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Abstract

In order to improve the applicability of Hoek-Brown failure criterion and reduce subjectivity in the determination of geological strength index (GSI) value for anisotropic rocks, a modified Hoek-Brown failure criterion is proposed, which considers the variation of the GSI value with the bedding angle of anisotropic rocks. Triaxial compression test data of anisotropic rocks with different bedding angles were first collected. The results show that the peak strength of anisotropic rocks exhibits a U-shaped relation with bedding angle α . Then, the rock specimen with the bedding angle $\alpha = 0^\circ$ is defined as the intact rock. The uniaxial compressive strength σ_c and material parameter m_i of intact rocks are obtained from data fitting using Hoek-Brown failure criterion. The corresponding GSI values under different bedding angles are calculated. The relationship between GSI and bedding angle α is fitted using Gaussian function, and a new strength model of anisotropic rocks is established based on Hoek-Brown failure criterion. Finally, the proposed model is verified by comparing the peak strength obtained from the GSI-softening model. It is found that the modified Hoek-Brown failure criterion is suitable for predicting the strength of anisotropic rocks with different bedding angles and under various confining pressure. The physical meaning of the new parameters in the model is also discussed.

Keywords

anisotropic rock, bedding angle, Hoek-Brown failure criterion, GSI-strength model

Authors

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Strength model of anisotropic rocks based on Hoek-Brown criterion

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Abstract: In order to improve the applicability of Hoek-Brown failure criterion and reduce subjectivity in the determination of geological strength index (GSI) value for anisotropic rocks, a modified Hoek-Brown failure criterion is proposed, which considers the variation of the GSI value with the bedding angle of anisotropic rocks. Triaxial compression test data of anisotropic rocks with different bedding angles were first collected. The results show that the peak strength of anisotropic rocks exhibits a U-shaped relation with bedding angle β . Then, the rock specimen with the bedding angle $\beta = 0^\circ$ is defined as the intact rock. The uniaxial compressive strength σ_c and material parameter m_i of intact rocks are obtained from data fitting using Hoek-Brown failure criterion. The corresponding GSI values under different bedding angles are calculated. The relationship between GSI and bedding angle β is fitted using Gaussian function, and a new strength model of anisotropic rocks is established based on Hoek-Brown failure criterion. Finally, the proposed model is verified by comparing the peak strength obtained from the GSI-softening model. It is found that the modified Hoek-Brown failure criterion is suitable for predicting the strength of anisotropic rocks with different bedding angles and under various confining pressure. The physical meaning of the new parameters in the model is also discussed.

Keywords: anisotropic rock; bedding angle; Hoek-Brown failure criterion; GSI-strength model

1 Introduction

Anisotropic rock masses are commonly encountered in tunnel engineering. During tunnel excavation, various types of rock layers, especially the weak planes with special structural features, can lead to deformation and instability of rock masses, ultimately resulting in the failure of underground constructions^[1]. Therefore, studying the mechanical properties of anisotropic rocks is an important prerequisite for ensuring the safety of underground engineering projects. Among numerous yield criteria for rock masses, the Hoek-Brown (H-B) yield criterion has been widely applied to solving practical engineering problems due to its broad applicability. However, the traditional H-B criterion cannot effectively address the issue of rock strength in anisotropic rock masses such as shale^[2–3]. Therefore, it is necessary to modify the H-B criterion to analyze the strength characteristics of anisotropic rock masses.

To promote the application of the H-B criterion in anisotropic rock masses, numerous scholars have conducted exploration and research^[4–9]. For instance, Yao et al.^[10] established a numerical model for simulating the failure process of brittle anisotropic rocks based on the H-B criterion, using a method that generates an anisotropic rock using Voronoi grids and assigns anisotropic micro-parameters using a function. Shi et al.^[11] found that the single weak plane criterion overestimated the strength of anisotropic rocks at certain orientations, and the modified H-B criterion showed better agreement with experimental

data of anisotropic rocks. Cheng et al.^[12] demonstrated that the H-B strength criterion is a good predictor of the nonlinear increase in peak strength of composite rock specimens under different confining pressures through experimental studies on the anisotropy of anisotropic rock masses under triaxial compression. Li et al.^[13] proposed a rock anisotropy H-B criterion that considers the effect of critical confining pressure, which can be used for calculating the strength of anisotropic rocks.

The above-mentioned studies did not take into account the effect of bedding angle on the strength and anisotropy of rocks. However, anisotropic rocks undergo geological processes that result in different orientations of bedding planes, leading to mechanical anisotropy. Therefore, it is important to consider the effect of bedding angle on the strength anisotropy in anisotropic rocks when applying the H-B criterion. Hoek et al.^[14] introduced modifications to the parameters m and s in the H-B criterion to incorporate the bedding angle β , establishing a yield criterion for anisotropic rock masses. This suggests that it is possible to consider the anisotropy of rock strength by establishing functions for the parameters in the H-B criterion with respect to the bedding angle β . Many researchers^[15–19] have further modified the H-B yield criterion by considering variations in parameters such as σ_c , m , s , and a based on the bedding angle β . These modifications allow for a better simulation of the anisotropic characteristics of rock strength.

