\lambda_{GU}$ is the variable value of the reduction factor when considering only geological uncertainty, and $1-\lambda_{GUSV}$ is the variable value of the reduction factor when considering both geological uncertainty and spatial variability of soil parameters.

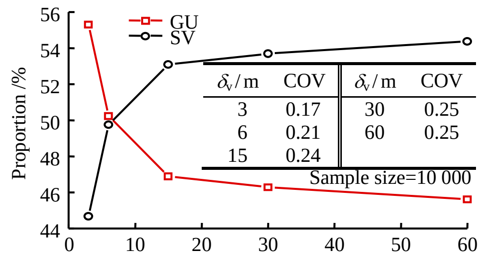
As depicted in Fig. 17(a), the proportion of bearing capacity reduction resulting from geological uncertainty decreases with an increase in the number of boreholes. In the BS3 borehole scheme, the effects of geological uncertainty and spatial variability of soil parameters on bearing capacity reduction are evenly balanced. Conversely, in the BS9 scheme, the reduction in shallow foundation bearing capacity is predominantly attributed to the spatial variability of soil parameters, accounting for 96.81%. This suggests that the calculation framework of the BS9 borehole scheme can be simplified to a lognormal random field calculation framework that solely considers the spatial variability of soil parameters.

In addition, the variation of the vertical correlation distance affects the model calculations. In this paper, only the BS3 scheme is selected for analysis and the variation pattern of the other schemes is similar to this scheme. As plotted in Fig. 17(b), with the increase of the vertical correlation distance, the effect of the spatial variability of soil parameters gradually enhances while the effect of the geological uncertainty gradually reduces and eventually tends to be stable.

It is important to highlight that at a depth of 3 m, the reduction in bearing capacity is significantly influenced by geological uncertainty, accounting for 55.32% of the reduction, while 44.68% is attributed to the spatial variability of soil parameters. As the vertical correlation distance increases, the influence of spatial variability of soil parameters on the calculation results gradually becomes more pronounced, whereas the influence of geological uncertainty gradually diminishes. In practical projects, boreholes are typically sparsely distributed, underscoring the significance of considering both geological uncertainty and spatial variability of soil parameters in the calculation of shallow foundation bearing capacity.

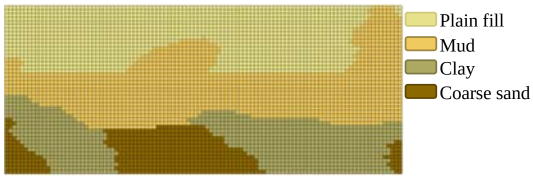


(a) Different borehole schemes

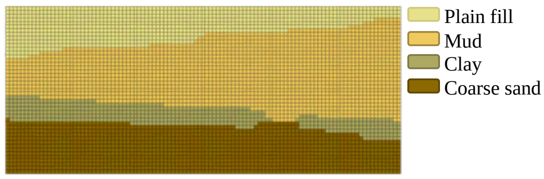


(b) Different vertical correlation distance

Fig. 17 Bearing capacity percentage curves



(a) Stratum distribution (BS3)



(b) Stratum distribution (BS9)

Fig. 18 Realization results of different schemes

To visually observe the impact of geological uncertainty and spatial variability of soil parameters, Fig. 18 presents a typical realization of stratum distribution under different borehole schemes. In Fig. 18(a), the stratum distribution appears chaotic in the BS3 scheme, emphasizing the significant influence of geological uncertainty on the bearing capacity calculation results. Conversely, Fig. 18(b) depicts the simulation results of the BS9 scheme, where the stratum distribution closely resembles the given simplified stratum, indicating that the reduction in bearing capacity is primarily attributed to the spatial variability of soil parameters. As the number of boreholes increases, the clarity of the stratum distribution improves, leading to weakened variability and dominance of spatial variability of soil parameters. In most result analyses, the influence of geological uncertainty is found to be smaller than that of spatial variability of soil parameters. This is due to the fact that the bearing capacity of shallow foundations largely depends on the near-surface soil layer, resulting in a relatively smaller influence of geological uncertainty and a relatively larger influence of spatial variability of soil parameters. It's important to note that this proportional relationship may change for deeply embedded geotechnical structures.

In actual engineering projects with a sparse number of boreholes, the impact of geological uncertainty on the results cannot be overlooked. However, in sites with sufficient boreholes, it is reasonable to focus solely on considering the spatial variability of soil parameters when calculating the bearing capacity of shallow foundations. Furthermore, the extent of influence of geological uncertainty and spatial variability of soil parameters on shallow foundation bearing capacity can be further validated through the acquisition of field monitoring data in future studies.

6 Conclusion

This paper explores the impact of geological uncertainty and spatial variability of soil parameters on the bearing capacity of shallow foundations. It introduces a stochastic computational framework utilizing the random finite difference method and conducts corresponding reliability analyses. Additionally, it proposes a reduction factor to streamline the consideration of geological uncertainty and spatial variability of soil parameters for engineering applications. Furthermore, it defines contribution indexes to quantitatively analyze the influence of these two types of variability on the calculation results, leading to three key conclusions.

(1) The findings suggest that the number of boreholes and the vertical correlation distance significantly impact the bearing capacity of shallow foundations. A small number of boreholes and a large vertical correlation distance lead to pronounced geological uncertainty and spatial variability of soil parameters, resulting in higher failure probabilities for the structure. The traditional deterministic calculation method may overestimate the bearing capacity of shallow foundations under these conditions.

(2) The paper introduces the reduction factor method to simplify the consideration of spatial variability by applying varying degrees of reduction to the deterministic analysis results. The reduction factor is the ratio of the confidence lower bound of the calculation result to the deterministic result of the shallow foundation bearing capacity. When the borehole scheme is BS3, the minimum reduction factor is approximately 0.535. It indicates that the deterministic result is reduced by almost half when the variability is taken into account. This underscores the significance of accounting for geological uncertainty and spatial variability of soil parameters, particularly when the number of boreholes is insufficient.

(3) Furthermore, the paper defines the contribution index to quantitatively assess the influence of geological uncertainty and spatial variability of soil parameters on the calculation results, in conjunction with the concept of the reduction factor. The contribution index is influenced by the number of boreholes and the vertical correlation distance, reflecting the impact of geological uncertainty and spatial variability of soil parameters. In the BS3 scheme, geological uncertainty contributes 55.32% to the reduction in bearing capacity, while spatial variability of soil parameters accounts for 44.68% of the reduction. However, in the BS9 scheme, the contribution of spatial variability of soil parameters to the bearing capacity reduction increases to 96.81%. This is attributed to the clearer stratum distribution enabled by a larger number of boreholes, validating the proposed calculation framework.

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