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This paper characterizes the strength characteristics of anisotropic rock masses using the geological strength index (GSI) value as a function of the bedding angle β . Firstly, the uniaxial compressive strength σ_c and constant m_i of intact rock specimens with bedding planes perpendicular to the axial direction ($\beta = 0$) are determined based on triaxial test results. It is assumed that the GSI value for specimens in this direction is 100. Secondly, for rock specimens with other bedding angles, the GSI values are calculated using the H-B criterion based on the uniaxial compressive strength σ_c and constant m_i of intact rock. This allows for the determination of the variation of GSI with the bedding angle β of rock masses, and a modified H-B yield criterion is proposed by considering the effect of the bedding angle β on the parameters m , s , and a . Finally, the proposed modified H-B yield criterion is validated using a large amount of experimental data. The results demonstrate that the modified H-B yield criterion effectively characterizes the strength characteristics of anisotropic rock masses.

2 Strength model of anisotropic rocks

2.1 Generalized Hoek-Brown yield criterion

Hoek and Brown^[14] (1980) first proposed the H-B criterion, which estimates the peak strength of rocks by substituting actual rock parameters. The specific expression is as follows:

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} \quad (1)$$

where σ_1 represents the maximum principal stress at rock failure, σ_3 represents the minimum principal stress, σ_c represents the uniaxial compressive strength of the rock, and m and s are semi-empirical parameters that represent the characteristics of rock masses.

In 1988, the H-B criterion was widely applied in engineering practice. Considering the effect of engineering disturbance on the mechanical properties of rocks in slope engineering, the concepts of disturbed rocks and undisturbed rocks were introduced. In addition, the relationship between the parameters (m_b and s) in the H-B criterion and the rock mass rating (RMR) was established^[20].

$$\sigma_1 = \sigma_3 + \sqrt{m_b\sigma_c\sigma_3 + s\sigma_c^2} \quad (2)$$

For undisturbed rocks:

$$m_b / m_i = e^{(RMR_{76}-100/28)} \quad (3)$$

$$s = e^{(RMR_{76}-100/9)} \quad (4)$$

For disturbed rocks:

$$m_b / m_i = e^{(RMR_{76}-100/14)} \quad (5)$$

$$s = e^{(RMR_{76}-100/6)} \quad (6)$$

where m_i represents the value of m_b for an intact rock block.

In 1992, Hoek et al.^[21] found that the H-B criterion tends to overestimate the tensile strength of rock masses when applied. To address this issue, they made modifications to the H-B criterion by introducing parameter a to reduce the tensile strength of rock masses towards zero. The specific expression for this modification is as follows:

$$\sigma_1 = \sigma_3 + \sigma_c(m_b\sigma_3 / \sigma_c)^a \quad (7)$$

Subsequent engineering applications have demonstrated that previously mentioned H-B criterion to high-quality rocks can result in overly conservative estimations. In light of this, a revision was made to the above results in 1995, introducing the generalized H-B strength criterion^[22–23].

$$\sigma_1 = \sigma_3 + \sigma_c(m_b\sigma_3 / \sigma_c + s)^a \quad (8)$$

For $GSI > 25$

$$m_b / m_i = e^{(GSI-100/28)} \quad (9)$$

$$s = e^{(GSI-100/9)} \quad (10)$$

$$a = 0.5 \quad (11)$$

For $GSI < 25$

$$s = 0 \quad (12)$$

$$a = 0.65 - GSI / 200 \quad (13)$$

In 2002, Hoek introduced the disturbance factor D to the H-B criterion to describe construction disturbances^[24].

$$m_b = m_i e^{\left(\frac{GSI-100}{28-14D}\right)} \quad (14)$$

$$s = e^{\left(\frac{GSI-100}{9-3D}\right)} \quad (15)$$

$$a = \frac{1}{2} + \frac{1}{6} \left(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}} \right) \quad (16)$$

In the H-B criterion expression, the parameters σ_c , m_i , GSI and D are relatively independent, while the parameters m_b , s , and a are calculated based on these four parameters. Among the parameters σ_c , m_i , GSI and D , the uniaxial compressive strength σ_c does not accurately reflect the unique layering structural characteristics of anisotropic rock masses. The parameter m_i also lacks a clear physical meaning. The disturbance coefficient D is a weakening factor that considers artificial or natural disturbances in rock masses, and therefore cannot reflect the specific layering characteristics of anisotropic rock masses. Moreover, the GSI system does not consider the effect of bedding angle β on rock mass strength. Therefore, these empirical models cannot represent the anisotropic strength of rock masses caused by bedding orientation. As an important

parameter in the H-B criterion, GSI realizes the quantitative calculation of rock mass parameters. Compared to other classification methods, it is the only method that can directly determine the mechanical parameters of rock masses. Furthermore, GSI can reflect the structural characteristics of engineering rock masses and can macroscopically describe the layering structural characteristics of rock samples. Considering the limitations mentioned above, it seems feasible to modify the H-B yield criterion by incorporating the variation of GSI with the bedding angle β to quantify the parameters, instead of the previous consideration of parameters σ_c , m , s , and a with the bedding angle β .

2.2 Geological strength index (GSI)

Hoek believed that the main basis for evaluating rock mass quality is the structural characteristics and structural plane state of the rock mass. In 1994, he proposed the geological strength index GSI^[22]. Since then, the GSI system has been revised and improved several times^[23,25], leading to significant developments in its engineering applicability and quantitative scoring. It has been widely applied in the fields of rock and geological engineering.

The GSI is a quantitative index used to assess the strength of rock masses. Although it is not theoretically a rock mass classification method, as an important parameter of the Hoek-Brown strength criterion, it enables the quantitative calculation of rock mass strength parameters. It is also the only method that directly determines the mechanical parameters of rock masses compared to other classification methods. However, it also has certain limitations in its application. In practical engineering, the accurate determination of GSI values heavily relies on the user's experience.

To reduce the subjectivity in determining GSI values, many scholars^[26–31] have established relationships with GSI from different perspectives, providing a basis for its quantitative determination.

Based on indicators that characterize rock mass structures^[32], such as the rock quality designation (RQD), volumetric joint count (J_v), P-wave velocity (V_p), P-wave velocity ratio (B_v), intactness coefficient (K_v), and fracture spacing (d), the structural characteristics and state of structural planes of rock masses can be quantitatively characterized. In addition, GSI can be determined based on the state of structural planes and rock mass structures. Wang et al.^[33] found that GSI can be characterized by K_v , where the intactness coefficient of rock masses is defined as the square of the ratio between the longitudinal wave velocity of the rock mass and that of the rock blocks. Furthermore, previous studies^[34–36] have indicated that the bedding angle of shale affects the longitudinal wave velocity. Therefore, it can be concluded that the K_v of anisotropic rocks is affected by the bedding angle. From the above discussion, it can be seen that correcting the GSI- β curve to modify the Hoek-Brown yield criterion is feasible.

2.3 Strength model

Based on the research ideas mentioned above, we collected triaxial test data of anisotropic rocks with different bedding angles under various confining pressures. The collected rock samples include Qinghai tawny layered sandstone^[37], Sichuan Renshou layered yellow sandstone^[38], typical layered quartz sandstone from the Three Gorges reservoir area^[39], shale from Daegu region in South Korea^[40], and layered slate from a slate quarry^[41]. All rock samples collected were cylinders with a height-to-diameter ratio of 2:1. Triaxial test results of typical rocks with different bedding angles are shown in Fig. 1.

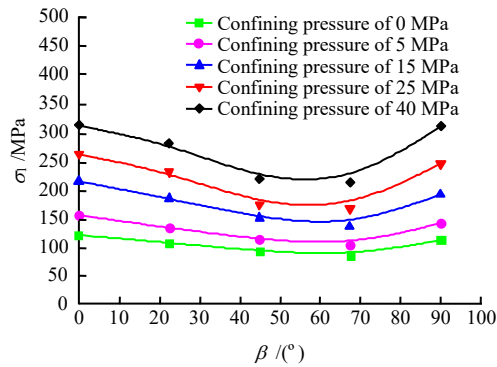
It can be observed from Fig. 1 that the peak strength of different anisotropic rocks under different confining pressures tends to decrease first and then increase as the bedding angle increases. This is because as the bedding angle increases, the tangential stress along the bedding planes due to axial loading stress increases. The strength and friction of the bedding planes become the dominant factors affecting the rock strength, resulting in a decrease in the peak strength of the specimens. When the bedding angle exceeds 60°, although the tangential stress continues to increase, the effects of the strength and friction of the bedding planes on the peak strength of rock specimens weakens. Instead, the strength of the rock matrix between the bedding planes becomes the dominant factor. As a result, the peak strength increases with the bedding angle. In the H-B criterion, σ_c is defined as the uniaxial compressive strength of intact rock specimens. For anisotropic rocks with different bedding angles, the uniaxial compressive strength determined by the triaxial test varies. However, when the bedding angle of the specimen is parallel to the end surface, the axial loading stress is perpendicular to the bedding planes, and the rock strength is mainly determined by its own strength, with the least effect from the bedding plane strength. Therefore, we assume that the rock sample with the bedding plane perpendicular to the axial direction is defined as the intact rock sample, i.e., $\beta = 0$. We also assume that the GSI of the rock specimen along this direction is 100, indicating an undisturbed state, i.e., $D = 0$. Based on this assumption, we can calculate the uniaxial compressive strength σ_c and the constant m_i . For rocks with other bedding angles, we keep the uniaxial compressive strength σ_c and the constant m_i unchanged, and calculate the GSI based on the triaxial test results of specimens with different bedding angles. This allows us to obtain the variation of GSI with the bedding angle of anisotropic rock masses.

2.3.1 Determination of the uniaxial compressive strength σ_c and material parameter m_i of rocks

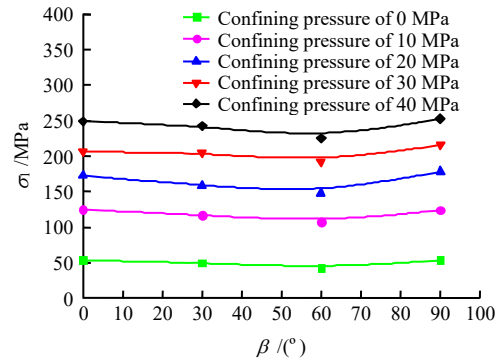
The tawny layered sandstone in Qinghai Province is presented as an example (Fig. 1(a)), its uniaxial compressive strength and material parameter of intact rocks can be determined through curve fitting, as illustrated in Fig. 2, where it is clear that the peak strength σ_1 of the sandstone increases with the increasing confining pressure σ_3 . With

a confining pressure increase of 40 MPa, the peak strength of the sandstone has increased by nearly 200 MPa, indicating

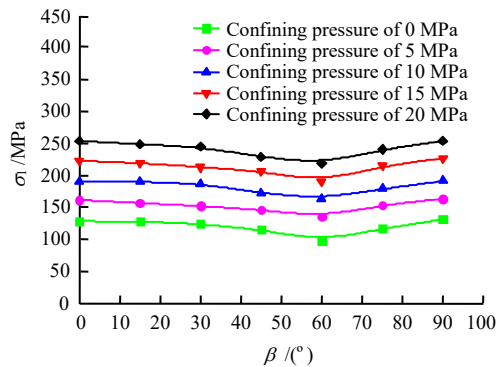
a significant effect of confining pressure on the mechanical properties of rocks.



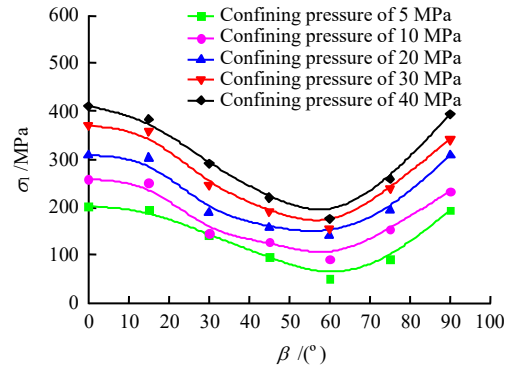
(a) Qinghai tawny layered sandstone



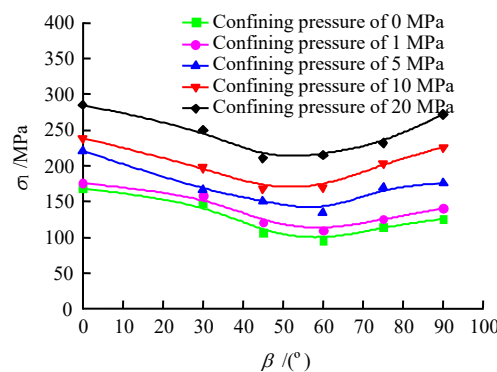
(b) Sichuan Renshou layered yellow sandstone



(c) Typical layered quartz sandstone from the Three Gorges reservoir area



(d) Shale from Daegu region in South Korea



(e) Layered slate from a slate quarry

Fig. 1 Triaxial test results of typical rocks with different bedding angles

According to the fitting curve, the uniaxial compressive strength of the sandstone is determined to be $\sigma_c = 121.6 \pm 4.1$ MPa, and the material parameter is $m_i = 12.5 \pm 0.9$. The correlation coefficient R^2 is 0.995 58.

2.3.2 Determination of GSI for different bedding angles

By substituting the uniaxial compressive strength σ_c and the constant m_i of intact rocks into the generalized H-B model, we can calculate the GSI values of rocks with different bedding angles. Fig. 3 shows the experimental data of the Qinghai tawny sandstone. The GSI decreases

initially and then increases with the increasing bedding angle, exhibiting a U-shaped trend. This trend is consistent with the variation of peak strength of anisotropic rocks.

Peng et al.^[42] proposed using the GSI weakening model to determine the GSI value of thermally damaged rock samples. The GSI weakening model can quantify the GSI value of rocks with different bedding angles under zero confining pressure based on the uniaxial compressive strength.

Taking inspiration from this approach, the present

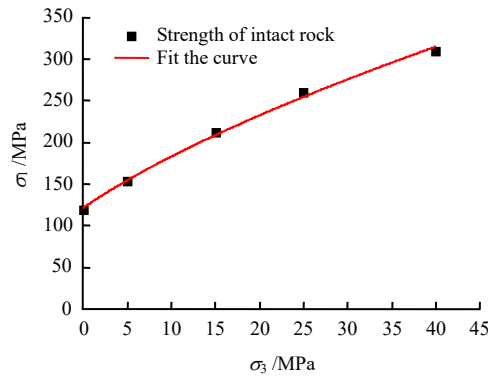


Fig. 2 Fitting curves of Qinghai tawny sandstone

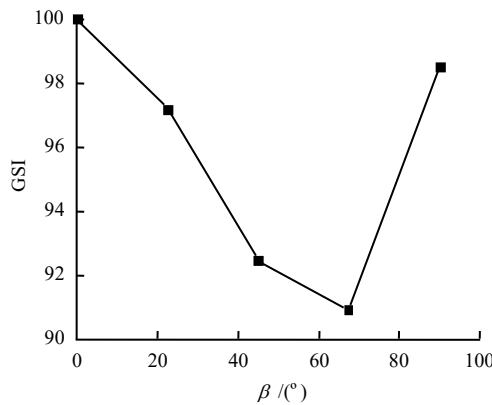


Fig. 3 Variation of GSI with bedding angle of Qinghai tawny sandstone

study employs the GSI weakening method to determine the GSI values of anisotropic rocks with different bedding angles. Using the Qinghai tawny sandstone, σ_c is the experimental peak strength at zero confining pressure ($\sigma_3 = 0$) and zero bedding angle ($\beta = 0^\circ$). By substituting the peak strength of rocks with different bedding angles under zero confining pressure ($\sigma_3 = 0$) into Eq. (17), the corresponding GSI values were calculated, revealing the variation of GSI with respect to the bedding angle in the GSI weakening model.

$$\sigma_3 = 0, \quad \sigma_1 = \sigma_c s^a \quad (17)$$

The results are presented in Fig. 4. It can be observed that the value of the weakened GSI model is greater than the calculated GSI overall. There is a significant discrepancy in the GSI values determined using different methods for the same rock with the same bedding angle.

Figure 5 compares the peak strength under different confining pressures obtained by substituting the calculated GSI and weakened GSI into the H-B criterion with the experimental results, and the data points represent the test values. From Fig. 5, it can be observed that the peak strength obtained by substituting the calculated GSI into the H-B criterion generally align more closely with the test values compared to that obtained by using the GSI weakened model. When the confining pressure is 0 MPa, the predictions of the weakened model are closer to the

test values. This may be due to the fact that the uniaxial compressive strength σ_c used in this study is obtained through fitting and there is an error between the fitted value and the test value. However, as the confining pressure increases, the proposed model shows better predictive performance. This is because the GSI determined by the weakened model only matches with the strength under zero confining pressure, while the GSI calculated by the H-B strength model in this study takes into account the matching with strength under different confining pressures. Therefore, considering only the GSI that matches the strength under a single confining pressure may overestimate the GSI.

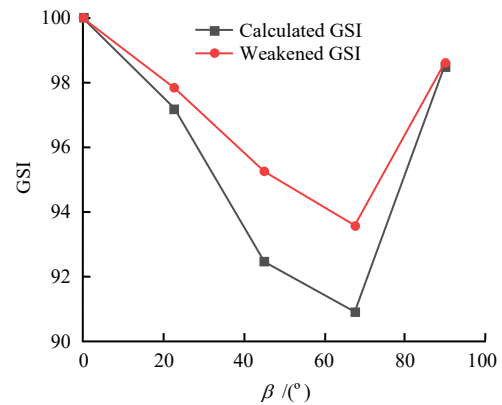


Fig. 4 Variations of the GSI using the weakened model as well as the calculated GSI in response to the bedding angle of Qinghai tawny sandstone

2.3.3 Relationship between GSI and bedding dip β

According to Fig. 3, the GSI of sandstone shows a decreasing trend initially, followed by an increasing trend, as the bedding angle increases, exhibiting a distinct U-shape. The GaussAmp graph also follows a similar pattern of first decreasing and then increasing. Therefore, based on the trend of GSI with respect to the bedding angle, a Gaussian model is adopted to represent the relationship between GSI and bedding angle. Additionally, the Gaussian model is modified based on the meaning of input parameters and the definition of Gaussian function.

$$y = \text{GSI}_0 + \lambda e^{-\frac{(\beta - \beta_m)^2}{2\omega^2}} \quad (18)$$

where GSI_0 is the GSI of intact rock, λ is the amplitude of GSI changing with bedding angle, β_m is the bedding angle corresponding to the minimum GSI, and ω is the curvature of the curve of GSI changing with the bedding angle.

The GSI fitting results of Qinghai tawny sandstone with different bedding angles using the proposed model are shown in Fig. 6.

From the above results, it is evident that the fitting effect of the Qinghai tawny sandstone is good. To further improve the fitting accuracy, the parameter assignment fitting is performed in combination with the original function

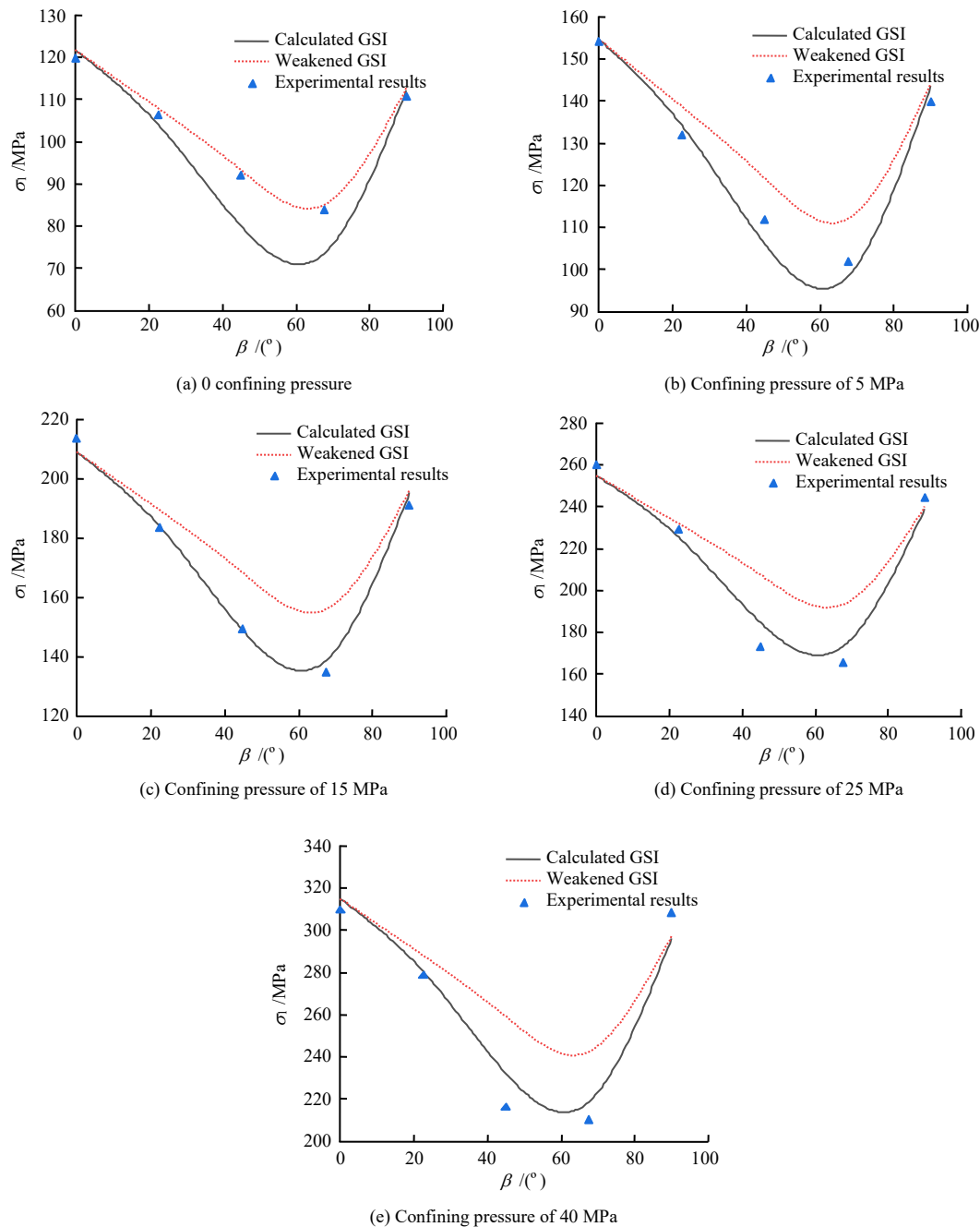


Fig. 5 Comparison of peak strength between substituting calculated GSI and weakened GSI to H-B criterion for Qinghai tawny sandstone

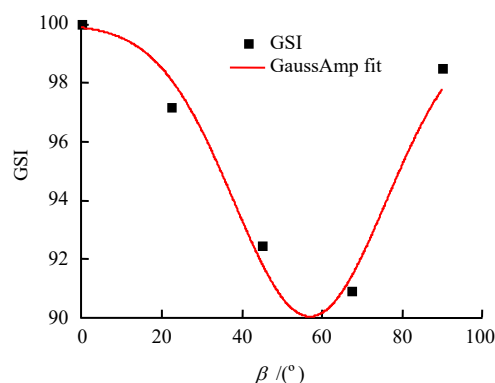


Fig. 6 Fitting curve of GSI for different bedding angles of Qinghai tawny sandstone based on Gaussian function

parameter meaning and the fitting results. Finally, the relationship between the GSI of the Qinghai tawny sandstone and the bedding angle can be expressed as $GSI = 100 + (-10) \exp[-0.5 \times ((\beta - 57) / 19)^2]$. The Gaussian function can be preliminarily determined as a model to characterize the GSI with respect to the bedding angle, and further validation will be conducted using the collected data.

3 Verification of the modified Hoek-Brown yield criterion

All the test results of rocks are summarized in Table 1.

Table 1 includes the determined uniaxial compressive strength σ_c of rocks and the material parameters m_i , as

well as the parameter values obtained from Gaussian function fitting. Correlation coefficient R^2 indicates that the fitting effect is good. Equation (18) can be rewritten as follows:

$$GSI = 100 + \lambda e^{\frac{(\beta - \beta_m)^2}{2\omega^2}} \quad (19)$$

Based on Eq. (19), the GSI of rocks with different bedding angles can be quantified by replacing GSI in Eqs. (9)–(16) with Eq. (19). The H-B criterion, which represents GSI with bedding angles, is the modified version that accounts for the combined effects of bedding angle and confining pressure. Based on the fitting parameters in Table 1 and following the aforementioned method, a modified H-B criterion applicable to other anisotropic rocks

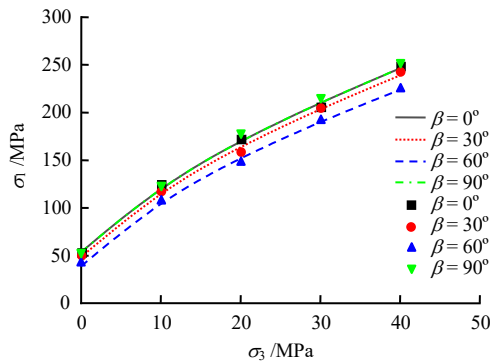
with varying bedding angles and confining pressures can be obtained. The detailed process is not elaborated here. We present the modified H-B criterion based on the collected data and incorporate it into the analysis, resulting in peak strength that closely matches the experimental results.

The results of the remaining four typical anisotropic rocks^[38–41] are shown in Fig. 7.

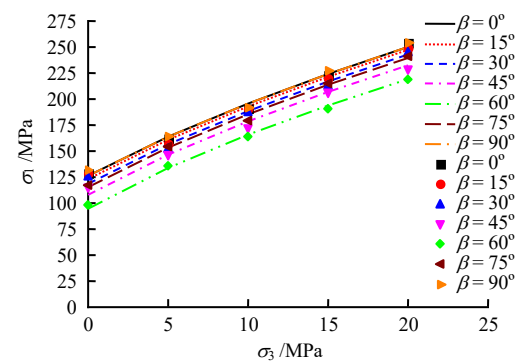
The curves in Fig. 7 represents the calculated results using the modified H-B criterion, while the points represent the collected experimental data. It can be observed that the curves closely match the experimental data. This indicates that the modified H-B criterion is capable of accurately predicting the peak strength of anisotropic rocks.

Table 1 Test results of rocks in previous literature

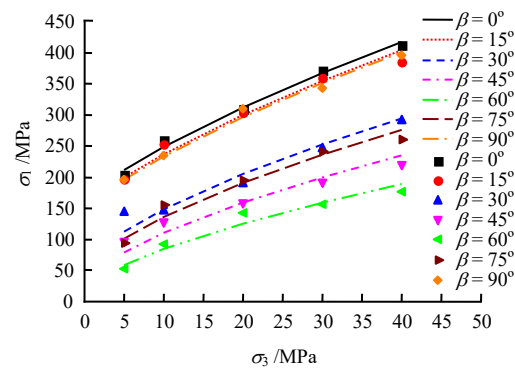
Rock	σ_c /MPa	m_i	R^2	GSI_0	λ	β_m	ω	R^2	Data sources
Qinghai tawny layered sandstone	121.6±4.1	12.5±0.9	0.995 58	100	-10.0±-0.6	57	19	0.967 41	Ref.[37]
Sichuan Renshou layered yellow sandstone	53.6±3.2	18.5±1.4	0.998 04	100	-48.4±6.6	46.6±0.1	6.5±0.2	1.000 00	Ref.[38]
Typical layered quartz sandstone from the Three Gorges reservoir area	126.3±3.4	14.7±1.0	0.993 46	100	-5.2±0.5	57.3±1.3	12.1±1.2	0.941 41	Ref.[39]
Shale from Daegu region in South Korea	168.4±10.8	16.8±2.1	0.988 34	100	-44.4±3.5	55.7±1.5	16.3±1.5	0.948 97	Ref.[40]
Layered slate from a slate quarry	173.3±6.1	12.4±1.8	0.963 82	100	-10.3±0.6	56.8±1.3	20.5±1.5	0.966 95	Ref.[41]



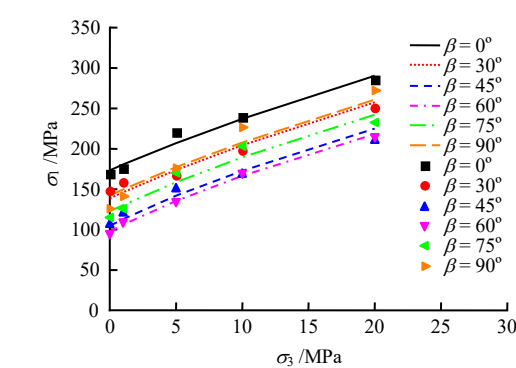
(a) Sichuan Renshou layered yellow sandstone



(b) Typical layered quartz sandstone from the Three Gorges reservoir area



(c) Shale from Daegu region in South Korea



(d) Layered slate from a slate quarry

Fig. 7 Comparison of peak strength calculated by modified H-B criterion and previous experimental results

4 Sensitivity analysis of parameters

Based on the definition of Gaussian function parameters and experimental data, Qinghai tawny sandstone in Table 1 is taken as an example. GSI_0 should be the initial value of the function. We define GSI at $\beta = 0$ as 100, and the fitting result is very close to 100. By assigning GSI_0 a value of 100 and performing Gaussian fitting again, the result is significantly better than direct fitting. Therefore, GSI_0 can be defined as the GSI value (intact rock) at $\beta = 0$, i.e., $GSI_0 = 100$.

Regarding the meaning of λ , the original function defines it as the difference between the peak value and the initial value. The fitting result is the difference between the lowest point and the initial value of 100. This is consistent with the meaning of the original function parameters. The value of λ can be defined as the minimum GSI minus 100, i.e., the negative range, which is also the error of the fitting function, $-\lambda$.

Based on multiple experimental verifications (as shown in Fig. 8), it has been found that assigning different values to ω ($\omega = 10, \omega = 13, \omega = 16, \omega = 19$) within a reasonable range can change the values of GSI_0 and λ . However, the value of β_m fluctuates within a very small range, indicating that the angle corresponding to the minimum GSI value for each type of anisotropic rock remains constant. In other words, β_m can be defined as the angle at which GSI is at its minimum. Based on the data from uniaxial and triaxial tests, β_m can also be defined as the angle at which the strength of the rock is at its minimum, indicating that the rock is most susceptible to failure when the bedding angle is at β_m , and its compressive capacity is at its weakest. According to the collected data, the fitted values of β_m for sandstone, shale, and slate are concentrated in the ranges of 57° – 60° , 57° – 58° , and 56° – 58° , respectively.

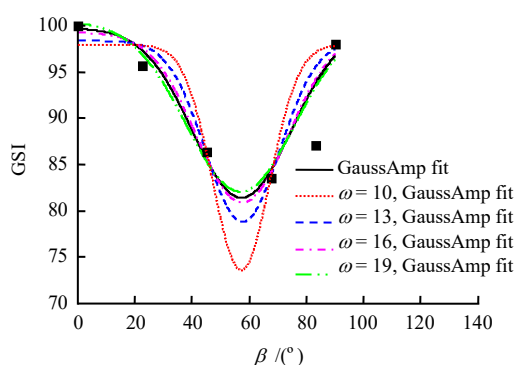


Fig. 8 Variation curves of β with GSI for different values of ω

The effect of ω on the λ is the most significant, followed by GSI_0 , and on the β_m is the least. According to the definition above, for a specific rock with $GSI_0 = 100$, β_m does not change with the variation of other parameters. As shown in Fig. 9, the change in ω ($\omega = 10, \omega = 13,$

$\omega = 16, \omega = 19$) will alter the opening size of the fitting function. Within a certain range, a larger ω will result in a larger opening of the function and a higher value of λ , which corresponds to the lowest GSI (at β_m). The value of ω directly affects the magnitude of GSI, thus ω can be defined as a parameter for anisotropic rock materials.

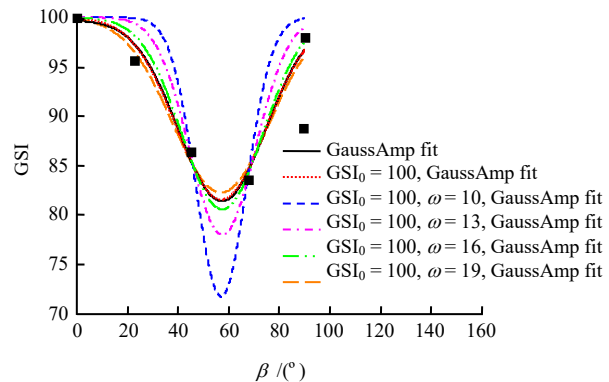


Fig. 9 Variation curves of GSI with β for different values of ω when $GSI_0 = 100$

5 Conclusions

(1) Considering the GSI variation of anisotropic rocks with the bedding angle, a modified Hoek-Brown yield criterion for anisotropic rocks is proposed. The modified H-B model can predict the peak strength of anisotropic rocks with different bedding angles under confining pressures, exhibiting good applicability in characterizing the strength characteristics of anisotropic rocks.

(2) Using the model proposed in this study, it is found that for a given bedding angle of anisotropic rocks, the peak strength increases with increasing confining pressure, while it initially decreases and then increases with increasing bedding angle under a given confining pressure, showing a U-shaped trend.

(3) Layered sandstone exhibits the lowest peak strength and weakest compressive ability within the bedding angle of 57° to 60° . Shale falls within the range of 57° to 58° , while slate falls within the range of 56° to 58° .

(4) This study proposes a model for the GSI variation with the bedding angle. Further research is needed to determine the specific parameters (λ and ω) for anisotropic rocks, which will be the focus of future studies.

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